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# DEFINITIONS

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10th class Math (English med)



*by*

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Note are available at: <https://www.mathcity.org>

**Chapter #1****Quadratic equation****Quadratic equation:**

An equation that contain the square of the unknown (variable) quantity, but not higher power is called a quadratic equation or an equation of the second degree. Standard form of quadratic Equation is  $ax^2 + bx + c = 0$ .

**Pure Quadratic equation:**

If  $b=0$  in quadratic equation  $ax^2 + bx + c = 0$  then it is called a pure quadratic equation.

i.e  $ax^2 + 0x + c = 0$  e.g.  $x^2 - 16 = 0$

**Methods to solve the quadratic equation:**

To find solution set of a quadratic equation following methods are use:

- i. Factorization
- ii. completing square
- iii. Quadratic formula

**Reciprocal equation:**

An equation is said to be a reciprocal equation if it remain unchanged, when x is replace by  $\frac{1}{x}$  for example  $x + \frac{1}{x} = 2$

**Exponential equation:**

An equation in which variable occurs in exponent is called exponential equation. e.g  $2^x - 1 = 0$

**Radical equation:**

An equation involving expression of the variable under radical sign is called radical equation. For example  $\sqrt{x+1} = 2$

**Extraneous root:**

A root of an equation which do not satisfy the original equation is called extraneous root.

**Chapter# 2****Theory of quadratic equation****Discriminant:**

The nature of the roots of quadratic equation depends on the value of the expression  $b^2 - 4ac$ , which is called the discriminant of the quadratic equation  $ax^2 + bx + c = 0$ .

**Nature of the roots of a quadratic equation through discriminant:**

The roots of the quadratic equation  $ax^2 + bx + c = 0$  are  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  and its discriminant is  $b^2 - 4ac$ .

If  $b^2 - 4ac > 0$  (+ve) and is a perfect square then the roots are rational (real) and unequal.

If  $b^2 - 4ac > 0$  (+ve) and is not a perfect square then the roots are irrational (real) and unequal.

If  $b^2 - 4ac = 0$  then the roots are rational (real) and equal.

If  $b^2 - 4ac < 0$  (-ve) then the roots are imaginary (complex conjugates) and unequal.

**Sum and product of the roots:**

The sum and the product of the roots of an equation  $ax^2 + bx + c = 0$  are  $S = \alpha + \beta = -\frac{b}{a}$ ,  $P = \alpha\beta = \frac{c}{a}$  respectively.

**Symmetric functions:**

Symmetric functions are those functions in which the roots involved are such that the value of the expression involving them remain unaltered, when roots are interchanged.

**Synthetic division:**

Synthetic division is the process of finding the quotient and remainder when the polynomial is divided by a linear polynomial.

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**Simultaneous equations:**

A system of equation having a common solution is are called simultaneous equations.

**Chapter # 3****Variations****Ratio:**

A relation between two quantities of the same kind is called ratio. If  $a$  and  $b$  are two quantities of the same kind and  $b$  is not zero then ratio of  $a$  and  $b$  is written as  $a : b$  or in fraction  $\frac{a}{b}$

**Proportion:**

A proportion is a statement which is expressed as equivalence of two ratios. If two ratio  $a : b$  and  $c : d$  are equal then we can write  $a : b = c : d$

**Direct variation:** If two quantities are related in such a way that when one quantity increase the other will also increase, if one quantity will decrease the other will also decrease is called direct variation.

$$y \propto x$$

**Inverse variation:**

If two quantities are related is such a way that when one quantity increase the other decrease is called Inverse variation.

$$y \propto \frac{1}{x}$$

**Joint variation:**

A combination of direct and inverse variations of one or more than one variation forms joint variation.

If  $y \propto x$  and  $y \propto \frac{1}{z}$ , then  $y \propto \frac{x}{z}$  or  $y = k \frac{x}{z}$

**Theorem of invertendo:** if  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{b}{a} = \frac{d}{c}$

**Theorem of Alternando:** if  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{a}{c} = \frac{b}{d}$

**Theorem of componendo:** if  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{a+b}{b} = \frac{c+d}{d}$  or  $\frac{a}{a+b} = \frac{c}{c+d}$

**Theorem of dividendo:** if  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{a-b}{b} = \frac{c-d}{d}$  or  $\frac{a}{a-b} = \frac{c}{c-d}$

**Theorem of coponendo-dividendo:** if  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{a+b}{a-b} = \frac{c+d}{c-d}$  or  $\frac{a-b}{a+b} = \frac{c-d}{c+d}$

**Chapter # 4****Partial fractions****Fraction:**

The quotient of two numbers or algebraic expressions is called fraction the quotient is indicated by (-).

**Rational fraction:**

An expression of the form  $\frac{N(x)}{D(x)}$  where  $D(x) \neq 0$  and  $N(x)$  and  $D(x)$  are polynomials in  $x$  with real coefficient is called

a rational fraction. For example  $\frac{x}{x^2+1}$

**Proper Fraction:**

A rational fraction  $\frac{N(x)}{D(x)}$  where  $D(x) \neq 0$  is called a proper rational fraction if degree of the polynomial  $N(x)$ , in the numerator is less than the degree of the polynomial  $D(x)$  in the denominator for example.  $\frac{x}{x^2 + 1}$

**Improper fraction:**

A rational fraction  $\frac{N(x)}{D(x)}$  where  $D(x) \neq 0$  is called an improper fraction if the degree of the polynomial is  $N(x)$ , in the numerator is greater or equal to the degree of the polynomial  $D(x)$  in the denominator .For example  $\frac{x^2 + 1}{x + 1}$

**Partial fraction:**

Every proper fraction  $\frac{N(x)}{D(x)}$  where  $D(x) \neq 0$  can be resolved into and algebraic sum of component Fractions these components fraction of a resultant fraction are called its partial fraction.

**Identity:**

The equation that remains true for all values of variable is called Identity. e.g  $2(x + 1) = 2x + 2$

**Chapter# 5****Sets and functions****Set:**

A collection of well-defined distinct object is called set. It is denoted by capital letters  $A, B, C$  e.g  $A = \{1, 2, 3, 4, 5\}$

**Union of set:**

Union of two sets  $A$  and  $B$  denoted by  $A \cup B$  is the set containing elements which either belong to  $A$  or to  $B$  or to both.

**Intersection of set:** Intersection of two sets  $A$  and  $B$  written as  $A \cap B$  (read as  $A$  intersection  $B$ ) is the set consisting of all the common element of  $A$  and  $B$

**Difference of sets:**

The set difference of  $B$  and  $A$  denoted by  $A - B$  is the set of all those elements of  $A$  but do not belong in to  $B$

**Compliment of sets:**

If  $U$  is a universal set and  $A$  is a subset of  $U$  then the complement of  $A$  is the set of those elements of  $U$  which are not contained in  $A$  and is denoted by  $A'$  or  $A^c$ .

**DE Morgan Laws:**

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

**Venn diagram:**

British mathematician John Venn (1834-1923) introduced rectangle for a universal set  $U$  and its subsets  $A$  and  $B$  as closed figures inside this rectangle.

**Ordered pair:**

Any two numbers  $x$  and  $y$  written in the form of  $(x, y)$  is called ordered pair.

**Binary relation:**

In mathematics a binary relation on a set  $A$  is a collection of ordered pairs of elements of  $A$ . In other words, it is a subset of the Cartesian product  $A^2 = A \times A$ , more generally a binary relation between two sets  $A$  and  $B$  is a subset of  $A \times B$

**Function:**

Suppose  $A$  and  $B$  are two non-empty sets then relation  $f : A \rightarrow B$  is called a function if

1.  $\text{Dom } f = A$
2. every  $x \in A$  appears one and only one ordered pair in  $f$ .

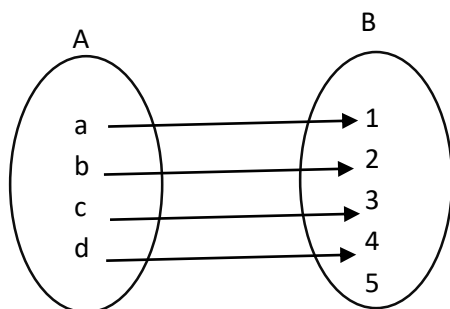
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**Domain and range of a function:**

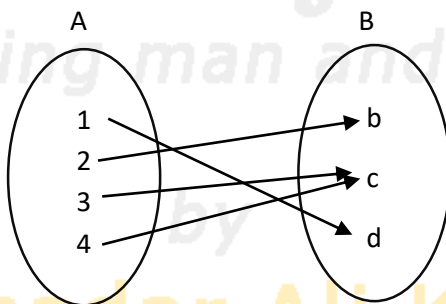
Domain  $f$  is the set consisting of all first element of each ordered pair in  $f$  and range  $f$  is the set consisting of all second elements of each ordered pair in  $f$ .

$$f = \{(0,1), (1,2), (2,3), (3,4)\} \quad \text{Dom } f = \{0,1,2,3\} \quad \text{Range } f = \{1,2,3,4\}$$

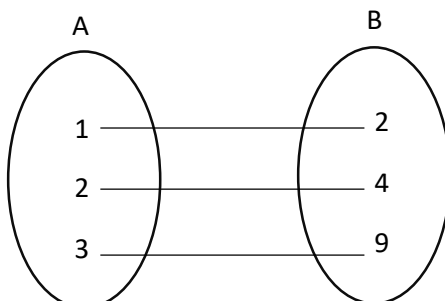
**Into function:** A function  $f : A \rightarrow B$  is called an into function, if at least one element in  $B$  is not an image of some element of set  $A$  i.e, Range of  $f \subsetneq B$

**Onto or surjective function:**

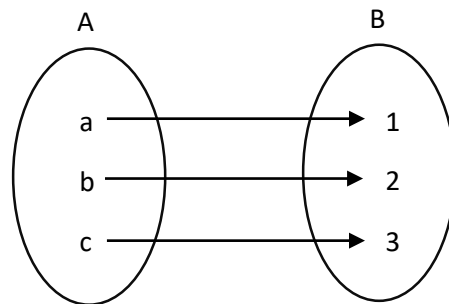
A function  $f : A \rightarrow B$  is called an onto function if every element of set  $B$  is an image of at least one element of set  $A$  i.e, Range of  $f = B$



**One—one function:** A function  $f : A \rightarrow B$  is called one—one function, if all distinct elements of  $A$  have distinct images in  $B$ , i.e



**Bijjective function:** A function  $f : A \rightarrow B$  is called bijjective function if function in one–one and onto.



**Constant function:** A function  $f : A \rightarrow A$  is called constant function if  $\forall x \in A$  there in an element  $c \in B$  such that  $f(x)=c$

**Subset:**

A & B are two sets and if the elements of set A are also the elements of set B then A is called the subset of set B and it is denoted as  $A \subseteq B$ .

$$A = \{1, 2, 3\}, B = \{1, 2, 3, 4\}$$

## Chapter#6

### Basic Statistics

**Frequency distribution:**

A frequency distribution is a tabular arrangement for classifying data into different groups.

**Frequency:**

The number of times a quantity occurs in the data is called its frequency.

**Cumulative frequency:**

The total frequency up to an upper class limit or boundary s called cumulative frequency.

**Grouped data:**

the data presented in the form of frequency distribution is called group data.

**Un-grouped data:**

When the data is collected from any source and record as it stand is called ungrouped data.

Example: the number of student collected from each class of a school 10,15,20,14,15,18

**Class Limits:**

The minimum and the maximum values defined for a class or group are called class limits.

**Mid-point or class mark:**

The average of each class is obtained by dividing the sum of lower and upper class limits by 2 is called mid-point or class mark.

**Histogram:**

A histogram is a graph of adjacent rectangle constructed on XY-plane. It is a graph of frequency distribution.

**Deviation:**

A deviation is define as a difference of any value of the variable from any constant  $D = x_i - A$

**Measure of central tendency:**

The measure or technique that are used to determine this central value are called measure of central tendency. e.g Mean, Median Mode.

**Arithmetic mean:**

Arithmetic mean or simply mean is measure by dividing the sum of all values of the variable by their number of observation. We denoted Arithmetic mean by  $\bar{X}$

$$\bar{X} = \frac{\sum x}{n} = \frac{\text{Sum of all value of observation}}{\text{no of values}}$$

**Properties of Arithmetic mean:**

- i. Mean of a variable with similar observations say constant k is the constant k itself.
- ii. Mean is affected by change in origin.
- iii. Mean is affected by change in scale.
- iv. Sum of deviations of the variable x from its mean is always zero.

**Geometric mean:**

Geometric mean of a variable X is the  $n^{\text{th}}$  positive root of the product of the  $x_1, x_2, x_3, \dots, x_n$  observations.

Mathematically,  $G.M = (x_1 \times x_2 \times x_3 \dots \times x_n)^{\frac{1}{n}}$  or  $G.M = \text{anti log} \left( \frac{\sum f \log X}{n} \right)$  (for un-grouped data)

$$G.M = \text{anti log} \left( \frac{\sum f \log X}{\sum f} \right) \text{ (for grouped data)}$$

**Harmonic mean:**

Harmonic mean refers to the value obtained by reciprocation the mean of the reciprocal of the  $x_1, x_2, x_3, \dots, x_n$  Observations. Mathematically

$$H.M = \frac{n}{\sum \frac{1}{x}} \text{ (for un-grouped data)}$$

$$H.M = \frac{\sum f}{\sum \frac{f}{x}} \text{ (for grouped data)}$$

**Mode:**

Mode is define as the most frequent occurring observation in the data.

$$\text{Mode} = L + \frac{f_m + f_1}{f_m - f_1 - f_2} \times h \text{ (for grouped data)}$$

**Median:**

Median is the middle most observation in an arranged data set. It divides the data set into two equal parts.

$$\text{Median} = L + \frac{h}{f} \left\{ \frac{n}{2} - c \right\} \text{ (for grouped data)}$$

**Dispersion:**

Dispersion means the spread or scatterness of observation in a data set.

**Measure of dispersion:**

The measure that are used to determine the degree or extent of variation in a data set are called measure of dispersion.

**Range:**

The difference between largest value and the small values is called range

OR

Range measures the extent of variation between two extreme observations of a data set. It formula is  $R = X_{\max} - X_{\min}$

**Variance:**

Variance is define as the mean of the square deviation of  $x_i$  ( $i=1,2,3,\dots,n$ ) observation from their arithmetic mean.

$$\text{Variance of } X = \text{var}(X) = S^2 = \frac{\sum (X - \bar{X})^2}{n} \quad (\text{for un-grouped data})$$

$$\text{Variance of } X = \text{var}(X) = S^2 = \frac{\sum f(X - \bar{X})^2}{\sum f} \quad (\text{for grouped data})$$

**Standard deviation:**

Standard deviation is define as the positive square root of variance.

$$\text{Standard deviation} = S = \sqrt{\frac{\sum (X - \bar{X})^2}{n}} \quad (\text{for un-grouped data})$$

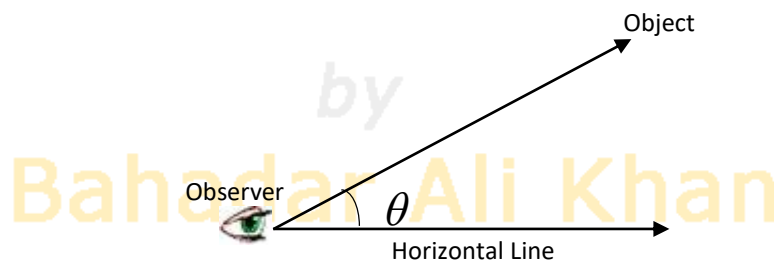
$$\text{Standard deviation} = S = \sqrt{\frac{\sum f(X - \bar{X})^2}{\sum f}} \quad (\text{for grouped data})$$

**Chapter#7*****Introduction to trigonometry***

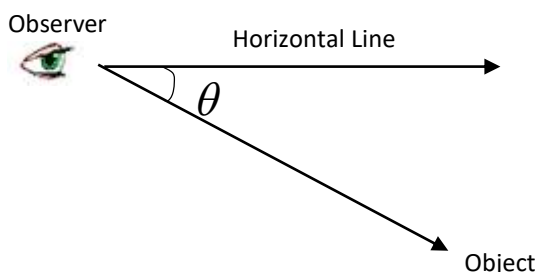
**Angle:** An angle is defined as the union of two non-collinear rays with some common starting point. The rays are called arms of the angle and the common starting point is known as vertex of the angle.

**Angle of elevation:**

The angle made between the horizontal line through eye and a line from the eye to the object above the horizontal line called an angle of elevation.



**Angle of Depression:** The angle made between the horizontal line through eye and a line from the eye to the object below the horizontal line called an angle of depression.

**Degree:**

If we divide the circumference of a circle into 360 equal arcs. Then the angle subtended at the center of the circle by one arc is called one degree and is denoted by  $1^\circ$



**Radian:**

The angle formed at the center of the circle by an arc, whose length is equal to the radius of the circle is called one radian.

**Co-terminal angle:**

Two or more than two angles with the same initial and terminal sides are called co-terminal angles.

**Quadrantal angle:**

An angle is called a quadrantal angle, if its terminal side lies on the x-axis or y-axis.

**Trigonometric ratios:** There are six fundamental trigonometric ratios (function) called sine, cosine, tangent, cotangent, secant, and cosecant,

**Relationship between radian and degree measure:**

$$1^\circ = \frac{\pi}{180} \text{ rad} = 0.0175 \text{ rad} \quad \text{and} \quad 1 \text{ rad} = \left(\frac{180}{\pi}\right)^\circ = 57.295^\circ$$

**Trigonometric identities**

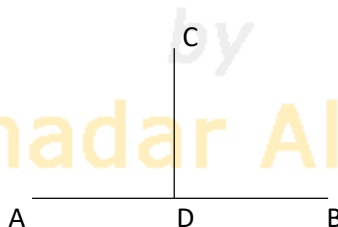
$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 + \sec^2 \theta = \tan^2 \theta$$

$$1 + \csc^2 \theta = \cot^2 \theta$$

**Chapter# 8****Projection of a side of triangle****Projection:**

The projection of a given point on a line segment is the foot of  $\perp$  drawn from the point on that line segment. If  $\overline{AB} \perp \overline{CD}$ , then evidently D is the foot of perpendicular CD from the point C on the line segment AB.

**Apollonius' Theorem:**

In any triangle the sum of the square on any two sides is equal to twice the square of one half the third side together with twice the square of the median which bisects the third side.

**Right Angle:**

The angle of measure equal to  $90^\circ$  is called right angle.

**Acute Angle:**

The angle of measure between  $0^\circ$  to  $90^\circ$  is called acute angle.

**Obtuse Angle:**

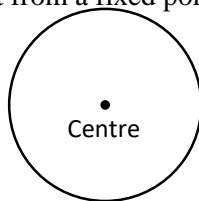
The angle of measure between  $90^\circ$  to  $180^\circ$  is called obtuse angle.

**Chapter# 9 + Chapter# 11**

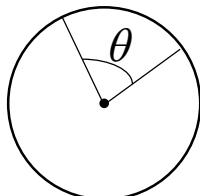
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**Chord of the circle + Chords and Arcs****Circle:**

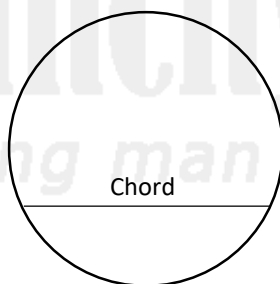
A circle is a set of point which are equidistant from a fixed point. The fixed point is called center of circle.

**Central angle:**

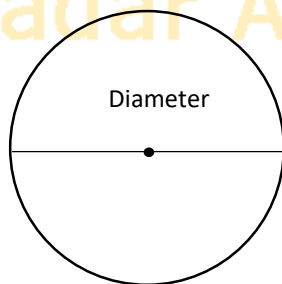
An angle whose vertex is at the center of the circle and its arms meet at the end points of an arc is called central angle.

**Chord of a circle:**

The line segment joining any two points of the circle with each other is called chord of the circle.

**Diameter:**

The chord passing through the center of the circle is called diameter of the circle.

**Segment of the circle:**

The circular region bounded by an arc and a corresponding chord is called segment of the circle. Evidently any chord divides a circle into two segments.

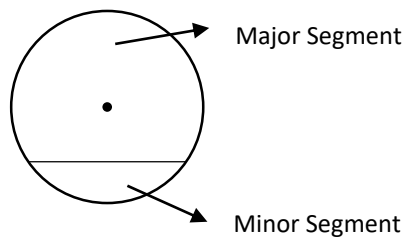
There two types of segments Major segment and minor segment.

**Major segments:**

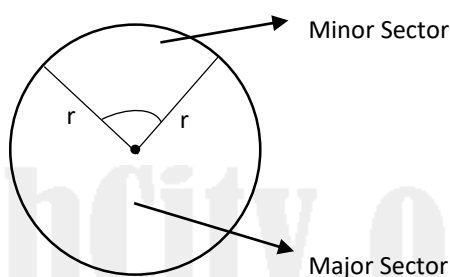
The circular region bounded by a major arc and a corresponding chord is called major segment.

**Minor segment:**

The circular region bounded by a minor arc and a corresponding chord is called minor segment.

**Sector of the circle:**

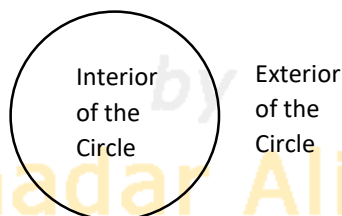
A sector of a circle is the plane figure bounded by two radii and the arc intercepted between them. There are two types of sectors. Major sector and minor sector

**Interior of the circle:**

The set of all the points lying inside the boundary of a circle is called interior of a circle.

**Exterior of the circle:**

The set of all the points lying outside the boundary of a circle is called exterior of a circle.

**Circular Area:**

Area of region enclosed by the boundary of circle is called circular area.

Circular area is calculated by the formula  $A = \pi r^2$

**Collinear points:**

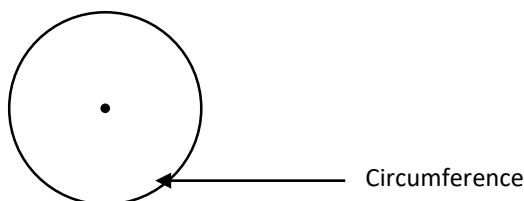
Two or more than two points lying on the same line are called collinear points.

**Non collinear points:**

Two or more than two point not lying on the same line are called non collinear points.

**Circumference of the circle:**

The length of the boundary of the circle is called circumference of the circle. Circumference is calculated  $C = 2\pi r$  here  $r$  is a radius and  $\pi$  is an irrational number.



**Arc of circle:** Any part of circumference of the circle is called arc of the circle.

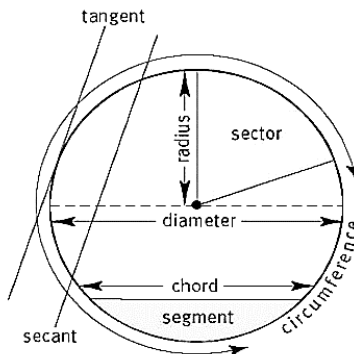
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**Radial segment of the circle:**

The line segment joining the center of a circle to any point of the circle is called radial segment.

**Parts of a circle:**



**Chapter #10**

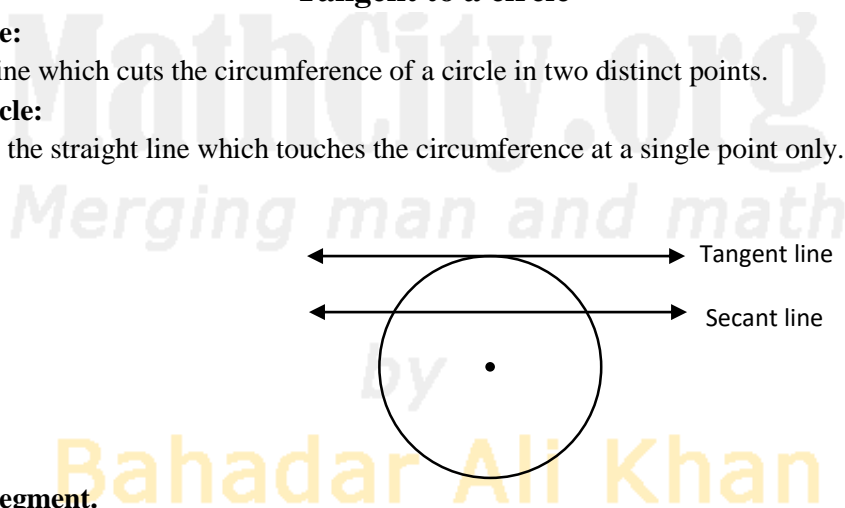
**Tangent to a circle**

**Secant Line of a circle:**

A secant is a straight line which cuts the circumference of a circle in two distinct points.

**Tangent Line of a circle:**

A tangent to a circle is the straight line which touches the circumference at a single point only.



**Length of a tangent segment.**

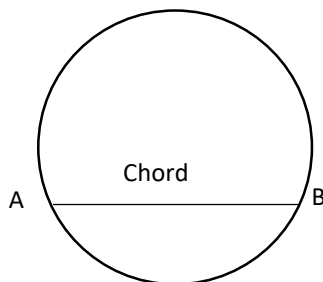
The distance between the given point outside the circle and point of tangent is called length of tangent segment.

**Chapter #12**

**Angle in a segment of a circle**

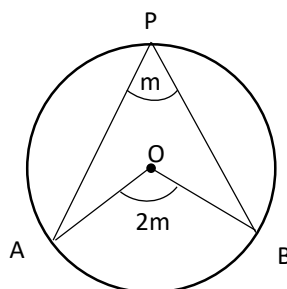
**Chord of a circle:**

The line segment joining any two points of the circle with each other is called chord of the circle.



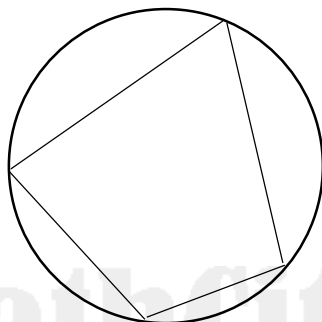
**Circum angle:**

A circum angle is subtended between any two chords of a circle having common point on its circumference. in the figure  $\angle APB$  is the circum angle.



**Cyclic quadrilateral:**

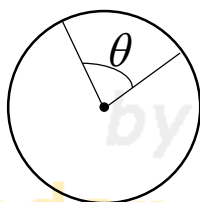
A quadrilateral is called cyclic when a circle can be draw through its four vertices.



**In centre:** in –centre of a triangle is the centre of a circle inscribe in a triangle.

**Central angle:**

An angle whose vertex is at the centre of the circle and its arms meet at the end points of an arc is called central angle.



**Chapter#13**

**Bahadar Ali Khan**

*Practical geometry*

**Geometry:**

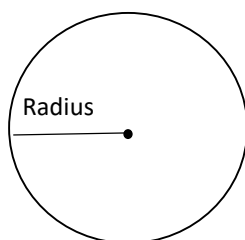
The word geometry is derived from two Greek words namely Geo (earth) and Metron (measurement).geometry is the branch of mathematics, which deal with the shape size and position of geometric figures.

**Polygon:**

A plane figure bounded with three or more sides is called a polygon.

**Radius:**

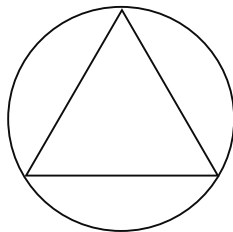
The distance from the centre of the circle to any point on the circle is called the radius of the circle.



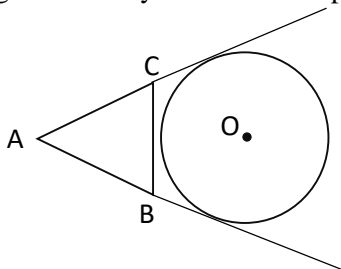
**Circumscribed circle:**

The circle passing through the three vertices of a triangle ABC is known as circumscribe circle its radius is called circum radius and center is called circum radius.

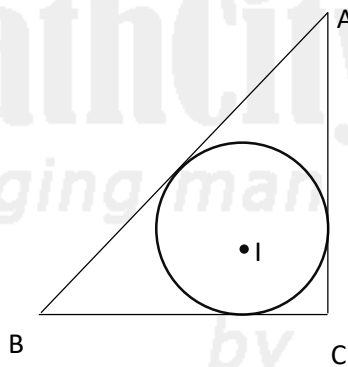
**Note are available at: <https://www.mathcity.org>**

**Escribed circle:**

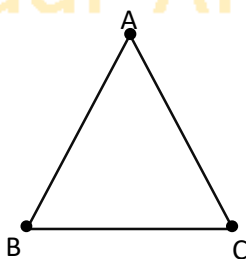
The circle touching one side of a triangle externally and other two produced sides internally is called escribed circle.

**Inscribed circle:**

A circle which touches the three sides of a triangle internally is known as inscribed circle, its radius is called in-radius and centre is called in-centre.

**Vertices:**

The corners of a polygon are called its vertices.

**Locus:**

The path an object moving according to some rule, is the locus of the object.

**Perimeter:**

The perimeter of a closed geometric figure is the sum of its sides.

**Triangle:**

A plane figure bounded with three sides is called a triangle. A triangle has six elements i.e. three sides and three angle.

**Regular polygon:**

A figure bounded by equal sides and which has all its angles equal is called a regular polygon.

**Segment of the circle:**

Note are available at: <https://www.mathcity.org>

The circular region bounded by an arc and a corresponding chord is called segment of the circle. Evidently any chord divides a circle into two segments.

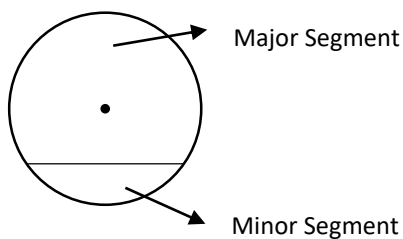
There two types of segments Major segment and minor segment.

**Major segments:**

The circular region bounded by a major arc and a corresponding chord is called major segment.

**Minor segment:**

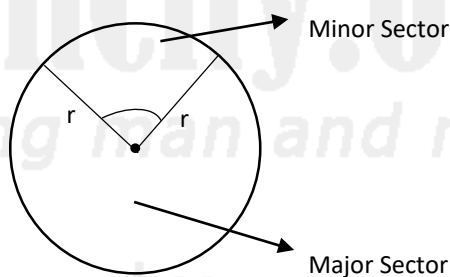
The circular region bounded by a minor arc and a corresponding chord is called minor segment.



**Sector of the circle:**

A sector of a circle is the plane figure bounded by two radii and the arc intercepted between them.

There are two types of sectors. Major sector and minor sector



**Tangent Line of a circle:**

A tangent to a circle is the straight line which touches the circumference at a single point only.