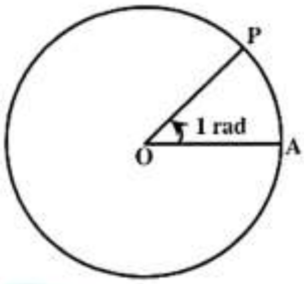
Chapter 7 class 10th

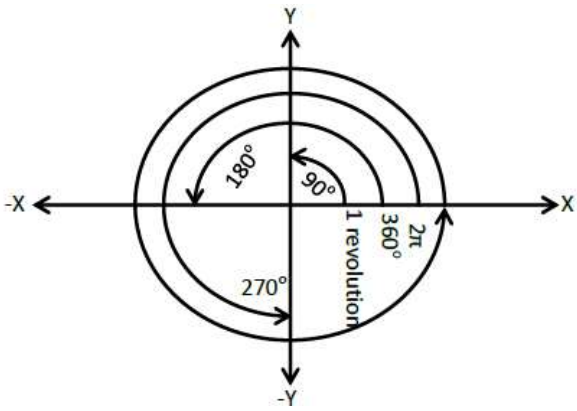
Radian:

The angle subtended at the Centre of the circle by an arc, whose length is equal to the radius of the circle is called one radian.



Relationship between radians and degree:

Things to know:



So,

$$1\,revolution = 2\pi\,radian = 360^0$$

$$2\pi\,radian = 360^0$$

$$to\ get\ 1\ radian\ divide\ by\ 2\pi$$

$$1\,radian = \frac{360^0}{2\pi}$$

$$1\,radian = \frac{180^0}{\pi}$$

To get 1 degree divide by 360

$$\frac{2\pi}{360}rad = 1^0$$

$$1^0 = \frac{\pi}{180}rad$$

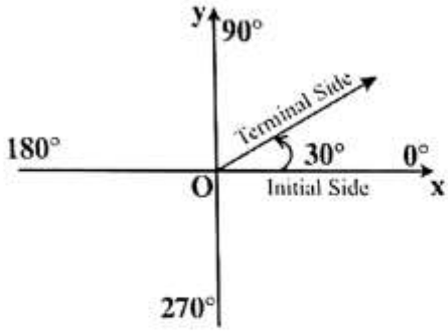
Remember that:

1 radian= $\frac{180}{3.1416} = 57.295795^o \approx 57^o17'45''$

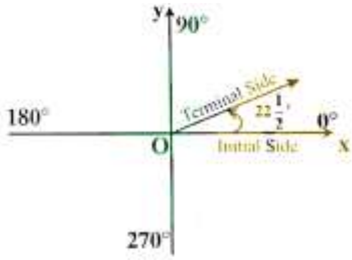
$$1^o = \frac{3.14516}{180} = 0.0175\,radians$$

Q.1: Locate the following angles:

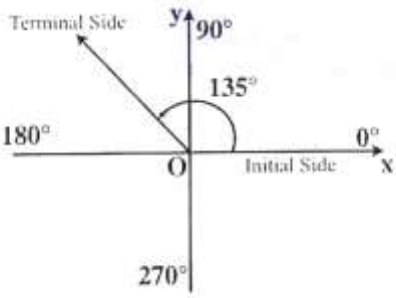
i. 30^0



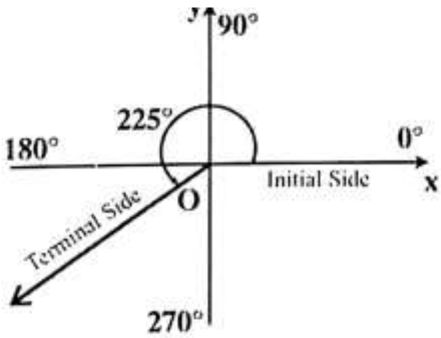
ii. $22\frac{1}{2}^0$



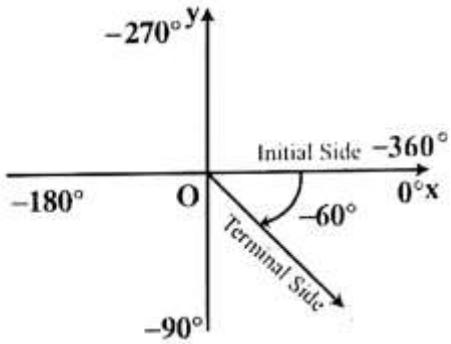
iii. 135^0



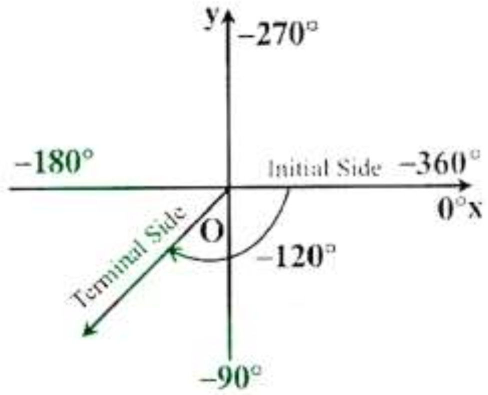
iv. -225^0



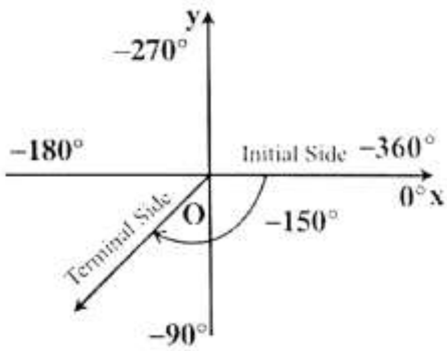
v. -60^0



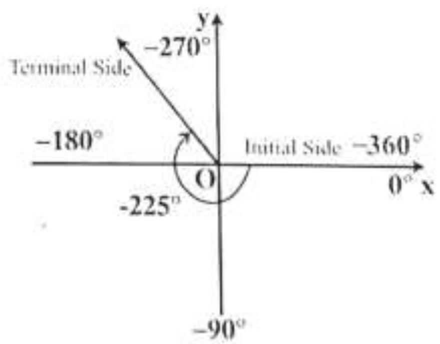
vi. -120^0



vii. -150^0



viii. -225^0



Q.2:

Express the following sexagesiml measures of angles in decimal form.

i. $45^0\ 30'$

Solution:

$$= 45^o + \frac{30^0}{60^0}$$

$$= 45^o + 0.5^0$$

$$= 45.5^o$$

ii. $60^0 30' 30''$

Solution:

$$= 60^0 + \frac{30^o}{60^0} + \frac{30^o}{60^o \times 60^o}$$

$$= 60^o + 0.5^o + 0.008^0$$

$$= 60.508^0$$

iii. $125^o 22' 50''$

Solution:

$$= 125^o + \frac{22^o}{60^o} + \frac{50^o}{60^o \times 60^o}$$

$$= 125^0 + 0.367^0 + 0.0139^o$$

$$= 125.3808^0$$

Q.3: Express the following in $D^o M' S''$:

i. 47.36^0

Solution:

$$= 47^0 + 0.36^0$$

$$= 47^0 + (0.36 \times 60)'$$

$$= 47^0 + 21' + (0.6 \times 60)''$$

$$= 47^0 + 21' + 36''$$

$$= 47^0 21' 36''$$

ii. 125.45^o

Solution:

$$= 125^0 + 0.45^0$$

$$= 125^0 + (0.45 \times 60)'$$

$$= 225^0 + 27'$$

$$225^0 27' 0''$$

iii. 225.75^o

Solution:

$$= 225^o + o.75^o$$

$$= 125^o + (0.75 \times 60)'$$

$$= 125^0 + 45'$$

$$= 225^o 45' 0''$$

iv. -22.5^0

Solution:

$$= -[22^0 + 0.5^0]$$

$$= -[22^0 + (0.5 \times 60)']$$

$$= -[22^o + 30']$$

$$= -22^0 30'$$

v. -67.58^0

Solution:

$$-(67^o + 0.58^0)$$
$$= -[67^0 + (0.58 \times 60)']$$
$$= -[67^0 + 34' + 0.8']$$
$$= [67^0 + 34' + (0.8 \times 60)'']$$
$$= -[67^0 + 34' + 48'']$$
$$= -67^o 34' 48''$$

vi. 315.18^0

$$= 315^o + 0.18^o$$
$$= 315^0 + (0.18 \times 60)'$$
$$= 315 + 10.8'$$
$$= 315^o + 10' + (0.8 \times 60)''$$
$$= 315^0 + 10' + 48''$$
$$= 315^o 10' 48''$$

Q.4: Express the following angles into radians.

i. 30^0

$$= 30\frac{\pi}{180} radians$$
$$= 30\frac{\pi}{30 \times 6} radians$$
$$= \frac{\pi}{6}\ radians$$

ii. 60^0

$$= 60\ \times \frac{\pi}{180}\ radian$$
$$= 60\frac{\pi}{60 \times 3} radian$$
$$= \frac{\pi}{3} radians$$

iii. 135^o

Solution:

$$225^0$$
$$= 225\ \frac{\pi}{180} radians$$
$$= 45 \times 3\frac{\pi}{45 \times 4} radians$$
$$= \frac{3\pi}{4} radians$$

iv. 225^0

Solution:225^o

$$= 225\ \frac{\pi}{180} radians$$
$$= 45 \times 5\frac{\pi}{45 \times 4} radians$$
$$= \frac{5\pi}{4} radians$$

v. -150^0

Solution:

$$-150^0$$
$$= -150\ \frac{\pi}{180} radians$$
$$= -5 \times 30\frac{\pi}{30 \times 6} radians$$
$$= \frac{-5\pi}{6} radians$$

vi. -225^0

Solution:

$$= -225\ \frac{\pi}{180} radians$$
$$= -5 \times 45\frac{\pi}{45 \times 4} radians$$
$$= \frac{-5\pi}{4} radians$$

vii. 300^0

Solution:

$$= 300\ \frac{\pi}{180} radians$$
$$= 60 \times 5\frac{\pi}{60 \times 3} radians$$
$$= \frac{5\pi}{3} radians$$

viii. 315^0

Solution:

$$= 315\ \frac{\pi}{180} radians$$
$$= 45 \times 7\frac{\pi}{45 \times 4} radians$$
$$= \frac{7\pi}{4} radians$$

Q.5: Convert each of the following to degrees.

i. $\frac{3\pi}{4}$

Solution:

$$\frac{3\pi}{4} radians$$
$$= \frac{3\pi}{4}\ \frac{180}{\pi}\ degree$$

$$= \frac{3\pi}{4} \frac{180}{\pi} \ degree$$
$$= 3 \times 45 \ degrees$$
$$= 135^o$$

ii. $\frac{5\pi}{6}$

Solution:

$$\frac{5\pi}{6} radians$$
$$= \frac{5\pi}{6} \frac{180}{\pi} \ degree$$
$$= \frac{5\pi}{6} \frac{180}{\pi} \ degree$$
$$= 5 \times 30 \ degrees$$
$$= 150^o$$

iii. $\frac{7\pi}{8}$

Solution:

$$\frac{7\pi}{8} radians$$
$$= \frac{7\pi}{8} \frac{180}{\pi} \ degree$$
$$= \frac{7 \times 180}{8} \ degree$$
$$= \frac{1260}{8} \ degrees$$
$$= 157.5^o$$

iv. $\frac{13\pi}{16}$

Solution:

$$\frac{13\pi}{16} radians$$
$$= \frac{13\pi}{16} \frac{180}{\pi} \ degree$$
$$= \frac{13 \times 180}{16} \ degree$$
$$= \frac{2340}{16} \ degrees$$
$$= 146.25^o$$

v. $3 \ radians$

Solution:

$$3 \ radians$$
$$= 3 \frac{180}{\pi} \ degree$$
$$= \frac{540}{\pi} \ degrees$$
$$= 171.887^o$$

vi. 4.5

Solution:

$$4.5 \ radians$$
$$= 4.5 \frac{180}{\pi} \ degree$$
$$= \frac{810}{\pi} \ degrees$$
$$= 257.831^0$$

vii. $-\frac{7\pi}{8}$

Solution:

$$-\frac{7\pi}{8} \ radians$$
$$= -\frac{7\pi}{8} \frac{180}{\pi} \ degree$$
$$= \frac{-1260}{8} \ degrees$$
$$= 157.5^o$$

viii. $-\frac{13}{16}\pi$

Solution:

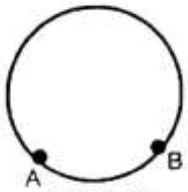
$$-\frac{13\pi}{16} \ radians$$
$$= -\frac{13\pi}{16} \frac{180}{\pi} \ degree$$
$$= \frac{-2340}{16} \ degrees$$

Type equation here.

$$146.25^o$$

Sector of a Circle:

i. **Arc of a circle:**
A part of the circumference of a circle is called an arc.

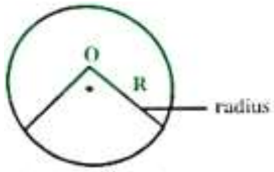


an arc

ii. Segment of a circle:
A part of the circular region bounded by an arc and a chord is called segment of a circle:

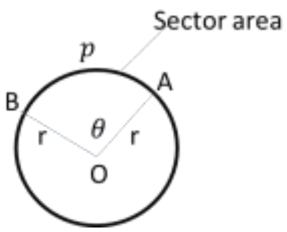
a segment

iii. Sector of a circle:
A part of a circular region bounded by the two radii and an arc is called sector of the circle.



a sector

Area of the Circular sector:
Consider a circle of radius and an arc of length units, subtended an angle θ at O.



Exercise 7.2

Question No.1 Find $\boldsymbol{\theta}$ when :

i. $\boldsymbol{I = 2cm,\ \ r = 3.5cm}$

Solution: using rule

$$I = r\theta$$
$$2 = 3.5\theta$$
$$\frac{2}{3.5} = \theta$$
$$\theta = 0.57\,radian$$

ii. $\boldsymbol{I = 4.5m\,,\ r = 2.5\,m}$

Solution: $\boldsymbol{using\ rule}$

$$I = r\theta$$
$$\frac{I}{r} = \theta$$
$$\frac{4.5}{2.5} = \theta$$
$$\theta = 1.8 radian$$

Question No.2 find $\boldsymbol{I\ \ when}$

i. $\boldsymbol{\theta = 180^0, r = 4.9cm}$

Solution:

As θ should be in radius so

$$\theta = 180^0$$
$$= 180\ \frac{\pi}{180}\ radian$$
$$= \pi\, radian$$

Using rule $I = r\theta$

$$= 4.9cm \times \pi$$
$$= 15.4cm$$

ii. $\boldsymbol{\theta = 60^o\ \ 30',\ \ r = 15mm}$

Solution:

As θ should be in radian, so

$$\theta = 60^o\ 30'$$
$$= 60^0 + \frac{30^0}{60^o}$$
$$= 60.5^o$$
$$= 60.5\ \frac{\pi}{180} radian$$
$$\theta = 1.056\, radian$$
$$\theta = 1.056\, radian$$
$$using\ rule\ I = r\theta$$
$$= 15mm \times 1.056$$
$$= 15.84mm$$

Questions No.3 find $r, when$

i. $I = 4cm,\ \ \theta = \frac{1}{4}\ \ radian$

Solution:

$$Using\ Rule\ I = r\ \theta$$
$$4cm = r\frac{1}{4}$$
$$4cm \times 4 = r$$
$$r = 16cm$$

ii. $I = 52cm\,, \theta = 45^o$

Solution: *As θ should be in radians.*

$$\theta = 45^o$$
$$= 45\frac{\pi}{180}\ radian$$

$$= \frac{\pi}{4} radian$$

Now using rule $I = r\theta$

$$52cm = r\frac{\pi}{4}$$

$$\frac{52cm \times 4}{\pi} = r$$

$$r = 66.21\ cm$$

Question No.4 In a circle of radius 12cm, find the length of an arc which subtends a Central angle $\theta = 1.5\ radian$

Solution:

$$Radius = r = 12cm$$

$$Arc\ length = ?$$

$$Central\ angle = \theta = 1.5\ radian$$

Using rule $I = r\theta$

$$I = 12m \times 1.5$$

$$I = 18m$$

Question No.5 In a circle of radius 10m, find the distance travelled by a point moving on this circle if the point makes 3.5 revolution.

Solution:

$$Radius = r = 10m$$

$$Number\ of\ revolutions = 3.5$$

$$Angle\ of\ one\ revolution = 2\pi$$

$$Angle\ of\ 3.5\ revolution = \theta$$

$$= 3.5 \times 2\ \pi radian$$

$$\theta = 7\pi\ radian$$

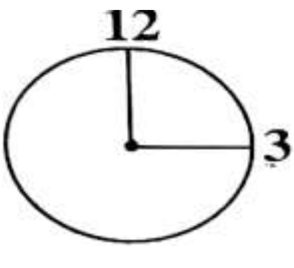
Distance travelled= $I = ?$

$$Using\ rule\ I = r\theta$$

$$I = 10m \times 7\pi$$

$$I = 220m$$

Question No.6 What is the circular measure of the angle between the hands of the watch at 3 O' clock?



Solution:

At 3 O' clock the minute hand will be at 12 and hour hand will be at 3 i.e the angle between the hands of watch will be one quarter of the central angle of full circle.

$$i.e = \frac{1}{4}\ of\ 360^o$$

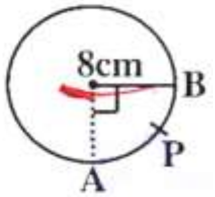
$$\frac{1}{4} \times 360^o$$

$$= 90^o$$

$$= 90\frac{\pi}{180} radian$$

$$= \frac{\pi}{2} radian$$

Question No.7 What is the length of arc APB?



Solution:

From the figure we see that

$$Radius = r = 8cm$$

$$Central\ angle = \theta$$

$$= 90^0$$

$$= \frac{\pi}{2} radian$$

Arc length $I = ?$

By rule $I = r\theta$

$$I = 8cm \times \frac{\pi}{2}$$

$$I = 4cm \times \pi$$

$$I = 12.57\ cm$$

So, length of arc APB is 12.57 cm

Question No.8 In a circle 12cm, how long an arc subtended a central angle of 84^0?

Solution:

Radius $= r = 12cm$

$$Arc\ length = I = ?$$

$$Central\ angle = \theta = 84^0$$

$$= 84\frac{\pi}{180}\ radian$$

$$= 1.466\ radian$$

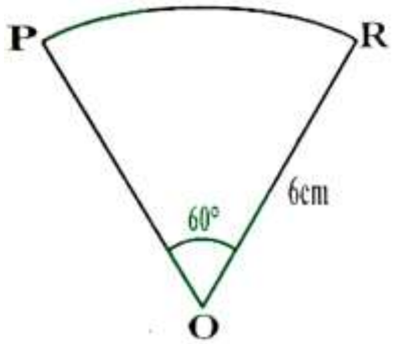
Now by rule $I = r\theta$

$$12cm \times 1.466$$

$$= \mathbf{17.6\ cm}$$

Question No.9 Find the area of sector OPR

(a)



Radius$= r = 6cm$

Central angle $= \theta = 60^o$

$$= 60\frac{\pi}{180}\ radian$$

$$= \frac{\pi}{3} radian$$

Area of sector =?

As area of sector $= \frac{1}{2}r^2\theta$

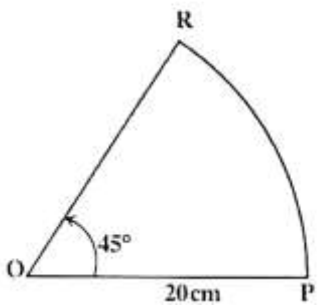
$$= \frac{1}{2} \times (6cm)^2 \times \frac{\pi}{3}$$

$$= \frac{1}{6} \times 36cm^2 \times \pi$$

$$= 6\pi\ cm^2$$

$$= 18.85cm^2$$

(b)



Radius $= r = 20cm$

$$Central\ angle = \theta = 45^0$$

$$= 45\frac{\pi}{180}\ radian$$

$$= \frac{\pi}{4} radian$$

Area of sector =?

Area of Sector $= \frac{1}{2}r^2\theta$

$$= \frac{1}{2}(20cm)^2 \times \frac{\pi}{4}$$

$$= \frac{400cm^2}{8} \times \pi$$

$$= 50\pi cm^2$$

$$= 157.1\ cm^2$$

Question No.10 Find area of sector inside a central angle of $\mathbf{20^0}$ $\boldsymbol{in\ a\ circle\ of\ radius\ 7m}$.

Solution:

Area of sector =?

$$Radius = r = 7m$$

Central angle$= \theta = 20^o$

$$= 20\frac{\pi}{180}\ radian$$

$$= \frac{\pi}{9} radian$$

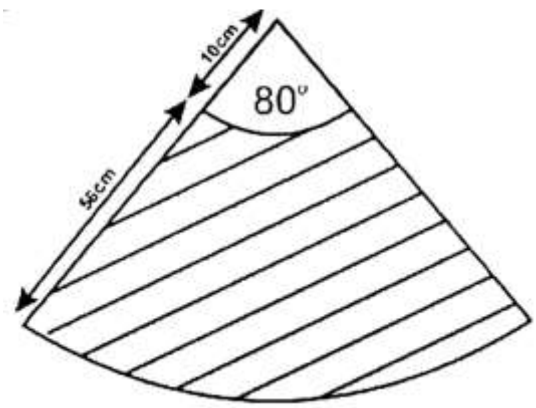
$$Area\ of\ sector = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2} \times (7m)^2 \times \frac{\pi}{9}$$

$$= \frac{49\pi}{18} m^2$$

$$= 8.55m^2$$

Question No.11 Sehar is making skirt. Each panel of this skirt is of the shape shown shaded in the diagram. How much material (cloth) is required for each panel?



Solution:

Central angle $= \theta = 80^0$

$$= 80\ \frac{\pi}{180} radian$$

$$= \frac{4\pi}{9}\ radian$$

Radius of bigger sector $= R = (65 + 10)cm$

$$R = 66cm$$

Radius of smaller sector$= r = 10cm$

$$Shaded\ area = ?$$

$$Area\ of\ bigger\ sector = \frac{1}{2}R^2\theta$$

$$= \frac{1}{2} \times (66cm)^2 \times \frac{4\pi}{9}$$

$$= 4356cm^2 \times \frac{2\pi}{9}$$

$$968\pi cm^2$$

Area of smaller $sector$ $= \frac{1}{2}\ r^2\theta$

$$=\frac{1}{2}r^2\theta$$
$$=\frac{1}{2}(10cm)^2\times\frac{4\pi}{9}$$
$$=\frac{200}{9}\pi cm^2$$
Shaded area $968\pi-\frac{200}{9}\pi$
$$=\frac{8712\pi-200\pi}{9}$$
$$=\frac{8512}{9}\pi cm^2$$
$$=2971.25cm^2$$

Question No.12 Find the area of a sector with central angle of $\frac{\pi}{5}$ radian in a circle of radius 10cm.

Solution:

Area of sector =?
$$Central\ angle=\theta=\frac{\pi}{5}\ radian$$
$$Radius=r=10cm$$
Area of sector $=\frac{1}{2}r^2\theta$
$$=\frac{1}{2}(10cm)^2\times\frac{\pi}{5}$$
$$=\frac{1}{10}\times 100cm^2\times\pi$$
$$=\frac{1}{10}\times 100cm^2\times\pi$$
$$=10\pi cm^2$$
$$=31.43\ cm^2$$

Question No.13 The area of sector with central angle $\boldsymbol{\theta}$ in circle of radius 2m is 10 square meter. Find $\boldsymbol{\theta\ in\ radius}$

Solution:

Area of sector $=10m^2$

Radius$=r=2m$

Central angle$=\theta=?$

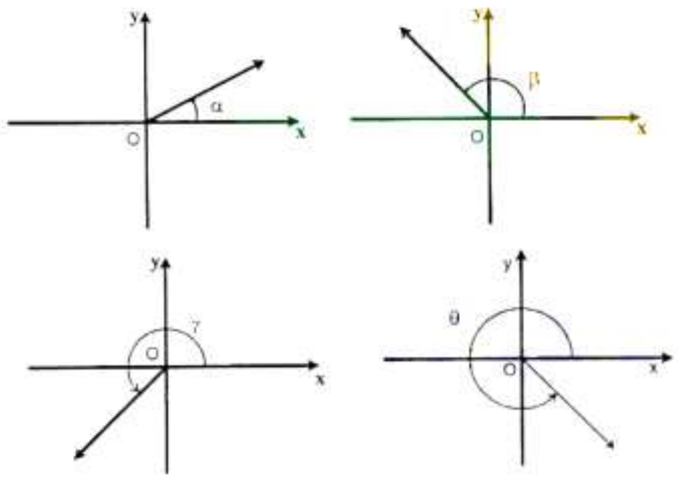
As $area\ of\ sector=\frac{1}{2}r^2\theta$
$$10m^2=\frac{1}{2}(2m)^2\theta$$
$$10m^2=\frac{1}{2}(4m^2)\theta$$
$$10m^2=20m^2$$
$$\theta=\frac{10m^2}{2m^2}$$
$$\theta=5\ radian$$

Angle in Standard position:

A general angle is said to be in standard position if its vertex is at the origin and its initial side is directed along the positive direction of the $x-axis$ of a rectangular coordinates system.

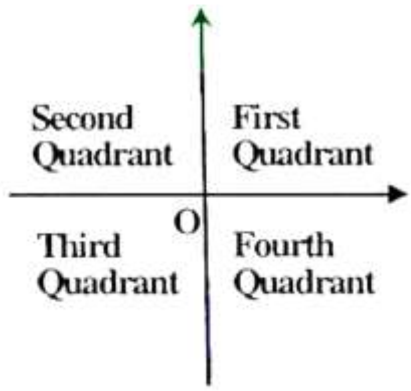
The position of the terminal side of a angle in standard position remains the same if measure of an angle is increased or decreased by a multiple of 2π

Some Standard angles are shown in the following figures.



The Quadrants and Quadrantal Angles:

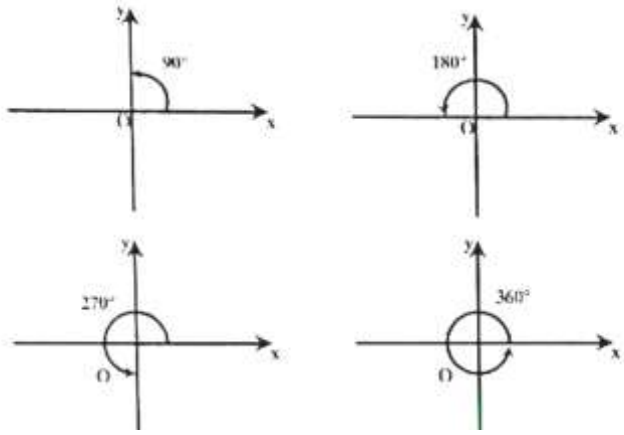
The $x-axis$ and $y-axis$ divides the plane in four regions. Called quadrants, When they intersect each other at right angle. The point of intersection is called origin and is denoted by O.



- Angle between O^o and 90^o are in the first quadrant.
- Angle between 90^o and 180^o are in the second quadrant.
- Angle between 180^o and 270^o are in the third quadrant.
- Angle between 270^o and 360^o are in the fourth quadrant.
- An angle in standard position is said to lies in that quadrant if its terminal side lies in that quadrant. Angles $\alpha,\beta,\gamma\ and\ \theta$ lie in $I, II\ , III\ and\ IV$ quadrant respectively.

Quadrantal Angles:

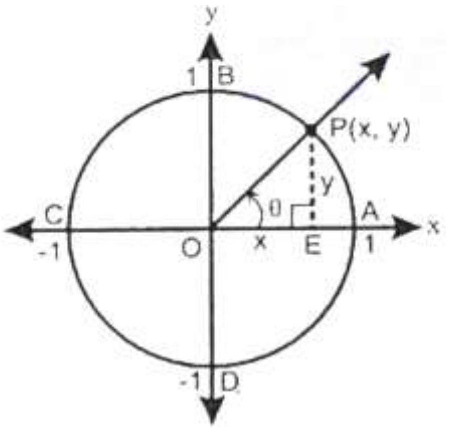
If the terminal side of an angle in standard position falls on $x-axis$ or $y-axis$ then it is called a quadrantal angles. The quadrantal angles are shown as below.'



Trigonometric Ratios and their reciprocals with the help of a unit circle:

There are six fundamental trigonometric ratios called sine, cosine, tangent, cotangent ,secant and cosecant. To define these functions we uses circular approaches which involves the unit circle.

Let θ be real number, which represents the radian measure of an angle in standard position. Let $P(x, y)$ Be any point on the unit circle lying on terminal side of θ as shown in figure.



We define sine of θ, *written as* $sin\theta$ *and cosine of* θ written as

$$sin\theta = \frac{EP}{OP} = \frac{y}{1} = sin\theta = y$$

$$cos\theta = \frac{OE}{OP} = \frac{x}{1} \quad cos\theta = x$$

$i.e\ \cos\theta\ and\ sin\theta\ are\ the\ x$ −coordinates and y −coordinates of the point P on the unit circle. The equations $x = cos\theta$ and $y = sin\theta$ are called circular or Trigonometric functions.

The remaining trigonometric functions tangent, cotangent, sector and cosecant will be denoted by $tan\theta$, $cot\theta, sec\theta, and\ cosec\theta\ for\ any\ real\ angle\ \theta$.

- $Tan\theta = \frac{EP}{OE} = \frac{y}{x} \quad tan\theta = \frac{y}{x}(x \neq 0)$

  $As\ y = sin\theta\ \ and\ x = cos\theta\ \ tan\theta = \frac{sin\theta}{cos\theta}$
- $cot\theta = \frac{x}{y}(y \neq 0) \quad cot\theta = \frac{cos\theta}{sin\theta}$
- $sec\theta = \frac{1}{x}(x \neq 0)\ \ and\ cosec\theta = \frac{1}{y}(y \neq 0)$
- $sec\theta = \frac{1}{cos\theta}\ \ and\ cosec\theta = \frac{1}{sin\theta}$

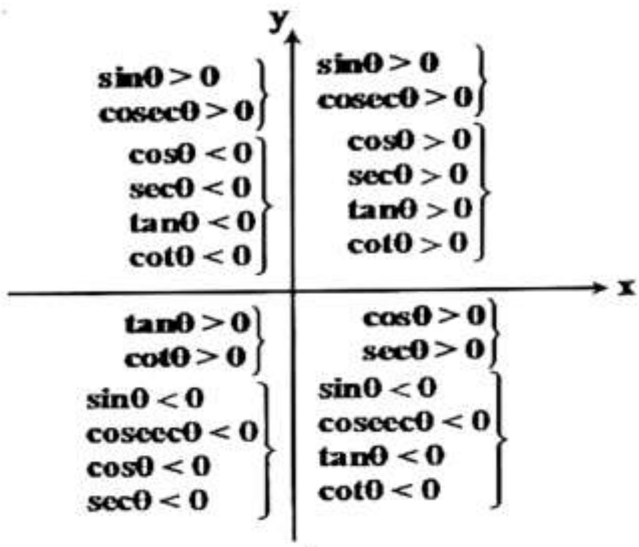
Reciprocal Identities:

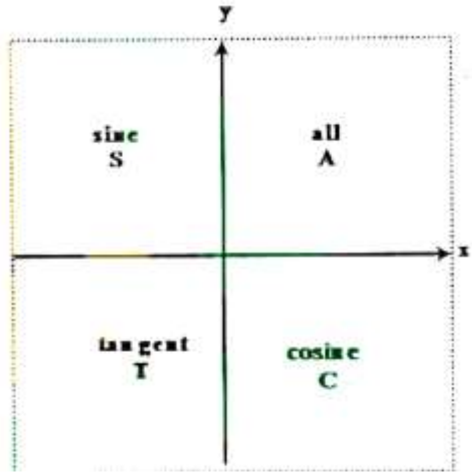
$\text{Sin}\,\theta = \frac{1}{cosec\theta}$ or $cosec\theta = \frac{1}{\sin\theta}$

$cos\theta = \frac{1}{sec\theta}$ or $sec\theta = \frac{1}{cos\theta}$

$tan\theta = \frac{1}{cot\theta}$ or $cot\theta = \frac{1}{tan\theta}$

Signs of trigonometric ratios in different Quadrants:





Allied angles:

$\sin(-\theta) = -sin\theta$	$\cos(-\theta) = cos\theta$
$\sin(90 - \theta) = cos\theta$	$\cos(90 - \theta) = sin\theta$
$\sin(90 + \theta) = cos\theta$	$\sin(90 + \theta) = -sin\theta$
$\sin(180 - \theta) = sin\theta$	$\cos(180 - \theta) = -cos\theta$
$\sin(180 + \theta) = -sin\theta$	$\cos(180 + \theta) = -cos\theta$

Exercise 7.3

Question No.1 Locate each of the following angles in standard position using a protector or fair free hand guess, also find a positive and a negative angle conterminal with each given angle:

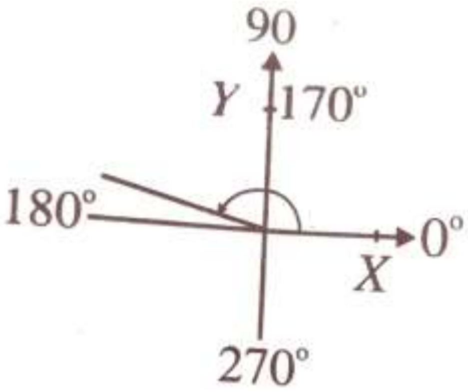
Solution:

i. 170^o

Positive coterminal angle $= 360^o + 170^o$

$= 530^o$

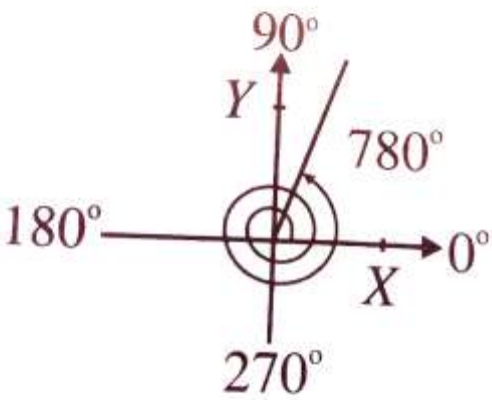
Negative coterminal angle $= -190^0$



ii. 780^0

Positive coterminal angle $780^0 + 2[360^o] = 60^0$

Negative coterminal angle $= -300^0$

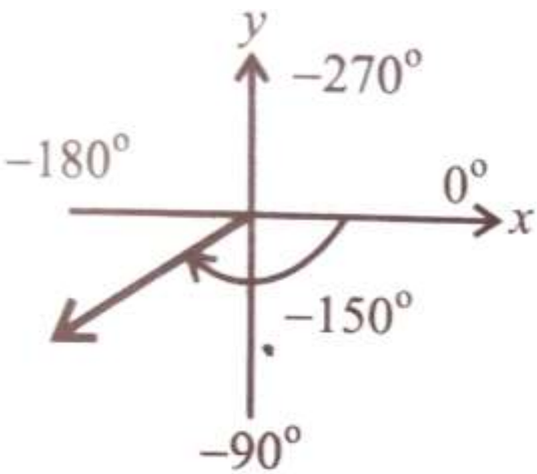


iii. -100^0

Positive coterminal angle 260^0

Negative coterminal angle $= -360^o - 100^o$

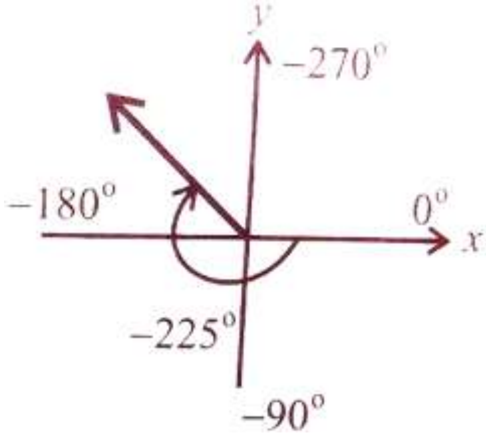
$= -460^0$



iv. -500^o

Positive coterminal angle $= 220^0$

Negative coterminal angle$= -140^o$



Question No.2 Identity closest quadrantile angles between which the following angles lie.

i. 156^0

Answer: $90^0 \quad and\ 180^0$

ii. 318^0

Answer: $270^0 \quad and\ 360^0$

iii. 572^0

Answer: $540^0 \quad and\ 630^0$

iv. -330^0

Answer: $0^o \quad and\ 90^o$

Question No.3 Write the closest quadrantal angles between which the angles lie. Write your answer in radian measure.

i. $\frac{\pi}{3}$

Answer : $o\ and\ \frac{\pi}{2}$

ii. $\frac{3\pi}{4}$

Answer: $\frac{\pi}{2} \quad and\ \pi$

iii. $-\frac{\pi}{2}$

Answer: $0 \ \ and\ -\frac{\pi}{2}$

iv. $-\frac{3\pi}{4}$

Answer : $-\frac{\pi}{2} \quad and - \pi$

Question No.4 in which quadrant θ lies, when

i. $sin\theta > 0, tan < 0$

Answer $II\ quadrant$

ii. $cos\theta < 0, sin\theta < 0$

Answer: $III\ quadrant$

iii. $sec\theta > 0, sin\theta < 0$

Answer : $IV\ quadrant$

iv. $co\theta < 0\ , tan\theta < 0$

Answer:$II\ quadrant$

v. $cosec\theta > 0, cos\theta > 0$

Answer:$I\ quadrant$

vi. $sin\theta < 0, sec\theta < 0$

Answer: $III\ quadrant$

Question No.5 Fill in the blanks:

i. $\cos(-150^0) =$ _______ $cos150^o$

ii. $\sin(-310^0) =$ _______ $sin310^o$

iii. $\tan(-210^0) =$ _______ $tan210^o$

iv. $\cot(-45^0) =$ _______ $cot45^0$

v. $\sec(-60^0) =$ _______ $sec60^o$

vi. $\text{cosec}(-137^0) =$ _______ $cosec137^o$

Answers:

i. $+ve$ ii. $-ve$ iii. $-ve$

iv. $-ve$ v. $+ve$ vi. $-ve$

Question No.6 The given point p lies on the terminal side of θ, Find quadrant of θ and all six trigonometric ratios.

i. $(-2,3)$

$we\ have\ x = -2$ and $y = 3, so\ \theta$ lies in quadrant II.

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-2)^2 + (3)^2}$$
$$= \sqrt{4+9}$$
$$= \sqrt{13}$$

Thus,

$$sin\theta = \frac{y}{r} = \frac{3}{\sqrt{13}} \qquad cosec\theta = \frac{\sqrt{13}}{3}$$
$$cos\theta = \frac{x}{r} = -\frac{2}{\sqrt{13}} \qquad sec\theta = -\frac{\sqrt{13}}{2}$$
$$tan\theta = \frac{y}{x} = -\frac{3}{2} \qquad cot\theta = -\frac{2}{3}$$

ii. $(-3,4)$

$we\ have\ x = -3$ and $y = 4, so\ \theta$ lies in quadrant III.

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-3)^2 + (4)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5$$

Thus,

$$sin\theta = \frac{y}{r} = \frac{-4}{5} \qquad cosec\theta = \frac{-5}{4}$$
$$cos\theta = \frac{x}{r} = \frac{-3}{5} \qquad sec\theta = -\frac{5}{3}$$
$$tan\theta = \frac{y}{x} = \frac{4}{3} \qquad cot\theta = \frac{3}{4}$$

iii. $(\sqrt{2}, 1)$

We have $x = \sqrt{2}$ and $y = 1$ so θ lies in quadrant II.

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{\left(\sqrt{2}\right)^2 + (1)^2}$$
$$= \sqrt{2+1}$$
$$= \sqrt{3}$$

Thus,

$$sin\theta = \frac{y}{r} = \frac{1}{\sqrt{3}} \qquad cosec\theta = \sqrt{3}$$
$$cos\theta = \frac{x}{r} = \frac{\sqrt{2}}{\sqrt{3}} \qquad sec\theta = \frac{\sqrt{3}}{\sqrt{2}}$$
$$tan\theta = \frac{y}{x} = \frac{1}{\sqrt{2}} \qquad cot\theta = \sqrt{2}$$

Question No.7 if $\cos\theta = -\frac{2}{3}$ and terminal arm of the angle θ is in quadrant II, find the valves of remaining trigonometric functions.

In any right triangles XYZ

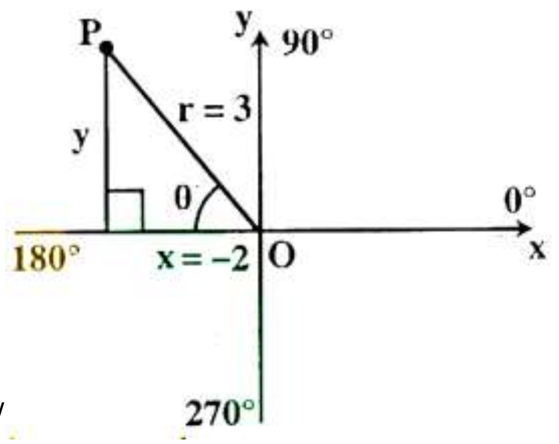
$cos\theta = -\frac{2}{3} = \frac{x}{r}\ \ then\ x = -2\ \ and\ r = 3$

Also,

$$sec\theta = \frac{1}{cos\theta} = -\frac{3}{2}$$

As we know

$$r^2 = x^2 + y^2$$
$$(3)^2 = (-2)^2 + y^2$$
$$9 = 4 + y^2$$
$$y^2 = 5$$
$$y = \pm\sqrt{5}\ \ so\ \ y = \sqrt{5}$$



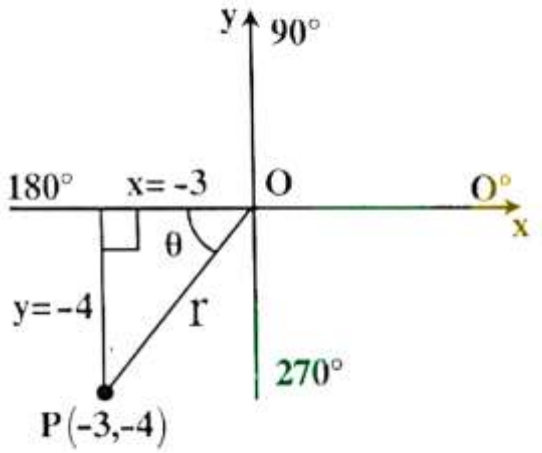
Now

$$sin\theta = \frac{y}{r} = \frac{\sqrt{5}}{3} \qquad cosec\theta = \frac{r}{y} = \frac{3}{\sqrt{5}}$$
$$cos\theta = \frac{x}{r} = \frac{-2}{3} \qquad sec\theta = \frac{r}{x} = \frac{-3}{2}$$
$$tan\theta = \frac{y}{x} = \frac{-\sqrt{5}}{2} \qquad cot\theta = \frac{-2}{\sqrt{5}}$$

Question No.8 $if\ tan\theta = \frac{4}{3}\ and\ sin\theta < 0, find\ the$ valves of other trigonometric functions at θ

Solution:

As $\tan\theta = \frac{3}{4}\ \ and\ sin\theta\ is - ve, which$ is possible in quadrant III only. We complete the figure.



From the figure $x = -3$ and $y = -4$

By Pythagorean theorem

$$r^2 = x^2 + y^2$$
$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{(-3)^2 + (-4)^2}$$
$$r = \sqrt{9 + 6}$$
$$r = \sqrt{25}$$
$$r = 5$$

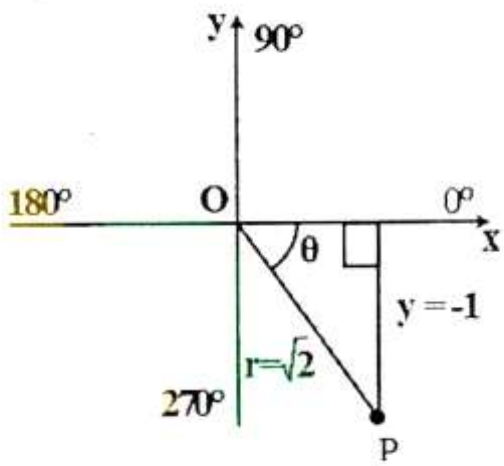
Now,

$sin\theta = \frac{y}{r} = -\frac{4}{5}$	$cosec\theta = \frac{r}{y} = \frac{-5}{4}$
$cos\theta = \frac{x}{r} = \frac{-3}{5}$	$sec\theta = \frac{r}{x} = \frac{-5}{3}$
$tan\theta = \frac{y}{x} = \frac{4}{3}$	$cot\theta = \frac{3}{4}$

Question No. 9 $\boldsymbol{if\ sin\theta = -\frac{1}{\sqrt{2}}}$, and terminal side of the angle is not in quadrant $\boldsymbol{III}$, find the valves of $\boldsymbol{tan\theta}$, $\boldsymbol{sec\theta}$ and $\boldsymbol{cosec\theta}$.

Solution:

As $\sin = -\frac{1}{\sqrt{2}}$ $and\ terminal$ side of angle is not in III quadrant, so it lies in quadrant IV.



From the figure $y = -1$ and $r = \sqrt{2}$

By Pythagorean theorem

$$r^2 = x^2 + y^2$$
$$x^2 = r^2 - y^2$$
$$x = \sqrt{r^2 - y^2}$$
$$r = \sqrt{\left(\sqrt{2}\right)^2 - (-1)^2}$$
$$r = \sqrt{2 - 1}$$
$$r = \sqrt{1}$$
$$r = 1$$

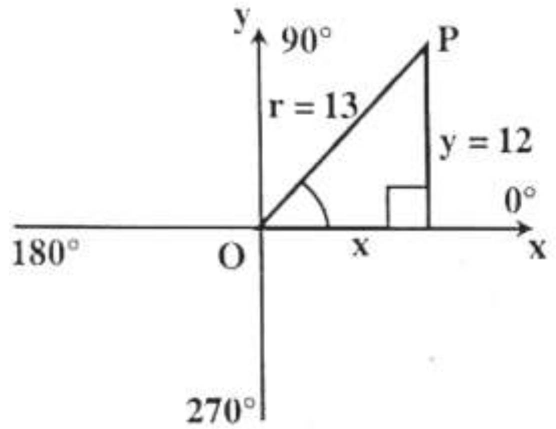
Now,

$$Tan\theta = \frac{y}{x} = -\frac{1}{1} = -1$$
$$sec\theta = \frac{r}{x} = \frac{\sqrt{2}}{1} = \sqrt{2}$$
$$cosec\theta = \frac{r}{y} = \frac{\sqrt{2}}{1} = -\sqrt{2}$$

Question No.10 If $\boldsymbol{cosec\theta = \frac{13}{12}}$ $\boldsymbol{and\ sec\theta > 0\ find}$ The remaining trigonometric functions.

Solution:

As, $cosec\theta = \frac{13}{12}$ $and\ also\ sec\theta\ is + ve$, which is only possible in quadrant I



From the figure $y = 12$ and $r = 13$

By Pythagorean theorem

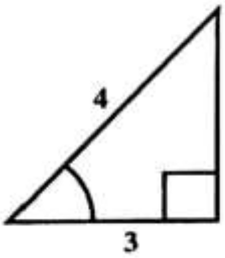
$$r^2 = x^2 + y^2$$
$$x^2 = r^2 - y^2$$
$$x = \sqrt{r^2 - y^2}$$
$$r = \sqrt{(13)^2 - (12)^2}$$
$$r = \sqrt{169 - 144}$$
$$r = \sqrt{25}$$
$$r = 5$$

Now,

$sin\theta = \frac{y}{r} = \frac{12}{13}$	$cosec\theta = \frac{r}{y} = \frac{13}{12}$
$cos\theta = \frac{x}{r} = \frac{5}{13}$	$sec\theta = \frac{r}{x} = \frac{13}{5}$
$tan\theta = \frac{y}{x} = \frac{12}{5}$	$cot\theta = \frac{5}{12}$

Question No.11 Find the valves of trigonometric functions at the indicated angles θ in the right triangles.

i.



From the figure Hypotenuse $= 4$ *and Base* $= 3$

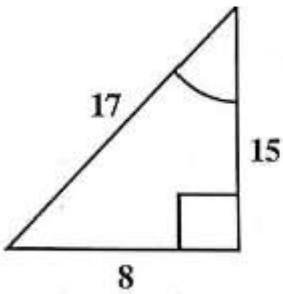
By Pythagorean theorem we can find perpendicular.

$$(Perp)^2 + (Base)^2 = (Hyp.)^2$$
$$(perp.)^2 + (3)^2 = (4)^2$$
$$(perp)^2 = 16 - 9$$
$$(perp)^2 = 7$$
$$perpendicual = \sqrt{7}$$

Now

$$sin\theta = \frac{Per.}{Hyp.} = \frac{\sqrt{7}}{4} \qquad cosec\theta = \frac{Hyp.}{Per.} = \frac{4}{\sqrt{7}}$$
$$cos\theta = \frac{Base}{Hyp.} = \frac{3}{4} \qquad sec\theta = \frac{Hyp.}{Base} = \frac{4}{3}$$
$$tan\theta = \frac{Per.}{Base} = \frac{\sqrt{7}}{3} \qquad cot\theta = \frac{Base}{Per.} = \frac{3}{\sqrt{7}}$$

ii. From the figure



$Hypertenous = 17$

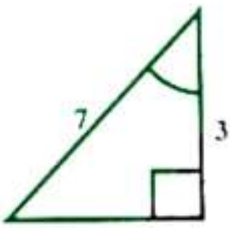
$Perperdicular = 8$

$Base = 15$

Now

$$sin\theta = \frac{Per.}{Hyp.} = \frac{8}{17} \qquad cosec\theta = \frac{Hyp.}{Per.} = \frac{17}{8}$$
$$cos\theta = \frac{Base}{Hyp.} = \frac{15}{17} \qquad sec\theta = \frac{Hyp.}{Base} = \frac{15}{17}$$
$$tan\theta = \frac{Per.}{Base} = \frac{8}{15} \qquad cot\theta = \frac{Base}{Per.} = \frac{15}{8}$$

iii. From the figure



$$hypotenous = 7 \;\; Base = 3$$

we can find perpendicular by Pythagorean theorem.

$$(Base)^2 + (PerP)^2 = (Hyp.)^2$$
$$(Perp.)^2 + (3)^2 = (7)^2$$
$$(perp.)^2 = 40 - 9$$
$$(perp.)^2 = 40$$
$$Perp. = \sqrt{40}$$
$$Perp. = \sqrt{4 \times 10}$$

Now.

$$sin\theta = \frac{Per.}{Hyp.} = \frac{2\sqrt{10}}{7} \qquad cosec\theta = \frac{Hyp.}{Per.} = \frac{2\sqrt{10}}{\sqrt{7}}$$
$$cos\theta = \frac{Base}{Hyp.} = \frac{3}{7} \qquad sec\theta = \frac{Hyp.}{Base} = \frac{7}{3}$$
$$tan\theta = \frac{Per.}{Base} = \frac{2\sqrt{10}}{3} \qquad cot\theta = \frac{Base}{Per.} = \frac{3}{2\sqrt{10}}$$

Question No.12 Find the value of the trigonometric functions. Do not use trigonometric table or calculator.

Solution:

we know that $2k\pi + \theta = \theta, where\ k \in Z$

i. $tan30^o$

$$30^o = 30\ \frac{\pi}{180}\ radian = \frac{\pi}{6} radian$$
$$tan30^o = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

ii. $tan330^o$

$$tan330^o = \tan(360^o - 30^o)$$
$$= tan2\pi - \frac{\pi}{6}$$
$$= \tan\left(-\frac{\pi}{6}\right)$$
$$= -\tan\frac{\pi}{6}$$
$$= -\frac{1}{\sqrt{3}}$$

iii. $sec\ 330^0$

$$sec330^o = \sec(360^0 - 30^0)$$
$$= \sec 2\pi - \frac{\pi}{6}$$
$$= sec - \frac{\pi}{6}$$
$$= \sec\frac{\pi}{6}$$
$$= \frac{2}{\sqrt{3}}$$

iv. $\cot\frac{\pi}{4}$

$$= \frac{1}{\tan\frac{\pi}{4}}$$
$$= \frac{1}{tan45^o} = \frac{1}{1} = 1$$

v. $\cos\frac{2\pi}{3}$

$$\cos(120^0) = -\frac{1}{2}$$

vi. $cosec\frac{2\pi}{3}$

$$cosec\frac{2\pi}{3} = cosec120^0 = \frac{1}{\sin(120^0)} = \frac{1}{\frac{\sqrt{3}}{2}}$$
$$= \frac{2}{\sqrt{3}}$$

vii. $\cos(-450^0)$

$$\cos(-450^0) = \cos(-360^0 - 90^0)$$
$$cos - 2\pi - \frac{\pi}{2}$$
$$= cos2(-1)\pi - \frac{\pi}{2}$$
$$cos\frac{\pi}{2} = 0$$

viii. $\tan(-9\pi)$

$$\tan(-9\pi) = \tan(-8\pi - \pi)$$
$$= \tan[2(-4)\pi - \pi]$$
$$= \tan\big(-8\pi + (-\pi)\big)$$
$$= \tan(-\pi)$$
$$= 0$$

ix. $cos\left(-\frac{5\pi}{6}\right)$

$$= \cos\left(-\frac{5\pi}{6}\right)$$
$$= -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

x. $\sin\frac{7\pi}{6}$

$$\sin\frac{7\pi}{6} = \sin\left(2\pi - \frac{5\pi}{6}\right)$$
$$= \sin\left[2\pi + \left(-\frac{5\pi}{6}\right)\right]$$
$$= \sin\left(-\frac{5\pi}{6}\right) = \sin(-150^0) = -\frac{1}{2}$$

xi. $\cot\frac{7\pi}{6}$

$$\cot\frac{7\pi}{6} = \cot\left[2\pi + \left(-\frac{5\pi}{6}\right)\right]$$
$$= \cot\left(-\frac{5\pi}{6}\right)$$
$$= \frac{1}{\tan\left(-\frac{5\pi}{6}\right)} = \frac{1}{\tan(-150^0)} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$

xii. $\cos 225^0$

$$\cos(225^0) = \cos(180^0 + 45^0)$$
$$= \cos\pi + \frac{\pi}{4}$$
$$= -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

Things to know:

$$\cos^2\theta + \sin^2\theta = 1$$
$$1 + \tan^0\theta = \sec^2\theta$$
$$1 + \cot^2\theta = cosec^2\theta$$

In problem $1 - 6$ simplify each expression to a single trigonometric functions.

1. $\frac{\sin^2 x}{\cos^2 x}$

Solution:

$$\because \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$

2. $tanxsinxsecx$

Solution:

$$tanxsinxsecx = tanxsinx\left(\frac{1}{cosx}\right)$$
$$= \frac{sinx}{cosx}\sin x\frac{1}{cosx}$$
$$= \frac{\sin^2 x}{\cos^2 x}$$
$$= \tan^2 x$$

3. $\frac{tanx}{secx}$

Solution:

$$\frac{tanx}{secx} = \frac{\frac{sinx}{cosx}}{\frac{1}{\cos x}} = \frac{sinx}{cosx} \times \frac{cosx}{1} = sinx$$

4. $1 - \cos^2 x$

Solution:

$$1 - \cos^2 x = \cos^2 x + \sin^2 x - \cos^2 x = \sin^2 x$$

5. $\sec^2 x - 1$

Solution:

$$\sec^2 x - 1 = \sec^2 x - (\sec^2 - \tan^2 x)$$
$$= \sec^2 x - \sec^2 x + \tan^2 x$$
$$= \tan^2 x$$

6. $\sin^2 x\,.\cot^2 x$

Solution:

$$\sin^2 x\,.\cot^2 x = \sin^2 x\,.\frac{\cos^2 x}{\sin^2 x}$$
$$= \cos^2 x$$

in problem 7-24 verify the identities.

7. $(1 - sin\theta)(1 + sin\theta) = \theta$

Solution:

$$\boldsymbol{L.H.S = (1 - sin\theta)(1 + sin\theta)}$$
$$= 1 - \sin^2\theta$$
$$= \cos^2\theta$$
$$= R.H.S$$

8. $\frac{sin\theta + cos\theta}{cos\theta} = 1 + tan\theta$

Solution:

$$L.H.S = \frac{sin\theta + cos\theta}{cos\theta}$$
$$= \frac{sin\theta}{cos\theta} + \frac{cos\theta}{cos\theta}$$
$$= tan\theta + 1$$
$$= R.H.S$$

9. $(tan\theta + cot\theta)tan\theta = \sec^2\theta$

Solution:

$$L.H.S = (tan\theta + cot\theta)tan\theta$$
$$= \left(\frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}\right)\frac{sin\theta}{cos\theta}$$
$$= \left(\frac{\sin^2\theta + \cos^2\theta}{sin\theta cos\theta}\right)\frac{sin\theta}{cos\theta}$$
$$= \left(\frac{1}{sin\theta cos\theta}\right)\frac{sin\theta}{cos\theta}$$
$$= \frac{1}{\cos^2\theta}$$
$$= \sec^2\theta$$

10. $(cot\theta + cosec\theta)(tan\theta - sin\theta) = sec\theta - cos\theta$

Solution:

$$L.H.S = (cot\theta + cosec\theta)(tan\theta - sin\theta)$$
$$= \left(\frac{cos\theta}{sin\theta} + \frac{1}{sin\theta}\right)\left(\frac{sin\theta}{cos\theta} - sin\theta\right)$$
$$= \left(\frac{cos\theta + 1}{sin\theta}\right)\left(\frac{sin\theta - sin\theta cos\theta}{cos\theta}\right)$$
$$= \left(\frac{1 + cos\theta}{sin\theta}\right)\left(\frac{sin\theta(1 - cos\theta)}{cos\theta}\right)$$
$$= (1 + cos\theta)\frac{(1 - \cos\theta)}{cos\theta}$$
$$= \frac{1 - \cos^2\theta}{cos\theta}$$
$$= \frac{1}{cos\theta} - \frac{\cos^2\theta}{cos\theta}$$
$$= sec\theta - cos\theta$$
$$= R.H.S$$

11. $\frac{sin\theta + cos\theta}{\tan^2\theta - 1} = \frac{\cos^2\theta}{sin\theta - cos\theta}$

Solution:

$$L.H.S = \frac{sin\theta + cos\theta}{\tan^2\theta - 1}$$
$$= \frac{sin\theta + cos\theta}{\frac{\sin^2\theta}{\cos^2\theta} - 1}$$
$$= \frac{sin\theta + cos\theta}{\frac{\sin^2\theta - \cos^2\theta}{\cos^2\theta}}$$
$$= \frac{sin\theta + cos\theta}{\sin^2\theta - \cos^2\theta} \times \cos^2\theta$$
$$= \frac{sin\theta + cos\theta}{(sin\theta + cos\theta)(sin\theta - cos\theta)} \times \cos^2\theta$$
$$= \frac{1}{sin\theta - cos\theta} \times \cos^2\theta$$
$$= \frac{\cos^2\theta}{sin\theta - \cos\theta}$$
$$= R.H.S$$

12. $\frac{\cos^2\theta}{sin\theta} + sin\theta = cosec\theta$

Solution:

$$L.H.S = \frac{\cos^2\theta}{sin\theta} + sin\theta$$
$$= \frac{\cos^2\theta + \sin^2\theta}{sin\theta}$$
$$= \frac{1}{sin\theta} = cosec\theta$$

13. $sec\theta - cos\theta = tan\theta sin\theta$

Solution:

$$L.H.S = sec\theta - cos\theta$$
$$= \frac{1}{cos\theta} - cos\theta$$
$$= \frac{1 - \cos^2\theta}{cos\theta}$$
$$= \frac{\sin^2\theta}{cos\theta}$$
$$= \frac{sin\theta}{cos\theta} \times sin\theta$$
$$= tan\theta sin\theta$$

14. $\frac{\sin^2\theta}{cos\theta} + cos\theta = sec\theta$

Solution:

$$L.H.S = \frac{\sin^2\theta}{cos\theta} + cos\theta$$
$$= \frac{\sin^2\theta + \cos^2\theta}{cos\theta}$$
$$= \frac{1}{cos\theta}$$
$$= sec\theta$$

15. $tan\theta + cot\theta = sec\theta cosec\theta$

Solution:

$$L.H.S = tan\theta + cot\theta$$
$$= \frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{cos\theta sin\theta}$$
$$= \frac{1}{sin\theta cos\theta}$$
$$= \frac{1}{sin\theta} \times \frac{1}{\cos\theta}$$
$$= sec\theta cosec\theta$$

16. $(tan\theta + cot\theta)(cos\theta + sin\theta) = sec\theta + cosec\theta$

Solution:

$$L.H.S = (tan\theta + cot\theta)(cos\theta + sin\theta)$$
$$= \left(\frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}\right)(cos\theta + sin\theta)$$
$$= \left(\frac{\sin^2\theta + \cos^2\theta}{cos\theta sin\theta}\right)(cos\theta + sin\theta)$$
$$= \left(\frac{1}{cos\theta sin\theta}\right)(cos\theta + sin\theta)$$
$$= \frac{cos\theta}{cos\theta sin\theta} + \frac{sin\theta}{cos\theta sin\theta}$$
$$= \frac{1}{sin\theta} + \frac{1}{cos\theta}$$
$$= cosec\theta + sec\theta$$
$$R.H.S$$

17. $sin\theta(tan\theta + cot\theta) = sec\theta$

Solution:

$$L.H.S = sin\theta(tan\theta + cot\theta)$$
$$= \sin\theta\left(\frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}\right)$$
$$= \left(\frac{\sin^2\theta + \cos^2\theta}{cos\theta sin\theta}\right)sin\theta$$

$$= \left(\frac{1}{cos\theta sin\theta}\right) sin\theta$$
$$= \frac{1}{cos\theta}$$
$$= sec\theta$$

18. $\frac{1+cos\theta}{sin\theta} + \frac{sin\theta}{1+cos\theta} = 2cosec\theta$

Solution:

$$L.H.S = \frac{1+cos\theta}{sin\theta} + \frac{sin\theta}{1+cos\theta}$$
$$= \frac{(1+cos\theta)^2 + \sin^2\theta}{sin\theta(1+cos\theta)}$$
$$= \frac{(1+2cos\theta+\cos^2\theta+\sin^2\theta)}{\sin\theta(1+cos\theta)}$$
$$= \frac{1+2cos\theta+1}{sin\theta}$$
$$= \frac{2+2cos\theta}{sin\theta(1+cos\theta)}$$
$$= \frac{2(1+cos\theta)}{sin\theta(1+cos\theta)}$$
$$= \frac{2}{sin\theta}$$
$$= 2cosec\theta$$
$$= R.H.S$$

19. $\frac{1}{1-cos\theta} + \frac{1}{1+cos\theta} = 2cosec^2\theta$

Solution:

$$L.H.S = \frac{1}{1-cos\theta} + \frac{1}{1+cos\theta}$$
$$= \frac{1+cos\theta+1-cos\theta}{(1-cos\theta)(1+cos\theta)}$$
$$= \frac{2}{1-\cos^2\theta}$$
$$= \frac{2}{\sin^2\theta}$$
$$= 2cosec^2\theta$$
$$= R.H.S$$

20. $\frac{1+sin\theta}{1-sin\theta} - \frac{1-sin\theta}{1+sin\theta} = 4tan\theta sec\theta$

Solution:

$$L.H.S = \frac{1+sin\theta}{1-sin\theta} - \frac{1-sin\theta}{1+sin\theta}$$
$$= \frac{(1+\sin\theta)^2 - (1-sin\theta)^2}{(1-sin\theta)(1+sin\theta)}$$
$$= \frac{1+2sin\theta+\sin^2\theta-1+2sin\theta-\sin^2\theta}{1-\sin^2\theta}$$
$$= \frac{4sin\theta}{\cos^2\theta}$$
$$= \frac{4sin\theta}{\cos\theta} \times \frac{1}{cos\theta}$$
$$= 4tan\theta sec\theta$$

21. $\sin^3\theta = sin\theta - sin\theta\cos^2\theta$

Solution:

$$R.H.S = sin\theta - sin\theta\cos^2\theta$$
$$= sin\theta(1-\cos^2\theta)$$
$$= sin\theta(\sin^2\theta)$$
$$= \sin^3\theta$$
$$= L.H.S$$

22. $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

Solution:

$$L.H.S = \cos^4\theta - \sin^4\theta$$
$$= (\cos^2\theta)^2 - (\sin^2\theta)^2$$
$$= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$$
$$= (\cos^2\theta - \sin^2\theta)(1)$$
$$= R.H.S$$

23. $\sqrt{\frac{1+cos\theta}{1-sin\theta}} = \frac{sin\theta}{1-cos\theta}$

$$L.H.S = \sqrt{\frac{1+cos\theta}{1-sin\theta}}$$
$$= \sqrt{\frac{(1+cos\theta)(1-cos\theta)}{(1-cos\theta)(1-cos\theta)}}$$
$$= \sqrt{\frac{1-\cos^2\theta}{(1-cos\theta)^2}}$$
$$= \sqrt{\frac{\sin^2\theta}{(1-cos\theta)^2}}$$
$$= \frac{sin\theta}{1-cos}$$
$$= R.H.S$$

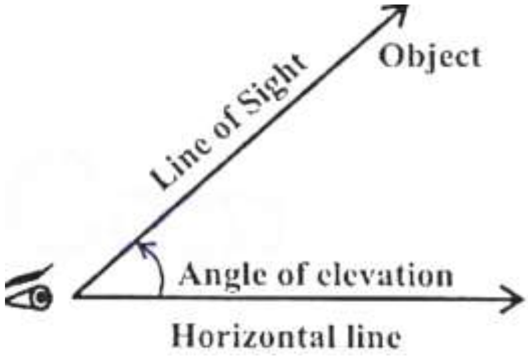
24. $\sqrt{\frac{sec\theta+1}{sec\theta-1}} = \frac{sec\theta+1}{tan\theta}$

Solution:

$$= \sqrt{\frac{sec\theta+1}{sec\theta-1}}$$
$$= \sqrt{\frac{(sec\theta+1)(sec\theta+1)}{(sec\theta-1)(sec\theta+1)}}$$
$$= \sqrt{\frac{(sec\theta+1)^2}{\sec^2\theta-1}}$$
$$= \sqrt{\frac{(sec\theta+1)^2}{\tan^2\theta}}$$
$$R.H.S$$

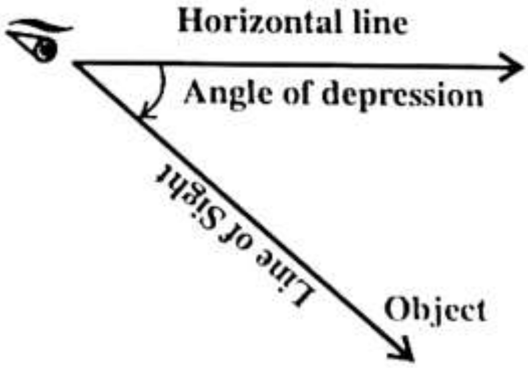
Angle of Elevation:

The angle between the horizontal line through eye and a lie from eye to the object above the horizontal line is called angle of elevation.



Angle of depression:

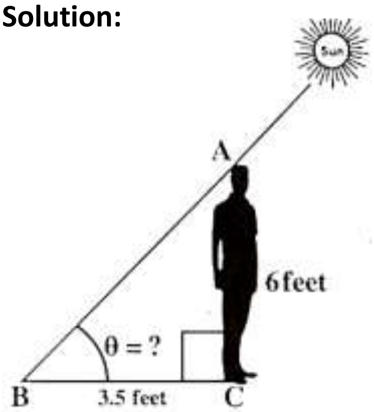
The angle between the horizontal line through eye and a line from eye to the object below the horizontal line is called angle of depression.



Exercise 7.5

Question No.1 Find the angle of elevation of the sun if a 6feet man casts a 3.5 feet shadow.

Solution:



From figure we have

$$tan\theta = \frac{AB}{BC}$$

$$tan\theta = \frac{6}{3.5}$$

$$tan\theta = 1.714$$

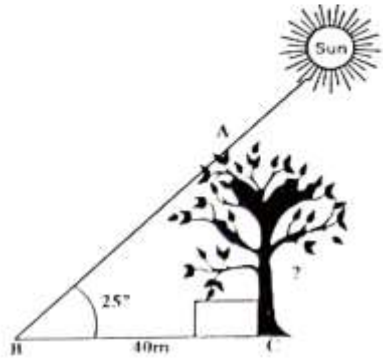
$$\theta = \tan^{-1}(1.7143)$$

$$\theta = 59.7437^0$$

$$\theta = 59^0 44' 37''$$

Question No.2 A true casts a 40 meter shadow when the angle of elevation of the sun is 25^0. Find the height of the tree.

Solution:



From the figure

Height of tree $= m\overline{AC} = ?$

Length of shadow$= m\overline{BC} = 40m$

Angle of Elevation $= \theta = 25^0$

Angle of fact that

$$tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$

$$tan\theta = \frac{m\overline{AC}}{40}$$

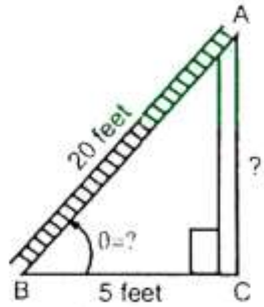
$$m\overline{AC} = 40 \times tan25^0$$

$$m\overline{AC} = 18.652m$$

So, height of tree is $18.652\ m$

Question No.3. A feet long ladder is learning against a wall. The bottom of the wall. Find the acute angle (angle of elevation) the ladder makes with the ground.

Solution:



from the figure

Length of ladder $= m\,\overline{AB} = 20feet$

Distance of ladder from the wall$= m\,\overline{BC} = 5\ feet$

$$Angle\ of\ elevation = \theta = ?$$

Using the fact that

$$cos\theta = \frac{m\,\overline{BC}}{m\,\overline{AB}}$$

$$cos\theta = \frac{5ft.}{20ft.}$$

$$cos\theta = 0.25$$

$$\theta = \cos^{-1} 0.25$$

$$\theta = 75.5225$$

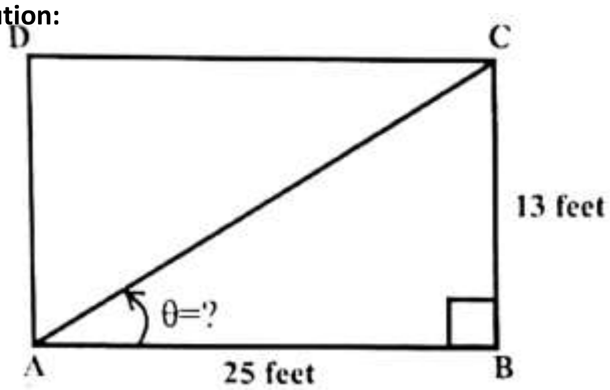
$$\theta = 75.5^0$$

$$or \quad \theta = 75^0 30'$$

So, angle of elevation is $75^0 30'$

Question No.4 The base of rectangular is 25 feet and the height of rectangular is 13 feet. Find the angle that diagram of the rectangular makes with the base.

Solution:



From the figure

Base of rectangular $= m\overline{BC} = 25feet$

Height of rectangular $= m\overline{BC} = 13feet$

Diagonal $\overline{AC}$ *is taken*

Angle between diagonal and base= θ

Using the fact that

$$tan\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$tan\theta = \frac{13}{25}$$

$$\theta = \tan^{-1}\frac{13}{25}$$

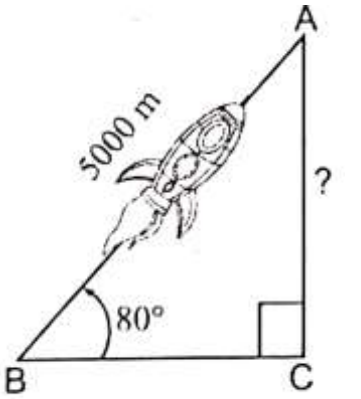
$$\theta = 27.4744$$

$$\theta = 27.47^0$$

So, angle between diagonal and base is 27.47^0

Question No.5 A rocket is launched and climbs at a constant angle of 80^0. Find the altitude of the rocket after it travels 5000 meter.

Solution:



From the figure

Distance travelled by rocket $= m\overline{AB} = 5000m$

$$Altitude\ of\ rocket = m\overline{AC} = ?$$

Angle of elevation $= \theta = 80^0$

Using

$$sin\theta = \frac{m\overline{AC}}{m\overline{AB}}$$

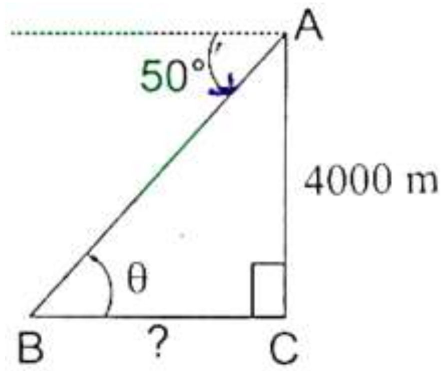
$$sin80^0 = \frac{m\overline{AC}}{5000}$$

$$m\overline{AC} = 5000 \times sin80^0$$

$$m\overline{AC} = 4924.04m$$

Question No.6 An aero plane pilot flying at an altitude of 4000m wishes to make an approaches to an airport at an angle of 50^0 *with the* plane be when the pilot begins to descend?

Solution:



From the figure

Altitude of aero plane= $m\overline{AC} = 4000m$

Distance of plane from airport= $m\overline{BC} = ?$

Angle of depression= 50^0

As the altimeter angles of parallel lines are equal, so angle

$$\theta = 50^0$$

Using the fact that

$$tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$

$$tan50^0 = \frac{4000m}{mBC}$$

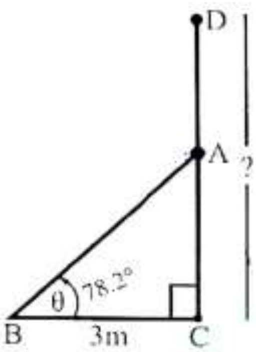
$$m\overline{BC} = \frac{4000m}{tan50^0}$$

$m\overline{BC} = 33356.4$m

So, the distance of aero plane from airport is 3356.4m

Question No.7 A guy wire (supporting wire) runs from the middle of a utility pole to ground. The wire makes an angle of 78.2^0 *with the ground* and touch the ground 3 meters from the base of the pole. Find the height of the pole.

Solution:



From the figure

Height of pole= $m\overline{CD} = ?$

Distance of wire from the base of the pole

$$= m\overline{BC} = 3m$$

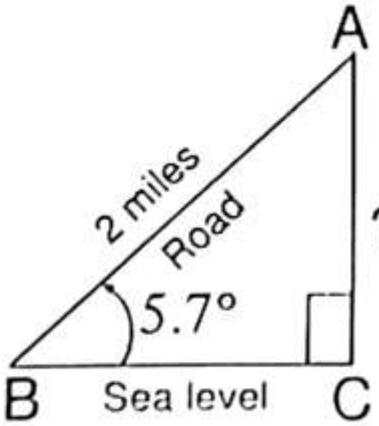
Angle of elevation= $\theta = 78.2^0$

As the wire is attached with the pole at its middle point A so first we find $m\overline{AC}$

Using the fact that

$$\tan\theta = \frac{m\overline{AC}}{m\overline{Bc}}$$
$$\tan 78.2 = \frac{m\overline{AC}}{3}$$
$$m\overline{AC} = 3m \times \tan 78.2^0$$
$$m\overline{AC} = 14.36m$$
So, Height of pole is $= m\overline{DC} = 2(m\overline{AC})$
$$= 2 \times 14.36m$$
$$= 28.72m$$
Question No.8 A road is inclined at an angle 5.7^0 suppose that we drive 2 miles up this road starting from sea level. How high above sea level are we?

Solution:



From the figure

Distance covered on road $= m\overline{AB} = 2miles$

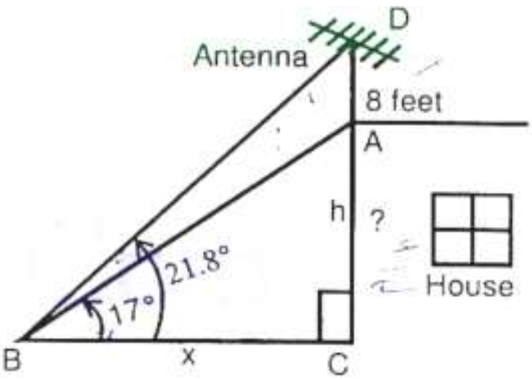
Angle of inclination $= \boldsymbol{\theta = 5.7^0}$

Height from sea level $= m\overline{AC} = ?$

Using the fact that,
$$\sin\theta = \frac{m\overline{AC}}{m\overline{AB}}$$
$$\boldsymbol{\sin 5.7^0} = \frac{m\overline{AC}}{\mathbf{2}}$$
$$m\overline{AC} = 2 \times \sin 5.7^0$$
$$m\overline{AC} = 0.199mile$$
Hence , we are at height of 0.199 mile from the sea level.

Question No.9 A television antenna of 8 feet height is point on the top of a house. From a point on the ground the angle of elevations to the top of the house is 17^0 And the angle of elevation to the top of antenna is 21.8^0. find the height of the house.

Solution:



From the figure

Distance of point from house $m\overline{BC} = x$

$\boldsymbol{Height\ of\ house} = m\overline{AC} = h = ?$

Height of antenna $= m\overline{AD} = 8feet$

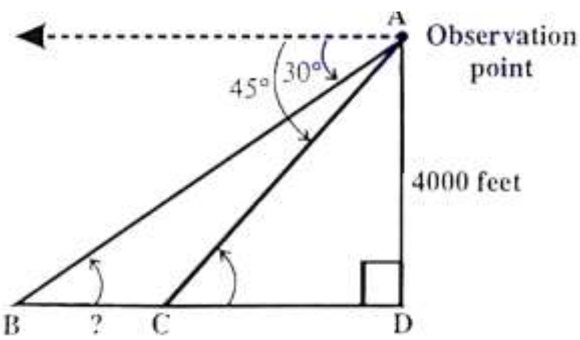
Angle of elevation of top of house $= 17^0$

Angle of elevation of top of antenna$= 21.8^0$

In right angled ΔABC
$$\tan 17^0 = \frac{m\overline{AC}}{m\overline{BC}}$$
$$\tan 17^0 = \frac{h}{x}$$
$$x = \frac{1}{\tan 17^0} \times h$$
$x = 3.271 \times h \rightarrow (i)$

Now in right angle ΔDBC
$$\tan 21.8 = \frac{m\overline{CD}}{m\overline{BC}}$$
$$\tan 21.8 = \frac{m\overline{AD} + m\overline{AC}}{m\overline{BC}}$$
$$\tan 21.8 = \frac{8+h}{x}$$
$$0.40 = \frac{8+h}{3.271h} \quad from\ (i)$$
$$0.40 \times 3.271h = 8 + h$$
$$1.3084h - h = 8$$
$$(1.3084 - 1)h = 8$$
$$0.3084h = 8$$
$$h = \frac{8}{0.3084}$$
$$h = \frac{8}{0.3084} = 25.94feet$$
question No.10 from an observation point, the angles of depression of two boats in line with this point are found to 30^0 *and* 45^0 find the distance between the two bosts if the point of observation is 4000 feet high.

Solution:



From the figure

Height of observation point $= m\overline{AD} = 4000feet$

Distance between boats $= m\overline{BC} = ?$

Angle of depression of points $B\ and\ C\ are\ 30^0\ and\ 45^0$ respectively from point A.

As the alternate angles of parallel lines are equal, so
$$m\angle B = 30^0 \quad and\ m\angle C = 45^0$$
Now in right angled ΔACD
$$\tan 45^0 = \frac{m\overline{AD}}{m\overline{CD}}$$

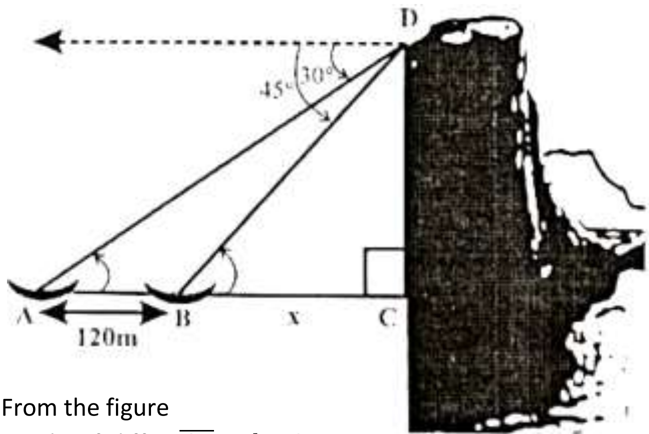
$$1 = \frac{4000}{m\overline{CD}}$$
$$m\overline{CD} = 4000feet$$
Now in right angled ΔACD
$$tan30^0 = \frac{m\overline{AD}}{m\overline{BD}}$$
$$\frac{1}{\sqrt{3}} = \frac{4000}{m\overline{BC} + m\overline{CD}}$$
$$\frac{1}{\sqrt{3}} = \frac{4000}{m\overline{BC} + 4000}$$
$$m\overline{BC} + 4000 = 4000\sqrt{3}$$
$$m\overline{BC} = 4000\sqrt{3} - 4000$$
$$m\overline{BC} = 6928.20 - 4000$$
$$m\overline{BC} = 2928.20feet$$
So, the distance between boats is 2928.2 feet.

Question No.11 Two ships, which are in lines with the base of a vertical cliff are 120 meters apart. The angles of depression from the top of the cliff to the ship are 30^0 and 45^0 as shown in the diagram.

(a) Calculate the distance BC

(b) Calculate the height CD of the cliff.

Solution:



From the figure

Height of cliff $= \overline{CD} = h = ?$

Distance$= \overline{BC} = x = ?$

Distance between boats $= \overline{AB} = 120m$

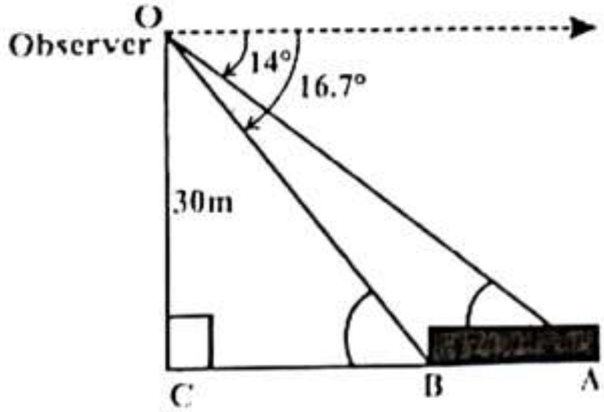
Angles of depression from point D to point A and B are 30^0 *and* 45^0 respectively.

As the altitude angles of parallel lines are equal, so $m\angle A = 30^0$ *and* $m\angle B = 45^0$

In right angled ΔBCD
$$tan45^0 = \frac{m\overline{CD}}{m\overline{BC}}$$
$$l = \frac{h}{x}$$
$$x = h \rightarrow (i)$$
Now in right angled ΔACD
$$tan30^0 = \frac{m\overline{CD}}{m\overline{AC}}$$
$$\frac{1}{\sqrt{3}} = \frac{h}{m\overline{BC} + m\overline{BC}}$$
$$\frac{1}{\sqrt{3}} = \frac{h}{120 + x}$$
$$120 + x = \sqrt{3}h$$
$$120 + h = \sqrt{3}h$$
$$120 = \sqrt{3}h - h$$
$$120 = h(\sqrt{3} - 1)$$
$$120 = h(1.7321 - 1)$$
$$120 = (0.7321h)$$
$$\frac{120}{0.7321} = h$$
$$h = 163.91m$$
As $x = h$ *, so*
$$x = 163.91m \ or\ 164m$$
Height of cliff$= m\overline{CD} = 164m$

Question No.12 Suppose that we are standing on a bridge 30 meter above a river watching a log (piece of wood) floating towards us. If the angle with the horizontal to the front of the log is 16.7^0 and angle with the horizontal to the back of the log is 14^0, *how long is the log*?

Solution:



Height of the observer position $= m\overline{OC} = 30m$

Length of log wood$= m\overline{AB} = x = ?$

Angles of depression from point O of the points A and B are 14^0 and $=16.7^0$ respectively

In right angled ΔOBC
$$tan16.7^0 = \frac{m\overline{OC}}{m\overline{BC}}$$
$$0.30 = \frac{30}{m\overline{BC}}$$
$$m\overline{BC} = \frac{30}{0.30}$$
$$m\overline{BC} = 100m$$
Now in right angled ΔOAC
$$tan14^0 = \frac{m\overline{OC}}{m\overline{AC}}$$
$$0.249 = \frac{30}{m\overline{AB} + m\overline{BC}}$$
$$0.249 = \frac{30}{(x + 100)}$$
$$0.249(x + 100) = 30$$
$$x + 100 = \frac{30}{0.249}$$
$$x + 100 = 120.482$$
$$x = 120.482 - 100$$
$$x = 20.482m$$
So the length of log is 20.48222m.