EXERCISE 4.1

Resolve into partial fractions.

1. $\frac{7x-9}{(x+1)(x-3)}$

Let $\frac{7x-9}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$ - - - - - - - - - - - - - - - - (i)

Multiplying both sides by (x+1) (x-3)

(x+1)(x-3) $\frac{7x-9}{(x+1)(x-3)} = \frac{A}{x+1}$(x+1)(x-3) + $\frac{B}{x-3}$(x+1)(x-3)

7x-9= A(x-3) +B(x+1) - - - - - - - - - - - - - - - - - (ii)

Equation (ii) is an identity, which holds good for all values of x and hence for

X=3 & x=-1

Put x=3 i.e. , x-3=0 on both sides of the equation (ii)

7(3)-9=A (3-3) +B (3+1)

21-9=A (0) +4B

12=4B

B=3

Put x=-1 i.e. , x+1=0 in eq(ii)

7(-1)-9=A (-1-3) +B (-1+1)

-7-9=-4A+B (0)

-16= -4A

A=4

Put values of A & B in (i)

Thus, $\frac{7x-9}{(x+1)(x-3)} = \frac{4}{x+1} + \frac{3}{x-3}$ are the required partial fractions.

2. $\frac{x-11}{(x-4)(x+3)}$

Let $\frac{x-11}{(x-4)(x+3)}=\frac{A}{x-4}+\frac{B}{x+3}$ ---- ---------- -(i)

Multiplying (x-4)(x+3) 0n b/s on eq(i)

(x-4)(x+3) $\frac{x-11}{(x-4)(x+3)}$=(x-4)(x+3) $\frac{A}{x-4}+\frac{B}{x+3}$(x-4)(x+3)

x-11=A(x+3)+B(x-4) --------------- (ii)

eq(ii) is identity, which holds good for all values of x , hence for x=-3 & x=4

put x=-3 i.e. , x+3=0 in eq (ii)

-3-11=A(-3+3)+B(-3-4)

-14=A(0)-7B

-14=-7B

B=2

Put x=4 i.e.,x-4=0 in eq(ii)

4-11=A(4+3)+B(4-4)

-7=7A +B(0)

A=-1

Put values of A & B in eq (i)

Thus, $\frac{x-11}{(x-4)(x+3)}=\frac{-1}{x-4}+\frac{2}{x+3}$ are the required partial fractions.

3. $\frac{3x-1}{x^2-1}$

$\frac{3x-1}{x^2-1}=\frac{3x-1}{(x+1)(x-1)}$

Let $\frac{3x-1}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1}$ ----------- (i)

Multiplying b/s by (x+1)(x-1)

(x+1)(x-1)$\frac{3x-1}{(x+1)(x-1)}$ =(x+1)(x-1) $\frac{A}{x+1}$ + $\frac{B}{x-1}$(x+1)(x-1)

3x-1=A(x-1)+B(x+1) - - - - - - - - - - - - - - - (ii)

Equation (ii) is identity, which holds good for all x and hence for x=-1 & x=1

Put x=-1 i.e., x+1=0 in eq (ii)

3(-1)-1=A(-1-1)+B(-1+1)

-3-1=-2A+B(0)

-4=-2A

A=2

Put x=1 i.e, x-1=0 in eq (ii)

3(1)-1=A(1-1)+B(1+1)

3-1=A(0)=2B

2=2B

B=1

Put values of A&B in eq (i)

Thus, $\frac{3x-1}{(x+1)(x-1)} = \frac{2}{x+1} + \frac{1}{x-1}$ are the required partial fractions.

4. $\frac{x-5}{x^2+2x-3}$

$\frac{x-5}{x^2+2x-3} = \frac{x-5}{x^2+3x-x-3}$

$\frac{x-5}{x^2+2x-3} = \frac{x-5}{x(x+3)-1(x+3)}$

$\frac{x-5}{x^2+2x-3} = \frac{x-5}{(x+3)(x-1)}$

Let $\frac{x-5}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$ - - - - - - - - - - - - -(i)

Multiplying eq (i) by (x+3)(x-1) on b/s

$(x+3)(x-1)\frac{x-5}{(x+3)(x-1)} = (x+3)(x-1)\frac{A}{x+3} + \frac{B}{x-1}(x+3)(x-1)$

x-5=A(x-1)+B(x+3) - - - - - - - - - - - - - - - - - (ii)

eq(ii) is identity which holds good for all values of x and hence for x=-3 & x=1

Put x=-3 i.e , x+3=0 in eq(ii)

-3-5=A(-3-1)+B(-3+3)

-8=-4A+0

A=2

Put x=1 i.e. , x-1=0 in eq(ii)

1-5=A(1-1)+B(1+3)

-4=A(0)+4B

B=-1

Put values of A & B in eq(i)

Thus, $\frac{x-5}{(x+3)(x-1)} = \frac{2}{x+3} - \frac{1}{x-1}$

Or $\frac{x-5}{x^2+2x-3} = \frac{2}{x+3} - \frac{1}{x-1}$ are the required partial fractions.

5. $\frac{3x+3}{(x-1)(x+2)}$

Let $\frac{3x+3}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$ - - - - - - - - - - - - - - - -(i)

Multiplying eq(i) by (x-1)(x+2) on b/s

$(x-1)(x+2)\frac{3x+3}{(x-1)(x+2)} = (x-1)(x+2)\frac{A}{x-1} + \frac{B}{x+2}(x-1)(x+2)$

3x+3 = A(x+2)+B(x-1) - - - - - - - - - - - - - - - - - -(ii)

Eq(ii) is an identity which holds good for all x and hence for x=1 & x=-2

Put x=1 i.e. , x-1=0 in eq(ii)

3(1)+3=A(1+2)+B(1-1)

3+3=3A+B(0)

6=3A

A=2

Put x=-2 i.e. ,x+2=0 in eq(ii)

3(-2)+3=A(-2+2)+B(-2-1)

-6+3=A(0)-3B

-3=-3B

B=1

Put values of A & b in eq(i)

Thus, $\frac{3x+3}{(x-1)(x+2)} = \frac{2}{x-1} + \frac{1}{x+2}$ are the required partial fractions.

6. $\frac{7x-25}{(x-4)(x-3)}$

Let $\frac{7x-25}{(x-4)(x-3)} = \frac{A}{x-4} + \frac{B}{x-3}$ - - - - - - - - - - - - -(i)

Multiplying eq(i) by (x-4)(x-3) on b/s

$(x-3)(x-4)\frac{7x-25}{(x-4)(x-3)} = (x-3)(x-4)\frac{A}{x-4} + \frac{B}{x-3}(x-3)(x-4)$

7x-25=A(x-3)+B(x-4) - - - - - - - - - - - - -(ii)

Eq(ii) is an identity which holds good for all x and hence for x=3 & x=4

Put x=3 i.e., x-3=0 in eq (ii)

7(3)-25=A(3-3)+B(3-4)

21-25=A(0)+B(-1)

-4=-B

B=4

Put x=4 i.e. , x-4=0 in eq(ii)

7(4)-25=A(4-3)+B(4-4)

28-25=A(1)+B(0)

Put values of A & B in eq(i)

Thus, $\frac{7x-25}{(x-4)(x-3)} = \frac{3}{x-4} + \frac{4}{x-3}$ are the required partial fractions.

7. $\frac{x^2+2x+1}{(x-2)(x+3)}$

Since given is an improper fraction, so we have to convert it into proper fraction by long division method.

$\frac{x^2+2x+1}{(x-2)(x+3)} = \frac{x^2+2x+1}{x^2+x+6}$

By long division,

$\frac{x^2+2x+1}{(x-2)(x+3)} = \frac{x^2+2x+1}{x^2+x+6}$

$\frac{x^2+2x+1}{(x-2)(x+3)} = 1+ \frac{x+7}{x^2+x+6}$

$\frac{x^2+2x+1}{(x-2)(x+3)} = 1+ \frac{x+7}{(x-2)(x+3)}$ - - - - - - - - - - - - - - -(A)

Consider, $\frac{x+7}{(x-2)(x+3)} = \frac{A}{(x-2)} + \frac{B}{(x+3)}$ - - - - - - - - - - - - - -(i)

Multiplying eq(i) with (x-2)(x+3) on b/s

(x-2)(x+3) $\frac{x+7}{(x-2)(x+3)}$ =(x-2)(x+3) $\frac{A}{(x-2)} + \frac{B}{(x+3)}$ (x-2)(x+3)

X+7=A(x+3)+B(x-2) - - - - - - - - - - - - - - - (ii)

Eq(ii) is an identity which holds good for all x and hence for x=-3 & x=2

Put x=-3 i.e. , x+3=0 in eq(ii)

-3+7=A(-3+3)+B(-3-2)

4=A(0)+B(-5)

4=-5B

B=$\frac{-4}{5}$

Put x=2 i.e. , x-2=0 in eq(ii)

2+7=A(2+3)+B(2-2)

9=5A+B(0)

9=5A

A=$\frac{9}{5}$

Put values of A & b In eq(i)

$\frac{x+7}{(x-2)(x+3)} = \frac{9/5}{(x-2)} + \frac{-4/5}{(x+3)}$

Thus, eq (A) ⇒

$\frac{x^2+2x+1}{(x-2)(x+3)} = 1 + \frac{9}{5(x-2)} - \frac{4}{5(x+3)}$ are the required partial fractions.

8. $\frac{6x^3+5x^2-7}{3x^2-2x-1}$

By long division method,

$\frac{6x^3+5x^2-7}{3x^2-2x-1}$ = 2x+3 + $\frac{8x-4}{3x^2-2x-1}$

$\frac{6x^3+5x^2-7}{3x^2-2x-1}$ = 2x+3 + $\frac{8x-4}{(3x+1)(x-1)}$ - - - - - - - -- - -(A)

Consider, $\frac{8x-4}{(3x+1)(x-1)} = \frac{A}{(3x+1)} + \frac{B}{(x-1)}$ - - - - - - - - - - - - -(i)

Multiplying eq (i) by (3x+1)(x-1) on b/s

(3x+1)(x-1) $\frac{8x-4}{(3x+1)(x-1)}$ =(3x+1)(x-1) $\frac{A}{(3x+1)} + \frac{B}{(x-1)}$(3x+1)(x-1)

8x-4=A(x-1)+B(3x+1) - - - - - - - - - - - - - - - -(ii)

Eq(ii) is an identity which holds good for all x and hence for x=1 & x=-1/3

Put x=1 i.e. , x-1=0 in eq (ii)

8(1)-4=A(1-1)+B(3(1)+1)

8-4= A(0)+ B(3+1)

4=4B

B=1

Put x=-1/3 i.e. , 3x+1=0 in eq(ii)

$8(\frac{-1}{3}) - 4 = A\ (\frac{-1}{3} - 1)$ +B $(3(\frac{-1}{3}) + 1)$

$\frac{-8-12}{3}$ =A $(\frac{-1-3}{3})$ + B (-1 +1)

$\frac{-20}{3}$ =A($\frac{-4}{3}$) +B (0)

$\frac{-20}{3} = \frac{-4}{3}A$

-20 = -4A

A=5

Put values of A & B in eq(i)

$\frac{8x-4}{(3x+1)(x-1)} = \frac{5}{(3x+1)} + \frac{1}{(x-1)}$

Thus, eq(A) ⇒ $\frac{6x^3+5x^2-7}{3x^2-2x-1}$ = 2x+3 + $\frac{5}{(3x+1)} + \frac{1}{(x-1)}$ are the required partial fractions.