Unit #9 Differential Equations

Differential equation: A differential equation is an equation that involves the derivatives of an unknown function (Dependent variable) of one or more variable (independent variable).

Note ① If the unknown function depends on only one variable, then the derivative is an ordinary derivative, and the equation called ordinary differential equation.

i) $\frac{dy}{dx} = xy$ $\quad y(x) = ?$

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ii) $\frac{dy}{dx} = x+y$ $\quad y(x) = ?$

iii) $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = 3$ $\quad y(x) = ?$

2) If the unknown function depends on more than one variable, then the derivative is Partial derivative, and the equation is called Partial differential equation.

Order of Differential equation: The order of a differential equation is the order of the highest-order derivative occurring in the equation.

Degree of a Differential equation: The degree of a differential equation is the Power of the highest-order derivative occurring in the equation.

Solution of a differential equation

A solution of an ordinary differential is any relation $y = f(x)$ or $f(x,y)$ which when substituted in the differential equation, it satisfies the equation.

General and Particular solution: The solution of a differential equation when depends on a single arbitrary constant quantity, is then called the general solution of the first order differential equation. If we give particular value to a single arbitrary constant quantity, then the solution to obtain is called the particular solution or specific solution or exact solution or actual solution of first order differential equation.

Note: General solution represents a family of curve & particular solution represents a particular curve chosen from a family of curves.

Linear Differential equation:

A differential equation is said to be Linear if

1) the dependent variable and its derivative occur to the first power only.
2) there are no product involving the dependent variable and its derivative.
3) there should be no non-linear functions of

the dependent variable, such as sine, log, exponential etc.

If one of the above condition is fail then Differential equation is called non-linear.

Formation of Differential equation

For the equation of a family of curves, we can form the corresponding differential equation by differentiating and then eliminating the arbitrary constants.

EXERCISE 9.1

① Find the order and degree of each of the following differential equations:

a) $\frac{dy}{dx} = x^2 + y$

Order = 1, because it is first order differential eq:

& Degree = 1, because Power of first order derivative is 1.

It is Linear differential equation, beause y & $\frac{dy}{dx}$ occur to the first Power, no term Contains $y.\frac{dy}{dx}$ & there is no non-linear function of y.

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b) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 11y = 3x$

Order = 1 ∵ it is 2nd. Order differential equation.

Degree = 1 ∵ Power of the 2nd Order derivative is 1

It is linear differential equation, because,

y & $\frac{dy}{dx}$ occur to the first Power, no term Contains

$y \cdot \frac{dy}{dx}$ & there is no non-linear function of y..

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c) $\frac{d^3y}{dx^3} + 2\left(\frac{dy}{dx}\right)^3 - y = 0$

Order = 3 ∵ it is 3rd order differential equation.

Degree = 1 ∵ Power of the 3rd order derivative is 1.

It is non-linear differential equation, because

Power of $\frac{dy}{dx}$ is 3.

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2) In each case, show that the indicated function is a solution of the differential equation.

a) $\boxed{y = e^x + e^{2x}}$, $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0 \longrightarrow ①$

$$\boxed{\frac{d}{dx}e^u = e^u \frac{du}{dx}}$$

Diff: w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}(e^x + e^{2x})$$

$$= e^x + e^{2x}(2)$$

$$\boxed{\frac{dy}{dx} = e^x + 2e^{2x}}$$

Diff: w.r.t. x

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[e^x + 2e^{2x}\right]$$

$$= e^x + 2\{e^{2x} \cdot 2 \cdot 1\}$$

$$\boxed{\frac{d^2y}{dx^2} = e^x + 4e^{2x}}$$

put value of y, $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ in ①

① ⇒ $(e^x + 4e^{2x}) - 3(e^x + 2e^{2x}) + 2(e^x + e^{2x}) = 0$

$$e^x + 4e^{2x} - 3e^x - 6e^{2x} + 2e^x + 2e^{2x} = 0$$

$$\cancel{3e^x} + \cancel{6e^{2x}} - \cancel{3e^x} - \cancel{6e^{2x}} = 0$$

$0 = 0$ (true)

Hence $y = e^x + e^{2x}$ is solution of D.E $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

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b) $\boxed{y = x - x\ln x}$, $x\frac{dy}{dx} + x - y = 0$ —— ①

Diff w.r.t x

$$\frac{dy}{dx} = 1 - \{x\frac{d}{dx}\ln x + \ln x \cdot \frac{d}{dx}x\}$$

$$= 1 - \{x \cdot \frac{1}{x} + \ln x \cdot 1\}$$

$$= \cancel{1} - \cancel{1} - \ln x$$

$$\boxed{\frac{dy}{dx} = -\ln x}$$

put value of y & $\frac{dy}{dx}$ in D.E ①

① ⇒ $x(-\ln x) + x - \{x - x\ln x\} = 0$

$$-\cancel{x\ln x} + \cancel{x} - \cancel{x} + \cancel{x\ln x} = 0$$

$0 = 0$ (true)

Hence $y = x - x\ln x$ is solution of D.E $x\frac{dy}{dx} + x - y = 0$

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c) $y = (x+c)e^{-x}$, $\frac{dy}{dx} + y = e^{-x}$ —— ①

Diff w.r.t x

$\frac{dy}{dx} = (x+c)\frac{d}{dx}e^{-x} + e^{-x}\frac{d}{dx}(x+c)$ (Product Rule)

$$\frac{dy}{dx} = (x+c)\left[e^{-x}(-1)\right] + e^{-x}(1+0)$$

$$\boxed{\frac{dy}{dx} = -(x+c)e^{-x} + e^{-x}}$$

put value of y & $\frac{dy}{dx}$ in D.E ①

$$① \Rightarrow \left[-(x+c)e^{-x} + e^{-x}\right] + (x+c)e^{-x} = e^{-x}$$

$$\cancel{-(x+c)e^{-x}} + e^{-x} + \cancel{(x+c)e^{-x}} = e^{-x}$$

$$e^{-x} = e^{-x} \quad (\text{true})$$

Hence $y = (x+c)e^{-x}$ is solution of D.E $\frac{dy}{dx} + y = e^{-x}$

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3) For each of the following equations, determine whether or not it becomes linear when divided by dx or dy.

a) $(x+y)dy = (x-y)dx$

÷ing by dx

$(x+y)\frac{dy}{dx} = x-y$ or $x\frac{dy}{dx} + y\frac{dy}{dx} = x-y$

It is non-linear because it contain Product $y\frac{dy}{dx}$

__2nd Part__ $\because (x+y)dy = (x-y)dx$

÷ing by dy

$$x+y = (x-y)\frac{dx}{dy}$$

$$x+y = x\frac{dx}{dy} - y\frac{dx}{dy}$$ (Here x is dependent variable)

It is non-linear Differential equation because it contain $x.\frac{dx}{dy}$

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b) $a\,dy + by\sin x\,dx = 0$

÷ing by dx

$a\frac{dy}{dx} + by\sin x = 0$ (Here y is dependent variable)

It is linear D.E because y & $\frac{dy}{dx}$ occur to the first Power, no term contains $y\frac{dy}{dx}$ & there is no non-linear function of y.

2nd Part ∵ $a\,dy + by\sin x\,dx = 0$

÷ing by dy

$a + by\sin x\frac{dx}{dy} = 0$ (Here x is dependent variable

It is non-linear differential equation, because x is dependent variable & D.E contains non-linear function $\sin x$.

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c) $3y\,dx + 2x\,dy = 0$

÷ing by dx

$3y + 2x\frac{dy}{dx} = 0$ (Here y is dependent variable

It is linear D.E because y & $\frac{dy}{dx}$ occur to the first Power, no term Contain $y\frac{dy}{dx}$ & there is no non-linear function of y.

2nd Part ∵ $3y\,dx + 2x\,dy = 0$

÷ing by dy

$3y\frac{dx}{dy} + 2x = 0$ (Here x is dependent variable

It is linear differential equation, because x & $\frac{dx}{dy}$ occur to the first power, no term contains $x\frac{dx}{dy}$ & there is no non-linear function of x.

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d) $e^x dy + xy^{1/3} dx = 0$

÷ing by dx

$e^x \frac{dy}{dx} + xy^{1/3} = 0$ (Here y is dependent variable)

It is non-linear differential equation, because it contain $y^{1/3}$

2nd Part ∵ $e^x dy + xy^{1/3} dx = 0$

÷ing by dy

$e^x + xy^{1/3} \frac{dx}{dy} = 0$ (x is dependent variable)

It is non-linear differential equation because it contain Product $x \frac{dx}{dy}$ and also non-linear function e^x

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4) In each case, use the initial condition and the general solution of the differential equation to determine a Particular solution:

a) $xy = c$ — ①

∵ $y(2) = 1$

so put $x=2, y=1$ in ①

$2(1) = C \Rightarrow C = 2$ put in ①

①⇒ $xy = 2$

which is the req: Particular solution

b) $y = x - x\ln x + c$ — ①

∵ $y(1) = 2$

so put $x=1$, $y=2$ in ①

$2 = 1 - 1.\ln 1 + C$

$2 = 1 - 0 + C$

$2 - 1 = C \Rightarrow \boxed{C=1}$ put in ①

①⇒ $y = x - x\ln x + 1$

which is the required Particular solution

c) $\sin(xy) + y = c \longrightarrow$ ①

∵ $y(\pi/4) = 1$

so put $y=1$ & $x = \pi/4$ in ①

①⇒ $\sin(1.\pi/4) + 1 = C$

$\frac{1}{\sqrt{2}} + 1 = C \Rightarrow C = 1 + \frac{1}{\sqrt{2}}$ put in ①

$\sin(xy) + y = 1 + \frac{1}{\sqrt{2}}$

or $\sin(xy) + y = 1.707$

which is the required Particular solution

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d) $\frac{y^2}{x} = \frac{x^2}{2} + c$ —①

$\because$ $y(1) = 1 \Rightarrow y = 1, x = 1$ put in ①

$\frac{1}{1} = \frac{1}{2} + c$

$1 - \frac{1}{2} = c \Rightarrow c = \frac{1}{2}$ put in ①

①$\Rightarrow$ $\frac{y^2}{x} = \frac{x^2}{2} + \frac{1}{2}$

which is the req. Particular solution

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5) Solve the following initial value problem

a) $\frac{dy}{dx} = \cos x$ $\boxed{y(0)=1}$

integrate w.r.t. x

$\int \left(\frac{dy}{dx}\right) dx = \int \cos x dx$

$y = \sin x + c$ —①

$\because$ $y(0) = 1$

So put $y=1$ & $x=0$ in ①

$1 = \sin 0 + c$

$1 = 0 + c \Rightarrow \boxed{c=1}$

put in ①

①$\Rightarrow$ $y = \sin x + 1$

which is the Req: Solution

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b) $\frac{dy}{dx} = x^2$ $y(0)=1$

integrate w.r.t x

$\int \left(\frac{dy}{dx}\right) dx = \int x^2 dx$

$y = \frac{x^3}{3} + c$ —①

$\because$ $y(0)=1 \Rightarrow y = 1$ & $x = 0$ put in ①

①$\Rightarrow$ $1 = \frac{0}{3} + c$

$1 = 0 + c \Rightarrow \boxed{c=1}$

put in ①

①$\Rightarrow$ $y = \frac{x^3}{3} + 1$ Ans

which is the req. solution

Note.

Seperable D.E :

Differential equation of the form $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ are called sperable D.E, because we can write it as $g(y)dy = f(x)dx$

To find solution just integrate both sides.

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c) $\frac{dy}{dx} = 2xy^2$, $\boxed{y(3)=-1}$

⇒ $dy = 2xy^2dx$

⇒ $\frac{dy}{y^2} = 2x\,dx$

integrate

$\int y^{-2}dy = \int 2x^{1}dx$

$\frac{y^{-1}}{-1} = \cancel{2}\frac{x^2}{\cancel{2}} + C$

$-\frac{1}{y} = x^2 + C$ —— ①

∵ $y(3) = -1 \Rightarrow y=-1$ & $x=3$

put in ①

① ⇒ $-\frac{1}{-1} = 3^2 + C$

$1-9 = C \Rightarrow \boxed{C=-8}$

put in ①

① ⇒ $-\frac{1}{y} = x^2 - 8$

×ing by -1

$\frac{1}{y} = \frac{-x^2+8}{1}$

taking Reciprocal

$y = \frac{1}{-x^2+8}$

$y = \frac{1}{8-x^2}$ Ans

d) $\frac{dy}{dx} + y = y^2$ $y(0) = \frac{1}{2}$

$\frac{dy}{dx} = y^2 - y$

$dy = y(y-1)dx$

$\frac{dy}{y(y-1)} = dx$

integrate

$\int \frac{1}{y(y-1)}dy = dx$

$\int\left(\frac{A}{y} + \frac{B}{y-1}\right)dy = \int dx$ —— ①

where $\frac{1}{y(y-1)} = \frac{A}{y} + \frac{B}{y-1}$

×ing by $y(y-1)$

$1 = A(y-1) + By$ —— ②

put $y=0$ in ②

$1 = A(0-1)+0$

$1 = -A \Rightarrow \boxed{A=-1}$

put $y-1=0 \Rightarrow y=1$ in ②

$1 = 0 + B.1$

$\Rightarrow \boxed{B=1}$

put $A=-1$ & $B=1$ in ①

$\int\left(\frac{-1}{y} + \frac{1}{y-1}\right)dy = \int dx$

$-\ln y + \ln(y-1) = x + C_1$

⇒ $\ln(y-1) - \ln y = x + C_1$

⇒ $\ln\frac{y-1}{y} = x + C_1$ $\left(\ln a - \ln b = \ln\frac{a}{b}\right)$

⇓

$\ln \frac{y-1}{y} = x + c_1$

taking Antilog

$\frac{y-1}{y} = e^{x+c_1}$ $\left(\because \ln x = y \Rightarrow x = e^y \right.$

$\frac{y-1}{y} = e^x e^{c_1}$

$e^{c_1} = c$

$\frac{y-1}{y} = e^x \cdot c$

$y-1 = ye^x c$ —①

$\because$ $y(0) = \frac{1}{2} \Rightarrow x=0$ & $y = \frac{1}{2}$ put in ①

①⇒ $\frac{1}{2} - 1 = \frac{1}{2} e^0 c$

$-\cancel{\frac{1}{2}} = \cancel{\frac{1}{2}} \cdot 1 \cdot c \Rightarrow \boxed{c = -1}$ put in ①

①⇒ $y - 1 = ye^x(-1)$

$y - 1 = -ye^x$

$y + ye^x = 1$

$y(1+e^x) = 1 \Rightarrow y = \frac{1}{1+e^x}$ Ans

e) $y\frac{dy}{dx} + xy^2 - x = 0$ $y(0) = -1$

$y\frac{dy}{dx} = x - xy^2$

$y\,dy = x(1-y^2)\,dx$

$\Rightarrow \frac{y\,dy}{1-y^2} = x\,dx$

integrate

$\int \frac{y\,dy}{1-y^2} = \int x\,dx$

$\int \frac{1}{1-u} du = \frac{x^2}{2}$

put $y^2 = u$

Diff: w.r.t. y

$2y\frac{dy}{dy} = \frac{du}{dy}$

$2y \cdot 1 = \frac{du}{dy} \Rightarrow \boxed{y\,dy = \frac{du}{2}}$

$\frac{\ln(1-u)}{-1} = \frac{x^2}{2} + c_1$

$-\ln(1-y^2) = \frac{x^2}{-2} + c_1$ $\left(\int \frac{1}{ax+b} dx = \frac{\ln(ax+b)}{a} + c \right.$

xing by -1

$\ln(1-y^2) = -\frac{x^2}{2} - c_1$ $\left(\ln x = y \Rightarrow x = e^y \right.$

$\Rightarrow 1-y^2 = e^{-x^2/2 - c_1}$

$1-y^2 = e^{-x^2/2} e^{-c_1}$

$1-y^2 = c e^{-x^2/2}$ ——— ① $\left(e^{-c_1} = c \right.$

$\because y(0) = -1 \Rightarrow y = -1, x = 0$ put in ①

$1-(-1)^2 = ce^0$

$1-1 = c \Rightarrow c = 0$ put in ①

①$\Rightarrow 1-y^2 = 0 \cdot e^{-x^2/2}$

$1-y^2 = 0 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1$

or $\boxed{y=-1}$ Ans $\left\{ \because \right.$ given that $y = -1$

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f) $2\frac{dy}{dx} = 4xe^{-x}$ $y(0) = 42$

$\cancel{2}dy = \overset{2}{\cancel{4}}xe^{-x}dx$

integrate

$\int 1\, dy = 2\int \underset{I}{x}\, \underset{II}{e^{-x}} dx$ Int: by Parts

$y = 2\left[x\int e^{-x}dx - \int\left(\frac{d}{dx}x\right)\left(\int e^{-x}dx\right)dx \right]$

$y = 2\left[x\frac{e^{-x}}{-1} - \int 1\left(\frac{e^{-x}}{-1}\right)dx \right]$

$y = 2\left[-xe^{-x} + \int e^{-x}dx \right]$

$y = 2\left[-xe^{-x} + \frac{e^{-x}}{-1} \right] + c$

$y = 2\left[-xe^{-x} - e^{-x} \right] + c$

$$y = -2xe^{-x} - 2e^{-x} + c \longrightarrow ①$$

∵ $y(0) = 42 \Rightarrow y = 42,\ x = 0$ put in ①

$$42 = 0 - 2e^{0} + c$$

$$42 = \underset{\downarrow}{-2(1)} + c \Rightarrow c = 42 + 2 \Rightarrow \boxed{c = 44}$$

put in ①

①⇒ $y = -2xe^{-x} - 2e^{-x} + 44$ Ans

Differential equations reducible to seperable form

If the solution of the differential equation is not possible by separable form, then the given differential equation can be reduced in separable form by substitution. This substitution changes the dependent variable form y to a new variable, say, u and keeps x as the independent variable.

For example $\frac{dy}{dx} = f(ax + by + c)$ put $ax + by + c = u$

Homogeneous Function: A function $f(x, y)$ is homogeneous function of degree n in variables x and y iff $f(\lambda x, \lambda y) = \lambda^{n} f(x, y)$, $n = 1, 2, 3 \ldots$

Homogeneous Differential equation:

The differential equation $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ is called a homogeneous differential equation, if it defines a homogeneous function of degree zero.

The homogeneous differential equation can be reduced to separable form by introducing a new variable: $u = y/x$ or $y = ux$

Orthogonal trajectories of the given family of curves

The two families of curves $F(x, y, c_1)$ and $G(x, y, c_2)$ are Perpendicular at a Point of intersection, if and only if their tangents are Perpendicular at the Point of intersection. If their tangent lines say L_1 & L_2 are Perpendicular, then the Product of their slopes equals -1:

$m_1 m_2 = -1$ m_1 & m_2 are the slopes of the two tangent lines L_1 & L_2

$$\Rightarrow m_1 = -\frac{1}{m_2}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{G} = \frac{-1}{\left(\frac{dy}{dx}\right)_F}$$

EXERCISE 9.2

1) Find general solution of the following D.E

a) $$\frac{2x\frac{dy}{dx} - 2y}{x^2} = 0$$

$$\Rightarrow 2x\frac{dy}{dx} - 2y = 0$$

$$\cancel{2}x\frac{dy}{dx} = \cancel{2}y$$

$$x\,dy = y\,dx$$

$$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$$

integrate

$$\int\frac{dy}{y} = \int\frac{dx}{x}$$

$$\int\frac{1}{y}dy = \int\frac{1}{x}dx$$

$$\ln y = \ln x + \ln c$$

$$\Rightarrow \ln y = \ln xc$$

taking antilog

$$\Rightarrow y = cx$$ Ans

b) Given that

$$\frac{dy}{x} + y\,dx = 2dx$$

$$\frac{dy}{x} = 2dx - y\,dx$$

$$\frac{dy}{x} = (2-y)\,dx$$

$$\Rightarrow \frac{dy}{2-y} = x\,dx$$

integrate

$$\int \frac{1}{2-y}\,dy = \int x\,dx$$

$$\frac{\ln(2-y)}{-1} = \frac{x^2}{2} + c_1 \quad \left(\because \int \frac{1}{ax+b}\,dx = \frac{\ln(ax+b)}{a} + c\right)$$

x'ing by −1

$$\ln(2-y) = -\frac{x^2}{2} - c_1$$

$$\Rightarrow 2-y = e^{-\frac{x^2}{2} - c_1} \quad \left(\because \ln x = y \Rightarrow x = e^y\right.$$

$$2-y = e^{-x^2/2} e^{-c_1}$$

$$2-y = e^{-x^2/2} c \quad \left(e^{-c_1} = c\right.$$

$$2 - c e^{-x^2/2} = y$$

or $y = 2 - c e^{-x^2/2}$ Ans

c) $$\left(\frac{dy}{dx}\right)^2 = 1 - y^2$$

Sq: root

$$\frac{dy}{dx} = \pm\sqrt{1-y^2}$$

$$\Rightarrow dy = \pm\sqrt{1-y^2}\,dx$$

$$\frac{dy}{\sqrt{1-y^2}} = \pm dx$$

integrate

$$\int \frac{1}{\sqrt{1-y^2}}\,dy = \pm\int dx$$

$$\sin^{-1} y = \pm(x+c)$$

$$y = \sin[\pm(x+c)]$$

$$y = \pm\sin(x+c) \quad \left(\because \sin(-\theta) = -\sin\theta\right.$$

Ans

d) $$e^x \frac{dy}{dx} + y^2 = 0$$

$$e^x \frac{dy}{dx} = -y^2$$

$$e^x dy = -y^2 dx$$

$$\Rightarrow \frac{dy}{y^2} = -\frac{1}{e^x}\,dx$$

integrate.

$$\int y^{-2}\,dy = -\int e^{-x}\,dx$$

$$\frac{y^{-2+1}}{-2+1} = +\frac{e^{-x}}{+1} + c$$

$$\frac{y^{-1}}{-1} = e^{-x} + c.$$

$$-\frac{1}{y} = e^{-x} + c$$

taking Reciprocal

$$-y = \frac{1}{e^{-x}+c}$$

$$\Rightarrow y = \frac{-1}{e^{-x}+c} \quad \text{Ans}$$

e) $\sqrt{1-x^2}\,dy = \sqrt{1-y^2}\,dx$

$$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$$

integrate

$$\int \frac{1}{\sqrt{1-y^2}}dy = \int \frac{1}{\sqrt{1-x^2}}dx$$

$$\sin^{-1}y = \sin^{-1}x + C$$

$$\Rightarrow y = \sin^{-1}(\sin^{-1}x + C)$$

f) $\operatorname{cosec}^2 x\,dy + \sec y\,dx = 0$

$$\operatorname{cosec}^2 x\,dy = -\sec y\,dx$$

$$\Rightarrow \frac{1}{\sec y}dy = -\frac{1}{\operatorname{cosec}^2 x}dx$$

integrate

$$\int \cos y\,dy = -\int \sin^2 x\,dx$$

$$\sin y = -\int \frac{1-\cos 2x}{2}dx$$

$$\sin y = -\frac{1}{2}\int (1-\cos 2x)\,dx$$

$$\sin y = -\frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + C$$

$$\sin y = -\frac{1}{2}x + \frac{\sin 2x}{4} + C$$

$$\Rightarrow y = \sin^{-1}\left(-\frac{x}{2} + \frac{\sin 2x}{4} + C\right)$$ Ans

2) Reduce the following D.Es in separable form and then solve

a) Given that

$$y' = (y+x)^2$$

$$\Rightarrow \frac{dy}{dx} = (y+x)^2$$

put $y+x=u$

Diff w.r.t x

$$\frac{dy}{dx} + 1 = \frac{du}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{du}{dx} - 1}$$

$$\frac{du}{dx} - 1 = u^2$$

$$\frac{du}{dx} = u^2+1$$

$$\Rightarrow du = (u^2+1)\,dx$$

$$\Rightarrow \frac{du}{u^2+1} = dx$$

$$\boxed{\int \frac{1}{1+x^2}dx = \tan^{-1}x}$$

integrate

$$\int \frac{1}{u^2+1}du = \int 1\,dx$$

$$\tan^{-1}u = x + C$$

$$\tan^{-1}(y+x) = x + C$$

$$\Rightarrow y+x = \tan(x+C) \Rightarrow \boxed{y = \tan(x+C) - x}$$

Ans

b) Given that

$$y' = \tan(x+y) - 1$$

$$\frac{dy}{dx} = \tan(x+y) - 1$$

put $x+y=u$

Diff w.r.t x

$$1 + \frac{dy}{dx} = \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{du}{dx} - 1$$

$$\frac{du}{dx} - \cancel{1} = \tan u - \cancel{1}$$

$$\frac{du}{dx} = \tan u$$

$$\frac{du}{\tan u} = dx$$

integrate

$$\int \cot u\,du = \int 1\,dx$$

$$\ln \sin u = x + C_1$$

$$\Rightarrow \sin u = e^{x+C_1}$$

$$\sin(x+y) = e^x e^{C_1}$$

$$\sin(x+y) = Ce^x$$ Ans

$(e^{C_1} = C$

c) Given that

$y' = (x+e^{y}-1)e^{-y}$

$\Rightarrow \frac{dy}{dx} = \frac{(x+e^{y}-1)}{e^{y}}$

$\Rightarrow e^{y}\frac{dy}{dx} = x+e^{y}-1$

put

$x+e^{y}-1 = u$

Diff w.r.t. x

$1+e^{y}\frac{dy}{dx} - 0 = \frac{du}{dx}$

$e^{y}\frac{dy}{dx} = \frac{du}{dx} - 1$

$\frac{du}{dx} - 1 = u$

$\frac{du}{dx} = u+1$

$\Rightarrow \frac{du}{u+1} = dx$

integrate

$\int \frac{1}{u+1}du = \int 1 dx$

$\ln(u+1) = x + c_1$

$\Rightarrow u+1 = e^{x+c_1}$

$\boxed{\because \ln x = y \Rightarrow x = e^{y}}$

$x + e^{y} \cancel{-1} \cancel{+1} = e^{x}e^{c_1}$

$e^{y} = ce^{x} - x$

Ans

$e^{c_1} = c$

d) $y\frac{dy}{dx} + xy^{2} - x = 0$

$y\frac{dy}{dx} = x - xy^{2}$

$y\frac{dy}{dx} = x(1-y^{2})$

$\Rightarrow y\,dy = x(1-y^{2})dx$

$\Rightarrow \frac{y}{1-y^{2}}dy = x\,dx$

integrate

$\int \frac{y}{1-y^{2}}dy = \int x\,dx$

x'ing by -2

$\int \frac{-2y}{1-y^{2}}dy = -2\int x\,dx$

$\ln(1-y^{2}) = -\cancel{2}\frac{x^{2}}{\cancel{2}} + c_1$

$\ln(1-y^{2}) = -x^{2} + c_1$

$\Rightarrow 1-y^{2} = e^{-x^{2}+c_1}$

$1-y^{2} = e^{-x^{2}}e^{c_1}$

$1-y^{2} = ce^{-x^{2}}$

Ans

$\int \frac{f'(x)}{f(x)}dx = \ln f(x) + c$

$\ln x = y \Rightarrow x = e^{y}$

$e^{c_1} = c$

---

e) Given that

$\frac{dy}{dx} = \frac{y}{x} + \frac{x^{2}}{y^{2}}$

$\frac{dy}{dx} = \frac{y}{x} + \left(\frac{x}{y}\right)^{2}$

put $\frac{y}{x} = u$

$\Rightarrow y = ux$

Diff w.r.t. x

$\frac{dy}{dx} = u\cdot 1 + x\frac{du}{dx}$

Product Rule

$\cancel{u} + x\frac{du}{dx} = \cancel{u} + \left(\frac{1}{u}\right)^{2}$

$x\frac{du}{dx} = \frac{1}{u^{2}}$

$x\,du = \frac{1}{u^{2}}dx$

$\Rightarrow u^{2}du = \frac{1}{x}dx$

integrate

$\int u^{2}du = \int \frac{1}{x}dx$

$\frac{u^{3}}{3} = \ln x + c_1$

x'ing by 3

$u^{3} = 3\ln x + 3c_1$

$\Rightarrow u^{3} = \ln x^{3} + 3c_1$

taking Antilog

$e^{u^{3}} = e^{\ln x^{3} + 3c_1}$

$e^{\ln \square} = \square$

$e^{u^{3}} = e^{\ln x^{3}}e^{3c_1}$

$e^{\left(\frac{y}{x}\right)^{3}} = x^{3}c \Rightarrow e^{\left(\frac{y}{x}\right)^{3}} = cx^{3}$

Ans

$$y = (x^3+1)\,dx$$

integrate

$$\int(y-3)\,dy = \int(x^3+1)\,dx$$

$$\frac{y^2}{2} - 3y = \frac{x^4}{4} + x + c_1$$ Ans

× ing by 8

$$4y^2 - 24y = 2x^4 + 8x + 8c_1$$

$$4y^2 - 24y = 2x^4 + 8x + C$$ $8c_1 = C$

Ans

Note In Homogeneous D.E put $\frac{y}{x} = u$ or $y = ux$

3) Solve the following homogeneous D.E.

a) $\frac{dy}{dx} = \frac{x+y}{x-y}$

Replace x by λx & y by λy in RHS

$$= \frac{\lambda x + \lambda y}{\lambda x - \lambda y} = \frac{\lambda(x+y)}{\lambda(x-y)} = \lambda^0\left(\frac{x+y}{x-y}\right)$$

$\Rightarrow \frac{x+y}{x-y}$ is Homogeneous function of degree zero

Hence $\frac{dy}{dx} = \frac{x+y}{x-y}$ is HDE put $y = ux$

Diff w.r.t x

$$\frac{dy}{dx} = u\cdot 1 + x\frac{du}{dx}$$

$$\downarrow$$

$$u + x\frac{du}{dx} = \frac{x+ux}{x-ux}$$

$$\Rightarrow x\frac{dy}{dx} = \frac{\cancel{x}(1+u)}{\cancel{x}(1-u)} - u$$

$$x\frac{dy}{dx} = \frac{1+u-u(1-u)}{1-u} = \frac{1+\cancel{u}-\cancel{u}+u^2}{1-u}$$

$$x\frac{du}{dx} = \frac{1+u^2}{1-u}$$

$$x\,du = \frac{1+u^2}{1-u}\,du$$

$$\Rightarrow \frac{1-u}{1+u^2}\,du = \frac{1}{x}\,dx$$

integrate

$$\int \frac{1-u}{1+u^2}\,du = \int \frac{1}{x}\,dx$$

$$\int\left[\frac{1}{1+u^2} - \frac{u}{1+u^2}\right]du = \ln x$$

→ M&D by 2

$$\int \frac{1}{1+u^2}\,du - \frac{1}{2}\int \frac{2u}{1+u^2}\,dx = \ln x$$

$$\tan^{-1}u - \frac{1}{2}\ln(1+u^2) = \ln x + \ln c$$

$$\tan^{-1}u - \ln(1+u^2)^{1/2} = \ln x + \ln c$$

$$\tan^{-1}u = \ln x + \ln c + \ln(1+u^2)^{1/2}$$

($\ln a + \ln b = \ln ab$

$$\tan^{-1}u = \ln\{xc(1+u^2)^{1/2}]$$

$$\tan^{-1}\frac{y}{x} = \ln\left[cx\left(1+\frac{y^2}{x^2}\right)^{1/2}\right]$$

$\because$ $y = ux$

$\Rightarrow$ $u = y/x$

$u^2 = y^2/x^2$

---

b) Given that

$$\frac{dy}{dx} = \frac{xy - y^2}{x^2}$$

In RHS Replace x by λx & y by λy

$$= \frac{(\lambda x)(\lambda y) - (\lambda y)^2}{(\lambda x)^2} = \frac{\lambda^2 xy - \lambda^2 y^2}{\lambda^2 x^2} = \frac{\lambda^2(xy - y^2)}{\lambda^2 x^2} = \lambda^0\frac{(xy-y^2)}{x^2}$$

$\Rightarrow$ $\frac{xy - y^2}{x^2}$ is Homogeneous function of degree zero

Hence $\frac{dy}{dx} = \frac{xy - y^2}{x^2}$ is HDE so put $y = ux$

Diff w.r.t. x

$$\frac{dy}{dx} = u.1 + x\frac{du}{dx}$$

$\downarrow$

$$u + x\frac{du}{dx} = \frac{x.ux - (ux)^2}{x^2}$$

↳

$$x\frac{du}{dx} = \frac{x^2u - u^2x^2}{x^2} - u$$

$$= \frac{\cancel{x^2}(u-u^2)}{\cancel{x^2}} - u = \cancel{u} - u^2 - \cancel{u}$$

$$x\frac{du}{dx} = -u^2$$

$$\Rightarrow \frac{du}{-u^2} = +\frac{1}{x}dx$$

integrate

$$-\int u^{-2}du = \int \frac{1}{x}dx$$

$\left(\int x^n dx = \frac{x^{n+1}}{n+1} + c\right.$

$$+\frac{u^{-1}}{+1} = \ln x + c_1 \quad \text{——} ①$$

$$\frac{1}{u} - c_1 = \ln x$$

$$\Rightarrow e^{1/u - c_1} = x$$

$\left(\because \ln x = y \Rightarrow x = e^y\right.$

$$e^{1/u}e^{-c_1} = x$$

$$e^{x/y}c = x$$

$\left(e^{-c_1} = c\right.$

$y = ux \Rightarrow u = \frac{y}{x}$

$\frac{1}{u} = \frac{x}{y}$

or $x = ce^{x/y}$

2nd method from ①

$$u^{-1} = \ln x + c$$

$$\frac{1}{u} = \ln x + c$$

$\because y = ux$

$\Rightarrow \frac{y}{x} = u$

$\Rightarrow \frac{1}{u} = \frac{x}{y}$

$$\frac{x}{y} = \ln x + c$$

taking Reciprocal.

$$\frac{y}{x} = \frac{1}{\ln x + c}$$

$$y = \frac{x}{\ln x + c}$$ Ans

c) Given that

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

On RHS Replace x by λx & y by λy.

$$= \frac{(\lambda x)^2+3(\lambda y)^2}{2(\lambda x)(\lambda y)} = \frac{\lambda^2 x^2+3\lambda^2 y^2}{2\lambda^2 xy} = \frac{\lambda^2(x^2+3y^2)}{\lambda^2(2xy)} = \lambda^0\left(\frac{x^2+3y^2}{2xy}\right)$$

$\Rightarrow \frac{x^2+3y^2}{2xy}$ is Homogeneous function of degree zero

Hence $\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$ is HDE so put $y = ux$

Diff w-r-t-x

$$\frac{dy}{dx} = u \cdot 1 + x\frac{du}{dx}$$

$\downarrow$

$$u + x\frac{du}{dx} = \frac{x^2+3(ux)^2}{2x(ux)}$$

$$\hookrightarrow x\frac{du}{dx} = \frac{\cancel{x^2}(1+3u^2)}{2\cancel{x^2}u} - u$$

$$= \frac{1+3u^2}{2u} - u = \frac{1+3u^2-2u^2}{2u}$$

$y = ux$

$\boxed{\frac{y}{x} = u}$

$$x\frac{du}{dx} = \frac{1+u^2}{2u}$$

$$\Rightarrow \frac{2u}{1+u^2}du = \frac{1}{x}dx$$

integrate

$$\int \frac{2u}{1+u^2}du = \int \frac{1}{x}dx \quad ①$$

$$\Rightarrow \ln(1+u^2) = \ln x + \ln c$$

$$\ln\left(1+\frac{y^2}{x^2}\right) = \ln cx$$

taking Anti log

$$1 + \frac{y^2}{x^2} = cx$$

$$\frac{x^3+y^3}{x^2} = cx$$

$$x^3+y^3 = cx^3 \quad \text{Ans}$$

OR 2nd method from eq ①

$$\ln(1+u^2) + \ln c = \ln x$$

$$\ln c(1+u^2) = \ln x$$

$$\Rightarrow c\left(1+\frac{y^2}{x^2}\right) = x$$

$$c\left(\frac{x^2+y^2}{x^2}\right) = x$$

$$c(x^2+y^2) = x^3$$

or $x^3 = c(x^2+y^2)$ Ans

(2.2

d) $\frac{dy}{dx} = \frac{\sqrt{x^2-y^2}+y}{x}$ In RHS. Replace x by λx

& Replace y by λy

$$= \frac{\sqrt{\lambda^2x^2-\lambda^2y^2}+\lambda y}{\lambda x} = \frac{\sqrt{\lambda^2(x^2-y^2)}+\lambda y}{\lambda x} = \frac{\lambda\sqrt{x^2-y^2}+\lambda y}{\lambda x} = \frac{\lambda\left[\sqrt{x^2-y^2}+y\right]}{\lambda x}$$

$$= \lambda^0\left(\frac{\sqrt{x^2-y^2}+y}{x}\right)$$

$\Rightarrow \frac{\sqrt{x^2-y^2}+y}{x}$ is Homogeneous function of degree zero

Hence $\frac{dy}{dx} = \frac{\sqrt{x^2-y^2}+y}{x}$ is HDE so put $y = ux$

Diff w.r.t x

$\frac{dy}{dx} = u\cdot 1 + x\frac{du}{dx}$

$$u + x\frac{du}{dx} = \frac{\sqrt{x^2-u^2x^2}+ux}{x}$$

$$x\frac{du}{dx} = \frac{\sqrt{x^2(1-u^2)}+ux}{x} - u$$

$$= \frac{x\sqrt{1-u^2}+ux}{x} - u = \frac{\cancel{x}\left[\sqrt{1-u^2}+u\right]}{\cancel{x}} - u$$

$$x\frac{du}{dx} = \sqrt{1-u^2} + \cancel{u} - \cancel{u}$$

$$\Rightarrow x\frac{du}{dx} = \sqrt{1-u^2}$$

$$\Rightarrow \frac{1}{\sqrt{1-u^2}}du = \frac{1}{x}dx$$

integrate

$$\int \frac{1}{\sqrt{1-u^2}}du = \int \frac{1}{x}dx$$

$\because y = ux$

$\Rightarrow \frac{y}{x} = u$

$$\sin^{-1}u = \ln x + c$$

$$u = \sin(\ln x + c)$$

$$\frac{y}{x} = \sin(\ln x + c)$$

$$y = x\sin(\ln x + c)$$ Ans

e) $\frac{dy}{dx} = \frac{xy+y^2}{x^2+xy+y^2}$ In RHS Replace x by λx & y by λy

$$\frac{dy}{dx} = \frac{(\lambda x)(\lambda y)+(\lambda y)^2}{(\lambda x)^2+(\lambda x)(\lambda y)+(\lambda y)^2} = \frac{\lambda^2xy+\lambda^2y^2}{\lambda^2x^2+\lambda^2xy+\lambda^2y^2} = \frac{\lambda^2(xy+y^2)}{\lambda^2(x^2+xy+y^2)}$$

$$= \lambda^0\left(\frac{xy+y^2}{x^2+xy+y^2}\right)$$

$\Rightarrow \frac{xy+y^2}{x^2+xy+y^2}$ is homogeneous function of degree zero.

Hence $\frac{dy}{dx} = \frac{xy+y^2}{x^2+xy+y^2}$ is HDE so put $y = ux$

Diff w.r.t. x

$$\frac{dy}{dx} = u.1 + x\frac{dy}{dx}$$

$$\downarrow$$

$$u + x\frac{du}{dx} = \frac{x.ux+u^2x^2}{x^2+x.ux+u^2x^2}$$

$$x\frac{du}{dx} = \frac{\cancel{x^2}(u+u^2)}{\cancel{x^2}(1+u+u^2)} - u$$

$$= \frac{u+u^2-u(1+u+u^2)}{1+u+u^2}$$

$$= \frac{\cancel{u}+\cancel{u^2}-u-\cancel{u^2}-u^3}{1+u+u^2}$$

$$x\frac{du}{dx} = -\frac{u^3}{1+u+u^2}$$

$$\Rightarrow \frac{1+u+u^2}{u^3}du = -\frac{1}{x}dx$$

integrate.

$$\int \frac{1+u+u^2}{u^3}du = -\int \frac{1}{x}dx$$

$$\int\left(\frac{1}{u^3}+\frac{u}{u^3}+\frac{u^2}{u^3}\right)du = -\ln x$$

$$\int\left(\frac{1}{u^3}+\frac{1}{u^2}+\frac{1}{u}\right)du = -\ln x$$

$$\int \left(u^{-3}+u^{-2}+\frac{1}{u}\right)du = -\ln x$$

$$\frac{u^{-2}}{-2}+\frac{u^{-1}}{-1}+\ln u = -\ln x + c$$

$$\underset{\hookrightarrow}{-\frac{1}{2u^2}}\underset{\hookrightarrow}{-\frac{1}{u}}+\ln u = \underset{\hookleftarrow}{-\ln x}\underset{\hookleftarrow}{+c}$$

$$\ln u + \ln x - c = \frac{1}{2u^2}+\frac{1}{u}$$

$\because y = ux$

$\Rightarrow \frac{y}{x} = u$

$\Rightarrow \frac{1}{u} = \frac{x}{y}$

$\frac{1}{u^2} = \frac{x^2}{y^2}$

$$\ln\frac{y}{x}+\ln x - c = \frac{x^2}{2y^2}+\frac{x}{y}$$

$$\ln\left[\frac{y}{\cancel{x}}\cdot\cancel{x}\right] - c = \frac{x^2}{2y^2}+\frac{x}{y}$$

×ing by $2y^2$

$$2y^2\ln y - 2y^2c = x^2+2xy$$

(① $\ln a+\ln b = \ln ab$
② $\ln a^b = b\ln a$

$$y^2\ln y^2 - 2y^2c = x^2+2xy$$

$$-2y^2c = x^2+2xy-y^2\ln y^2$$

÷ing by $(-2c)$

$$\Rightarrow \quad y^2 = \frac{-1}{2c}\left[x^2+2xy-y^2\ln y^2\right]$$

$$y^2 = c_1\left[x^2+2xy-y^2\ln y^2\right]$$

$\boxed{\frac{-1}{2c} = c_1}$

---

4) Reduce the following D.Es in the standard form of homogeneous form and then solve.

a) $x\frac{dy}{dx} = y+\sqrt{x^2+y^2}$ $\quad y(4)=3$

÷ing by x

$$\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$$

(H.D.E) so put $y = ux$

Diff w.r.t. x

$\frac{dy}{dx} = u.1 + x\frac{du}{dx}$

$$\underset{\hookrightarrow}{u}+x\frac{du}{dx} = \frac{ux+\sqrt{x^2+u^2x^2}}{x}$$

$$x\frac{dy}{dx} = \frac{ux+\sqrt{x^2(1+u^2)}}{x} - u$$

$x\frac{du}{dx} = \frac{ux + x\sqrt{1+u^2}}{x} - u$

$x\frac{du}{dx} = \frac{\cancel{x}(u+\sqrt{1+u^2})}{\cancel{x}} - u = \cancel{u} + \sqrt{1+u^2} - \cancel{u}$

$x\frac{du}{dx} = \sqrt{1+u^2}$

$\Rightarrow \frac{1}{\sqrt{1+u^2}}du = \frac{1}{x}dx$

integrate

$\left(\because \int \frac{1}{\sqrt{a^2+x^2}}dx = \ln(x+\sqrt{a^2+x^2})+C\right.$

$\int \frac{1}{\sqrt{1+u^2}}du = \int \frac{1}{x}dx$

$\ln(u+\sqrt{1+u^2}) = \ln x + \ln c$

$y = ux$

$\Rightarrow \left(u = \frac{y}{x}\right.$

$\ln a + \ln b = \ln ab$

$\ln\left(\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right) = \ln cx$

taking Anti log

$\Rightarrow \frac{y}{x}+\sqrt{\frac{x^2+y^2}{x^2}} = cx$

$\frac{y}{x}+\frac{\sqrt{x^2+y^2}}{x} = cx$

xing by x

$y+\sqrt{x^2+y^2} = cx^2$ —— ①

$\because y(4)=3 \Rightarrow$ $y=3$ & $x=4$ put in ①

$3+\sqrt{4^2+3^2} = c.4^2$

$3+\sqrt{16+9} = 16c$

$3+5 = 16c$

$\overset{1}{\cancel{8}} = \overset{2}{\cancel{16}}c \Rightarrow \boxed{c = 1/2}$ put in ①

①⇒ $y+\sqrt{x^2+y^2} = \frac{1}{2}x^2$

↳ $\sqrt{x^2+y^2} = \frac{x^2}{2} - y$

sq: B. sides

$$\left(\cancel{\sqrt{x^2+y^2}}\right)^{\cancel{2}} = \left(\frac{x^2}{2}-y\right)^2$$

$$x^2+\cancel{y^2} = \frac{x^4}{4}+\cancel{y^2}-\cancel{2}\left(\frac{x^2}{\cancel{2}}\right)y$$

$$x^2 = \frac{x^4}{4} - x^2y$$

$$\hookrightarrow \qquad \hookleftarrow$$

$$x^2y = \frac{x^4}{4} - x^2$$

÷ing by x^2

$$y = \frac{x^2}{4} - 1 \qquad \text{Ans}$$

---

b) $(x^4+y^4)\,dx = 2x^3y\,dy$ $\qquad y(1)=0$

or

$$2x^3y\,dy = (x^4+y^4)\,dx$$

÷ing by $2x^3y\,dx$

$$\frac{dy}{dx} = \frac{x^4+y^4}{2x^3y}$$

It is HDE so put $y=ux$

Diff w.r.t. x

$$\frac{dy}{dx} = u\cdot 1 + x\frac{du}{dx}$$

$$\downarrow$$

$$u + x\frac{du}{dx} = \frac{x^4+(ux)^4}{2x^3\cdot ux}$$

$$\hookrightarrow$$

$$x\frac{du}{dx} = \frac{x^4+u^4x^4}{2ux^4} - u$$

$$x\frac{du}{dx} = \frac{\cancel{x^4}(1+u^4)}{2\cancel{x^4}u} - u$$

$$x\frac{du}{dx} = \frac{1+u^4}{2u} - u$$

$$x\frac{du}{dx} = \frac{1+u^4-2u^2}{2u}$$

$$x\frac{du}{dx} = \frac{1^2+(u^2)^2-2u^2}{2u}$$

$$x\frac{du}{dx} = \frac{(1-u^2)^2}{2u}$$

$$\Rightarrow \frac{2u\,du}{(1-u^2)^2} = \frac{1}{x}dx$$

integrate

$$\int \frac{1 \cdot 2u\,du}{(1-u^2)^2} = \int \frac{1}{x}dx$$

put

$1-u^2 = t$

Diff w.r.t. u

$0 - 2u = \frac{dt}{du}$

$-2u = \frac{dt}{du}$

$\Rightarrow 2u\,du = -dt$

$$\Rightarrow \int \frac{1}{t^2}(-dt) = \ln x$$

$$-\int t^{-2}dt = \ln x$$

$$+\frac{t^{-1}}{\cancel{-}1} = \ln x + c$$

$$\frac{1}{t} = \ln x + c$$

$$\frac{1}{1-u^2} = \ln x + c$$

$y = ux$

$\Rightarrow u = y/x$

$$\frac{1}{1-\frac{y^2}{x^2}} = \ln x + c$$

$$\frac{1}{\frac{x^2-y^2}{x^2}} = \ln x + c$$

$$\frac{x^2}{x^2-y^2} = \ln x + c \longrightarrow ①$$

∵ $y(1) = 0 \Rightarrow y = 0$ & $x = 1$ put in ①

$$\frac{1}{1-0} = \ln 1 + c$$

$$1 = 0 + c \Rightarrow \boxed{c = 1}$$ put in ①

$$① \Rightarrow \frac{x^2}{x^2-y^2} = \ln x + 1$$

$$\frac{x^2}{\ln x + 1} = x^2 - y^2$$

$$y^2 = x^2 - \frac{x^2}{\ln x + 1}$$

$$y^2 = x^2 - \frac{x^2}{\ln x+1}$$

$$= \frac{x^2(\ln x+1) - x^2}{\ln x+1} = \frac{x^2 \ln x + \cancel{x^2} - \cancel{x^2}}{\ln x+1}$$

$$y^2 = \frac{x^2 \ln x}{\ln x+1}$$ Ans

---

⑤ Given that

$$\underset{\downarrow}{\text{slope}} = \frac{y-1}{1-x}$$

$$\frac{dy}{dx} = \frac{y-1}{1-x}$$

$$\Rightarrow \quad \frac{dy}{y-1} = \frac{dx}{1-x}$$

integrate

$$\int \frac{1}{y-1}\,dy = \int \frac{1}{1-x}\,dx$$

$$\left(\int \frac{1}{ax+b}\,dx = \frac{\ln(ax+b)}{a} + c \right.$$

$$\ln(y-1) = \frac{\ln(1-x)}{-1} + \ln c$$

$$\ln(y-1) = -\ln(1-x) + \ln c$$

$$\ln(y-1) = \ln c - \ln(1-x)$$

$$\ln(y-1) = \ln \frac{c}{1-x}$$

$$\left(\ln a - \ln b = \ln \frac{a}{b} \right.$$

taking Antilog

$$y-1 = \frac{c}{1-x} \quad \text{---} \; ①$$

∵ curve Passes through Point $P(4,-3)$ so put $x=4$, $y=-3$ in ①

$$-3-1 = \frac{c}{1-4}$$

$$-4 = \frac{c}{-3} \Rightarrow \boxed{c = 12}$$ put in ①

①⇒

$$y-1 = \frac{12}{1-x}$$

$$y = \frac{12}{1-x} + 1$$

$$y = \frac{12+1-x}{1-x} \Rightarrow \boxed{y = \frac{13-x}{1-x}} \text{ or } \boxed{y = \frac{\cancel{-}(x-13)}{\cancel{-}(x-1)}}$$

b) Given D.E is

$$xy\,y' = 3y^2+x^2$$

÷ing by xy

$$\frac{dy}{dx} = \frac{3y^2+x^2}{xy}$$

Replace x by λx
y by λy

$$= \frac{3(\lambda y)^2+(\lambda x)^2}{(\lambda x)(\lambda y)} = \frac{3\lambda^2y^2+\lambda^2x^2}{\lambda^2xy}$$

$$= \frac{\lambda^2(3y^2+x^2)}{\lambda^2xy} = \lambda^0\left(\frac{3y^2+x^2}{xy}\right)$$

⇒ $\frac{3y^2+x^2}{xy}$ is Homogeneous function of degree ze

Hence $\frac{dy}{dx} = \frac{3y^2+x^2}{xy}$ is HDE so put $y=ux$

Diff: w.r.t x

$$\frac{dy}{dx} = u\cdot 1 + x\frac{du}{dx}$$

$$u + x\frac{du}{dx} = \frac{3u^2x^2+x^2}{x\cdot ux}$$

$$x\frac{du}{dx} = \frac{\cancel{x^2}(3u^2+1)}{\cancel{x^2}u} - u$$

$$= \frac{3u^2+1-u^2}{u}$$

$$x\frac{du}{dx} = \frac{2u^2+1}{u}$$

⇒ $\frac{u}{2u^2+1}du = \frac{1}{x}dx$

integrate

$$\int \frac{u}{2u^2+1}du = \int \frac{1}{x}dx$$

put
$2u^2+1=t$
Diff: w.r.t u
$4u = \frac{dt}{du}$
⇒ $u\,du = \frac{dt}{4}$

$$\int \frac{1}{t}\frac{dt}{4} = \int \frac{1}{x}dx$$

$$\frac{1}{4}\ln t = \ln x + \ln c$$

$$\ln t^{1/4} = \ln cx$$

taking anti log

$\ln a + \ln b = \ln ab$

$$t^{1/4} = cx$$

$$\downarrow$$

$$(2u^2+1)^{1/4} = cx$$

$$\left(2\frac{y^2}{x^2}+1\right)^{1/4} = cx \text{ —— ①}$$

∵ $y = ux$

$\Rightarrow \frac{y}{x} = u$

∵ curve passes through point P(-1, 2) so

put $x=-1$, $y=2$ in ①

$$\left(\frac{2(2)^2}{(-1)^2}+1\right)^{1/4} = c(-1)$$

$$\left(\frac{8}{1}+1\right)^{1/4} = -c \Rightarrow 9^{1/4} = -c \Rightarrow \boxed{c = -9^{1/4}}$$

put in ①

$$① \Rightarrow \left(2\frac{y^2}{x^2}+1\right)^{1/4} = -9^{1/4}x$$

Ans

which is the req: curve

---

7) Given HDE is

$$t^2y' = y^2+2ty$$

÷ing by t^2

$$\frac{dy}{dt} = \frac{y^2+2ty}{t^2}$$

∵ it is HDE so put

$y = ut$

Diff w.r.t. t

$$\frac{dy}{dt} = u\cdot 1 + t\frac{dy}{dt}$$

$$u + t\frac{du}{dt} = \frac{u^2t^2+2t\cdot ut}{t^2}$$

$$t\frac{du}{dt} = \frac{u^2t^2+2t^2u}{t^2} - u$$

$$= \frac{\cancel{t^2}(u^2+2u)}{\cancel{t^2}} - u$$

$$= u^2+2u-u$$

$$t\frac{du}{dt} = u^2+u$$

$$\Rightarrow \frac{du}{u^2+u} = \frac{1}{t}dt$$

integrate

$$\int \frac{1}{u(u+1)}\,du = \int \frac{1}{t}\,dt$$

$$\int\left(\frac{A}{u}+\frac{B}{u+1}\right)du = \ln t \quad \text{——} ①$$

where $\frac{1}{u(u+1)} = \frac{A}{u}+\frac{B}{u+1}$

Xing by $u(u+1)$

$$1 = A(u+1) + Bu \quad \text{——} ②$$

put $u=0$ in ②

$1 = A(0+1)+B\cdot 0$

$1 = A+0$

$\boxed{A=1}$

put $u+1=0 \Rightarrow u=-1$ in ②

$1 = A(0)+B(-1)$

$1 = 0-B \Rightarrow \boxed{B=-1}$

put values of A & B in ①

$$\int\left(\frac{1}{u}+\frac{-1}{u+1}\right)du = \ln t$$

$$\ln u - \ln(u+1) = \ln t + \ln c$$

$\left(\ln a - \ln b = \ln \frac{a}{b}\right.$

$\ln a + \ln b = \ln ab$

$$\ln \frac{u}{u+1} = \ln tc$$

taking Antilog

$$\frac{u}{u+1} = tc$$

$\because y = ut$

$\Rightarrow \frac{y}{t} = u$

$$\frac{\frac{y}{t}}{\frac{y}{t}+1} = tc$$

$$\frac{\frac{y}{\cancel{t}}}{\frac{y+t}{\cancel{t}}} = tc$$

$$\frac{y}{y+t} = tc \quad \text{——} ①$$

$\because$ $y(1) = 2$ so $y=2$, $t=1$ put in ①

$\frac{2}{2+1} = 1 \cdot C \Rightarrow \boxed{C = 2/3}$ put in ①

① ⇒ $\frac{y}{y+t} = \frac{2}{3}t$

$3y = 2t(y+t)$

$3y = 2ty + 2t^2$

$3y - 2ty = 2t^2$

$y(3-2t) = 2t^2 \Rightarrow \boxed{y = \frac{2t^2}{3-2t}}$ Ans

which is the req. Particular Solution

Note ① $P(x, y)$
abscissa → ordinate.

② To convert Linear D.E of the form $\frac{dy}{dx} + Py = Q$ To Separable D.E just multiply it by integrating factor $e^{\int P dx}$

③ $u\frac{dv}{dx} + v\frac{du}{dx} = \frac{d}{dx}(uv)$

---

⑧ ∵ Velocity at any Point is equal to half its abscissa minus three times the time so its D.E is

$\frac{dx}{dt} = \frac{1}{2}x - 3t$

$\frac{dx}{dt} - \frac{1}{2}x = -3t$ —— ①

According to Standard form
$\boxed{\text{Here } P = -1/2 \quad Q = -3t}$

Integrating factor $= e^{\int P dt}$
$= e^{\int -\frac{1}{2} dt} = e^{-\frac{1}{2}t} = e^{-t/2}$

Now ×ing eq ① by $e^{-\frac{t}{2}}$

$$\underbrace{e^{-t/2}\frac{dx}{dt}-\frac{1}{2}xe^{-t/2}}_{\Downarrow \text{ Product Rule close}} = -3te^{-t/2}$$

$$\Rightarrow \frac{d}{dt}(e^{-t/2}.x) = -3te^{-t/2}$$

$$\boxed{\int e^{kx}dx = \frac{e^{kx}}{k}+c}$$

integrate w.r.t. t

$$\int \frac{d}{dt}(e^{-t/2}.x)\,dt = -3\int \underset{I}{t}\,\underset{II}{e^{-t/2}}dt$$

Int. by Parts

$$e^{-t/2}.x = -3\left[t.\frac{e^{-t/2}}{-1/2} - \int(1)\frac{e^{-t/2}}{-\frac{1}{2}}dt\right]$$

$$xe^{-t/2} = -3\left[-2te^{-t/2} + 2\int e^{-t/2}dt\right]$$

$$xe^{-t/2} = +6te^{-t/2} - 6\frac{e^{-t/2}}{-\frac{1}{2}} + c$$

$$xe^{-t/2} = 6te^{-t/2} + 12e^{-t/2} + c$$

X'ing $e^{t/2}$

$$x = 6t + 12 + ce^{t/2} \text{ ——— ①}$$

∴ at a time $t=2$, $x=-4$ so

$$-4 = 6(2)+12 + ce^{\cancel{2}/2}$$

$$-4 = 12 + 12 + ce^{1}$$

$$-28 = ce$$

X'ing by e^{-1}

$$-28e^{-1} = c \quad \text{put in ①}$$

$$① \Rightarrow \quad x = 6t + 12 - 28e^{-1}e^{t/2}$$

$$x = 6t + 12 - 28e^{t/2-1}$$ Ans

which is the req: equation of motion.

9) Given that

$$\frac{dx}{dt} = 1.2e^{0.04t}$$

$$\Rightarrow dx = 1.2e^{0.04t}\,dt$$

integrate

$$\int 1dx = 1.2\int e^{0.04t}\,dt$$

$$x = 1.2\frac{e^{0.04t}}{.04} + c$$

$$\left(\because \int e^{kx}dx = \frac{e^{kx}}{k} + c \right.$$

$$x = 30e^{0.04t} + c \text{ --- ①}$$

Initially when $t=0$ so $x=0$

$$0 = 30e^{0.04(0)} + c$$

$$0 = 30e^{0} + c$$

$$0 = 30(1) + c \Rightarrow \boxed{c = -30} \text{ put in ①}$$

①$\Rightarrow$ $x = 30e^{0.04t} - 30$ --- ②

Now consumption in $t=8$ years, put $t=8$ in ②

$$x = 30e^{0.04\times 8} - 30$$

$$x = 30e^{.32} - 30$$

$$= 30(1.3771) - 30 = 41.313 - 30$$

$$= 11.313 \text{ (billions of barrels)}$$

---

⑩ The rate of infection of disease is given by

$$\frac{dI}{dt} = \frac{100t}{t^2+1}$$

$$\Rightarrow dI = \frac{100t}{t^2+1}\,dt$$

integrate

$$\int 1.dI = \int \frac{100t}{t^2+1}\,dt$$

$$I = 100\int \frac{t}{t^2+1}\,dt$$ M&D by 2

$$I = \frac{100}{2}\int \frac{2t}{t^2+1}\,dt$$ $\because \int \frac{f'(x)}{f(x)}\,du = \ln f(x) + c$

$$I = 50\ln(t^2+1) + c \quad \text{——①}$$

Initially when $t=0$ then $I=0$ (infected peoples)

$$0 = 50\ln(0+1) + c$$

$$0 = 50\ln 1 + c$$

$$0 = 0 + c \Rightarrow \boxed{c=0}$$ put in ①

①⇒ $I = 50\ln(t^2+1) + 0$

$$I = 50\ln(t^2+1) \quad \text{——②}$$

Infected peoples over the first four months are

put $t=1$ in ②

$I = 50\ln(1^2+1) = 50\ln 2 = 34.66 \approx 35$ Peoples

put $t=2$ in ②

$I = 50\ln(2^2+1) = 50\ln(5) = 80.47 \approx 81$ Peoples

put $t=3$ in ②

$I = 50\ln(3^2+1) = 50\ln 10 = 115.13 \approx 116$ Peoples

put $t=4$ in ②

$I = 50\ln(4^2+1) = 50\ln 17 = 141.66 \approx 142$ Peoples

11) Given that

$$\frac{dR}{dx} = 2x^2e^{-x}$$

$\Rightarrow$ $dR = 2x^2e^{-x}\,dx$ $\qquad\qquad \int \text{I}\,\text{II}\,dx = \text{I}\int \text{II}\,dx - \int\left(\frac{d}{dx}\text{I}\right)\left(\int \text{II}\,dx\right)dx$

integrate

$$\int 1\,dR = 2\int \underset{\text{I}}{x^2}\,\underset{\text{II}}{e^{-x}}\,dx$$

Int. by Parts

$$R = 2\left[x^2\left(\frac{e^{-x}}{-1}\right) - \int (2x)\frac{e^{-x}}{-1}\,dx\right]$$

$$R = 2\left[-x^2e^{-x} + 2\int \underset{\text{I}}{x}\,\underset{\text{II}}{e^{-x}}\,dx\right]$$

Int: by Parts

$$= -2x^2e^{-x} + 4\left[x\left(\frac{e^{-x}}{-1}\right) - \int (1)\left(\frac{e^{-x}}{-1}\right)dx\right]$$

$$= -2x^2e^{-x} + 4\left[-xe^{-x} + 4\int e^{-x}\,dx\right]$$

$$= -2x^2e^{-x} - 4xe^{-x} + 4\frac{e^{-x}}{-1} + c$$

$$R = -2e^{-x}\left[x^2 + 2x + 2\right] + c \longrightarrow ①$$

Initially when (time) $x=0$ then $R=0$

$$0 = -2e^{0}\left[0+0+2\right] + c$$

$$0 = -2(2) + c \Rightarrow \boxed{c=4} \text{ put in } ①$$

$$① \Rightarrow R = -2e^{-x}\left[x^2 + 2x + 2\right] + 4$$

$$R = 4 - 2e^{-x}\left[x^2+2x+2\right] \longrightarrow ②$$

Now total Reaction to the drug from $x=1$ to $x=6$

At $x=1$ ② $\Rightarrow$ $R = 4 - 2e^{-1}(1^2+2(1)+2) = 4 - 2e^{-1}(5) = 4 - 10e^{-1} = 4 - 3.68$
$= 0.32\ \%$

At $x=2$ ② $\Rightarrow$ $R = 4 - 2e^{-2}(2^2+2(2)+2)$
$= 4 - 2e^{-2}(4+4+2) = 4 - 20e^{-2} = 4 - 2.71 = 1.29\%$

At $x=3$: ② ⇒ $R = 4-2e^{-3}(3^2+2(3)+2)$

$= 4-2e^{-3}(9+6+2) = 4-34e^{-3} = 4-1.69 = 2.31\%$

At $x=4$: ② ⇒ $R = 4-2e^{-4}(4^2+2(4)+2)$

$= 4-2e^{-4}(16+8+2) = 4-52e^{-4} = 4-0.95 = 3.05\%$

At $x=5$: ② ⇒ $R = 4-2e^{-5}(5^2+2(5)+2)$

$= 4-2e^{-5}(25+10+2) = 4-74e^{-5} = 4-0.5 = 3.5\%$

At $x=6$ ② ⇒ $R = 4-2e^{-6}(6^2+2(6)+2)$

$= 4-2e^{-6}(36+12+2) = 4-100e^{-6} = 4-0.25 = 3.75\%$

---

12) Given that

$$\frac{ds}{dx} = 0.38x + 0.04$$

⇒ $ds = (0.38x + 0.04)dx$

integrate

$\int 1\,ds = \int (0.38x + 0.04)dx$

$S = 0.38\frac{x^2}{2} + 0.04x + c$

$S = 0.19x^2 + 0.04x + c$ —— ①

Initially when $x=0$ then $S=0.25$ (given)

$0.25 = 0.19(0)^2 + 0.04(0) + c$ ⇒ $\boxed{c=0.25}$ put in ①

① ⇒ $S = 0.19x^2 + 0.04x + 0.25$ —— ②

Now Subscriber in 2004 (after 6 years) put $x=6$

$S = 0.19(6)^2 + 0.04(6) + 0.25$

$= 0.19(36) + .24 + 0.25$

$S = 6.84 + 0.24 + 0.25 = 7.33$ millions.

Note. To find curve of orthogonal trajectories

find $\frac{dy}{dx}$, say $\frac{dy}{dx} = \square$ (eleminate arbitrary constants)

Now D.E of O.T

$$\frac{dy}{dx} = -\frac{1}{\square}$$

Solve it & get curve of Orthogonal trajectories

---

13) a) Given eq: is

$y = cx^3 \Rightarrow \boxed{c = \frac{y}{x^3}}$

Diff w.r.t.x

$\frac{dy}{dx} = c \cdot 3x^2$

↓

$\frac{dy}{dx} = \frac{y}{x^{\cancel{3}}} \cdot 3\cancel{x^2}$

$\frac{dy}{dx} = \frac{3y}{x}$

Now D.E of O.T

$\frac{dy}{dx} = -\frac{x}{3y}$ (S.E)

$\Rightarrow 3y\,dy = -x\,dx$

integrate

$3\int y\,dy = -\int x\,dx$

$3\frac{y^2}{2} = -\frac{x^2}{2} + c_1$

X'ing by 2

$3y^2 = -x^2 + 2c_1$

$x^2 + 3y^2 = k$ Ans $(2c_1 = k)$

b) $xy = c$

Diff w.r.t.x

$\frac{d}{dx}(xy) = \frac{d}{dx}c$

$x\frac{dy}{dx} + y \cdot 1 = 0$

$x\frac{dy}{dx} = -y$

$\frac{dy}{dx} = -\frac{y}{x}$

D.E of O.T

$\frac{dy}{dx} = +\frac{x}{y}$

$\Rightarrow y\,dy = x\,dx$

integrate

$\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + c_1$

X'ing by 2

$y^2 = x^2 + 2c_1$

$y^2 - x^2 = k$ Ans $(2c_1 = k)$

c) $y = cxe^x \Rightarrow \boxed{c = \frac{y}{xe^x}}$

Diff w.r.t. x

$\frac{dy}{dx} = c\frac{d}{dx}(xe^x)$

$= c\left[x\frac{d}{dx}e^x + e^x\frac{d}{dx}x\right]$

$= c\{xe^x + e^x \cdot 1\}$

$= \underset{\downarrow}{c}\, e^x\{x+1\}$

$= \frac{y}{x\cancel{e^x}}\cancel{e^x}(x+1)$

$\frac{dy}{dx} = \frac{(x+1)y}{x}$

D.E of O.T

$\frac{dy}{dx} = -\frac{x}{(x+1)y}$

$\Rightarrow y\,dy = -\frac{x}{x+1}dx$

integrate

$\int y\,dy = -\int\frac{x}{x+1}dx$

$x+1\overline{)\cancel{x}}\;1$, $\underline{\cancel{x}\pm1}$, -1

$\frac{y^2}{2} = -\int\left(1 + \frac{-1}{x+1}\right)dx$

$= -\int\left(1 - \frac{1}{x+1}\right)dx$

$= -[x - \ln(x+1)] + \ln k$

$= -x + \ln(x+1) + \ln k$

$\frac{y^2}{2} = -x + \ln k(x+1)$ Ans

($\ln a + \ln b = \ln ab$)

---

d) $y^2 = x^2 + c$

Diff w.r.t. x

$\frac{d}{dx}y^2 = \frac{d}{dx}(x^2+c)$

$\cancel{2}y\frac{dy}{dx} = \cancel{2}x + 0$

$\Rightarrow \frac{dy}{dx} = \frac{x}{y}$

D.E of O.T

$\frac{dy}{dx} = -\frac{y}{x}$

$\Rightarrow \frac{dy}{y} = -\frac{1}{x}dx$

integrate

$\int\frac{1}{y}dy = -\int\frac{1}{x}dx$

$\ln y = -\ln x + \ln c$

$\ln x + \ln y = \ln c$

$\ln xy = \ln c$

taking Anti log

$xy = c$ Ans

which is req. eq. of curve

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e) $y = c\sin 2x \Rightarrow \boxed{c = \frac{y}{\sin 2x}}$

Diff: w.r.t. x

$\frac{dy}{dx} = \underset{\downarrow}{c}\cos 2x \cdot 2$

$= \frac{y}{\sin 2x}\cdot 2\cos 2x$

$\frac{dy}{dx} = 2y\frac{x}{\sin 2x}$

$\Rightarrow$ D.E of O.T

$\frac{dy}{dx} = -\frac{\sin 2x}{2y\cos 2x}$

$2y\,dy = -\tan 2x\,dx$

integrate

$2\int y\,dy = -\int\tan 2x\,dx$

($\int\tan x\,dx = -\ln\cos$)

($\ln a + \ln b = \ln ab$)

($\ln a^b = b\ln$)

$\cancel{2}\frac{y^2}{\cancel{2}} = -\left(-\frac{\ln\cos 2x}{2}\right) + \ln k$

$y^2 = +\frac{1}{2}\ln\cos 2x + \ln k$

$= \ln(\cos 2x)^{1/2} + \ln k$

$y^2 = \ln\left[k(\cos 2x)^{1/2}\right]$ Ans

f) Given that

$e^x \cos y = c$

Diff: w.r.t. x

$\frac{d}{dx}[e^x \cos y] = \frac{d}{dx} c$

$e^x(-\sin y \frac{dy}{dx}) + \cos y (e^x) = 0$

$-\cancel{e^x} \sin y \frac{dy}{dx} = -\cancel{e^x} \cos y$

$\Rightarrow \frac{dy}{dx} = \frac{\cos y}{\sin y}$

D.E of O.T

$\frac{dy}{dx} = -\frac{\sin y}{\cos y}$

$\Rightarrow \frac{\cos y}{\sin y} dy = -dx$

integrate

$\int \cot y \, dy = -\int dx$

$\ln \sin y = -x + c_1$

$\Rightarrow \sin y = e^{-x+c_1}$ $\because \boxed{\ln x = y \Rightarrow x = e^y}$

$\sin y = e^{-x} e^{c_1}$

×ing by e^x

$e^x \sin y = e^{c_1}$ $e^{c_1} = k$

$e^x \sin y = k$

g) $y = \sqrt{x+c}$

Sq: B. Sides

$y^2 = x + c$

Diff: w.r.t. x

$2y \frac{dy}{dx} = 1 + 0$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2y}$

D.E of O.T

$\frac{dy}{dx} = -2y$ (S.E)

$\Rightarrow \frac{dy}{y} = -2dx$

integrate

$\int \frac{1}{y} dy = -2 \int 1 \, dx$

$\ln y = -2x + c_1$

$\Rightarrow y = e^{-2x+c_1}$ $\left(\because \ln x = y \Rightarrow x = e^y\right.$

$y = e^{-2x} e^{c_1}$

$y = k e^{-2x}$ $\left(e^{c_1} = k\right.$

which is the req: eq:

h) $y = x^2 + c$

Diff: w.r.t x

$\frac{dy}{dx} = 2x + 0$

D.E of O.T

$\frac{dy}{dx} = -\frac{1}{2x}$

$\Rightarrow dy = -\frac{1}{2x} dx$

integrate

$\int 1 \, dy = -\frac{1}{2} \int \frac{1}{x} dx$

$y = -\frac{1}{2} \ln x + c$ Ans

i) Given that

$$e^x \sin y = c$$

Diff w.r.t. x

$$\frac{d}{dx}(e^x \sin y) = \frac{d}{dx}c$$

$$e^x \frac{d}{dx}\sin y + \sin y \frac{d}{dx}e^x = 0$$

$$e^x \cos y \frac{dy}{dx} + \sin y\, e^x = 0$$

$$\cancel{e^x}\cos y \frac{dy}{dx} = -\cancel{e^x}\sin y$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sin y}{\cos y}$$

D.E of O.T

$$\frac{dy}{dx} = +\frac{\cos y}{\sin y} \quad (S.E)$$

$$\Rightarrow \frac{\sin y}{\cos y}dy = dx$$

integrate

$$\int \tan y\, dy = \int dx$$

$$-\ln\cos y = x + c_1$$

$$\ln(\cos y)^{-1} = x + c_1$$

$$\Rightarrow (\cos y)^{-1} = e^{x+c_1}$$

$\because \ln x = y \Rightarrow x = e^y$

$$\frac{1}{\cos y} = e^x e^{c_1}$$

$$\frac{1}{e^{c_1}} = e^x \cos y$$

$$\Rightarrow e^x \cos y = k \quad \text{Ans}$$

let $k = \frac{1}{e^{c_1}}$

j) $\cos x \cosh y = c$

Diff: w.r.t. x

$$\frac{d}{dx}\cos x \cosh y = \frac{d}{dx}c$$

$$\cos x \frac{d}{dx}\cosh y + \cosh y \frac{d}{dx}\cos x = 0$$

$$\cos x \sinh y \frac{dy}{dx} + \cosh y(-\sin x) = 0$$

$$\cos x \sinh y \frac{dy}{dx} = +\sin x \cosh y$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin x \cosh y}{\cos x \sinh y}$$

D.E of O.T

$$\frac{dy}{dx} = -\frac{\cos x \sinh y}{\sin x \cdot \cosh y}$$

$$dy = -\frac{\cos x}{\sin x}\cdot\frac{\sinh y}{\cosh y}dx$$

$$\Rightarrow \frac{\cosh y}{\sinh y}dy = -\frac{\cos x}{\sin x}dx$$

integrate

$$\int \coth y\, dy = -\int \cot x\, dx$$

$$\ln\sinh y = -\ln\sin x + \ln k$$

$$\ln\sinh y + \ln\sin x = \ln k$$

$$\Rightarrow \ln(\sinh y \cdot \sin x) = \ln k$$

taking Antilog

$$\sinh y \cdot \sin x = k$$

or $\sin x \cdot \sinh y = k$ Ans