Standard Form

The **Standard Form** of a Quadratic Equation looks like this:

$$ax^2 + bx + c = 0, a \neq 0$$

Where a ,b ,c are real numbers **x** is variable or Unknown.

For example $7x^2 + x + 3 = 0$

Note:-

If a=0, it will not be a quadratic equation. The name Quadratic comes from "quad" meaning square, because the variable gets squared (like x2).It is also called an "Equation of Degree 2" (because of the "2" on the x)

Pure Quadratic Equations

The **Pure Form** of a Quadratic Equation looks like this:

$ax^2 + c = 0$, b=0 in Standard quadratic Equation $ax^2 + bx + c = 0$

For example $x^2 - 16 = 0$, $4x^2 = 7$ are pure form of quadratic equation.

How To Solve Quadratic Equation ?

There are three Methods to solve a Quadratic Equation.

1. Factorization

2. Completing the Square

3. By using Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise 1.1 10th-Class

1. Write the following Quadratic Equations in the standard form and point out pure Quadratic Equations.

(i) $(x+7)(x-3) = -7$

Soln. $(x+7)(x-3) = -7$

$x^2 + 4x - 21 = -7$

$x^2 + 4x - 21 + 7 = 0$

=> $x^2 + 4x - 14 = 0$ (Standard form)

(ii) $\frac{x^2+4}{3} - \frac{x}{7} = 1$

Soln. $\frac{x^2+4}{3} - \frac{x}{7} = 1$

Taking L.C.M of 3,7 (which is 21) and multiplying on both side

We get

$21\left(\frac{x^2+4}{3}\right) - 21\left(\frac{x}{7}\right) = 21(1)$

$7(x^2+4) - 3x = 21$

$7x^2 + 28 - 3x = 21$

$7x^2 + 28 - 3x - 21 = 0$

=> $7x^2 - 3x + 7 = 0$ (Standard form)

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(iii) $\boldsymbol{\frac{x}{x+1} + \frac{x+1}{x} = 6}$

Sol. $\frac{x}{x+1} + \frac{x+1}{x} = 6$

Multiply both sides by (x+1)(x).

(L.C.M Of (x+1) and (x)) We get

$(x+1)(x)\frac{x}{x+1} + (x+1)(x)\frac{x+1}{x} = 6(x+1)(x)$

$x^2 + (x+1)(x+1) = 6(x^2 + x)$

$x^2 + x^2 + 2x + 1 - 6x^2 - 6x = 0$

$-4x^2 - 4x + 1 = 0$ Taking (-) common we get

=> $4x^2 + 4x - 1 = 0$ (Standard form)

(iv) $\boldsymbol{\frac{x+4}{x-2} - \frac{x-2}{x} + 4 = 0}$

Sol. $\frac{x+4}{x-2} - \frac{x-2}{x} + 4 = 0$

Multiply both sides by (x-2)(x).

(L.C.M Of (x-2) and (x)) We get

$(x-2)(x)\frac{x+4}{x-2} - (x-2)(x)\frac{x-2}{x} + 4(x-2)(x) = 0(x-2)(x)$

$x(x+4) - (x-2)(x-2) + 4(x^2 - 2x) = 0$

$x^2 + 4x - (x^2 - 4x + 4) + 4x^2 - 8x = 0$

$4x^2 - 4 = 0$ Taking (4) common we get

=> $x^2 - 1 = 0$ (Pure quadratic Form)

(v) $\boldsymbol{\frac{x+3}{x+4} - \frac{x-5}{x} = 1}$

Do your self

Same As Part (iv) Ans. (Pure quadratic Form)

(vi) $\frac{x+1}{x+2} + \frac{x+2}{x+3} = \frac{25}{12}$

Sol. $\frac{x+1}{x+2} + \frac{x+2}{x+3} = \frac{25}{12}$

Multiply both sides by (x+2)(x+3)(12).

(L.C.M Of (x+2) , (x+3) and (12))

We get

$(x+2)(x+3)(12)\left(\frac{x+1}{x+2}\right) + (x+2)(x+3)(12)\left(\frac{x+2}{x+3}\right) = (x+2)(x+3)(12)\left(\frac{25}{12}\right)$

$(x+3)(12)(x+1) + (x+2)(12)(x+2) = (x+2)(x+3)(25)$

$(12)(x+3)(x+1) + (12)(x+2)(x+2) = (25)(x+2)(x+3)$

$12(x^2+4x+3) + 12(x^2+4x+4) = 25(x^2+5x+6)$ Simplifying we get

$(12+12-25)x^2 + (48+48-150)x + (38+48-150) = 0$

$-x^2 - 29x - 66 = 0$ **T**aking (-) common we get

=> $x^2 + 29x + 66 = 0$ (Standard form)

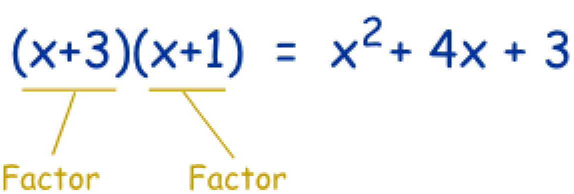
What is factorization?

Let us consider a simple example (Numbers)

12 = 3 × 4
i.e., 12 is product of 3 and 4.
3 and 4 are called factors or divisors of 12
12 is also equal to 2 × 6.

Similarly an Algebraic expression can be Factorized .

For Example $x^2 + 4x + 3 = 0$ have Factors as



Factorization

Expressing polynomials as product of other polynomials that cannot be further factorized is called Factorization

Q2. Solve by Factorization.

(i). $x^2 - x - 20 = 0$

Sol. $x^2 - x - 20 = 0$

$x^2 - 5x + 4x - 20 = 0$

$x(x-5) + 4(x-5) = 0$

$(x-5)(x+4) = 0$

$(x-5) = 0$ or $(x+4) = 0$

$=> x = 5$ or $x = -4$

Solution Set = {5,-4}

(ii) $3y^2 = y(y-5)$

Sol. $3y^2 = y(y-5)$

$3y^2 - y^2 + 5y = 0$

$2y^2 + 5y = 0$

Taking y common, we get

$y(2y+5) = 0$

$y = 0 \; or \; (2y+5) = 0$

$y = 0 \; or \; y = -\frac{5}{2}$

Solution Set = {0,-$\frac{5}{2}$}

(iii) $4 - 32x = 17x^2$

Sol. $4 - 32x = 17x^2$

$4 - 32x - 17x^2 = 0$ **T**aking (-) common

$17x^2 + 32x - 4 = 0$

$17x^2 + 34x - 2x - 4 = 0$

Some tips For Factorization.

$ax^2 + bx + c = 0$

In this Equation.

- If c is positive, then the factors you're looking for are either both positive or else both negative.
- (factors that you're looking for add to b.)

- If c is negative, then the factors you're looking for are of alternating signs; that is, one is negative and one is positive.

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$17x(x+2)-2(x+4)=0$

$(\mathrm{x}+2)(17\mathrm{x}-2)=0$

Thus, $(\mathrm{x}+2)=0$ or $(17\mathrm{x}-2)=0$

$\mathrm{x}=-2,\ 17\mathrm{x}=2$

$x=\frac{2}{17}$

Solution Set = {-2,$\frac{2}{17}$}

(iv) $x^2-11x=152$

Sol.$x^2-11x=152$

$x^2-11x-152=0$

$x^2-19x+8x-152=0$

$x(x-19)+8(x-19)=0$

$(x-19)(x+8)=0$

$Thus,\ x-19=0\ \ or\ \ x+8=0$

$\mathrm{x}=19,\ \mathrm{x}=-8$

Solution Set = {19,-8}

(v) $\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$

Sol. $\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$

Multiplying both sides by $12x(x+1)$

$\frac{x+1}{x}\times 12x(x+1)+\frac{x}{x+1}\times 12x(x+1)=\frac{25}{12}\times 12x(x+1)$

$12(x+1)(x+1)=12x(x)=25x(x+1)$

$12(x^2+2x+1)+12x^2=25(x^2+x)$

$12x^2+24x+12+12x^2=25x^2+25x$

$12x^2 + 24x + 12 + 12x^2 - 25x^2 - 25x = 0$

$12x^2 + 12x^2 - 25x^2 + 24x - 25x + 12 = 0$

$-x^2 - x + 12 = 0$ **T**aking (-) common we get

$or \quad x^2 + x - 12 = 0$

$x^2 + 4x - 3x - 12 = 0$

$x(x+4) - 3(x+4) = 0$

$(x+4)(x-3) = 0$

$Thus, x + 4 = 0 \ or \ x - 3 = 0$

$x = -4 \quad , x = 3$

Solution Set = {-4,3}

(vi) $\frac{2}{x-9} = \frac{1}{x-3} - \frac{1}{x-4}$

Sol. $\frac{2}{x-9} = \frac{1}{x-3} - \frac{1}{x-4}$

$\frac{2}{x-9} = \frac{(x-4)-(x-3)}{(x-3)(x-4)} \quad (taking\ L.C.M\ on\ R.H.S)$

$\frac{2}{x-9} = \frac{x-4-x+3}{(x-3)(x-4)}$

$\frac{2}{x-9} = \frac{-1}{(x-3)(x-4)}$

Now by **c**ross multiplication, we have

$2(x-3)(x-4) = -1(x-9)$

$2(x^2 - 7x + 12) = -x + 9$

$2x^2 - 14x + 24 + x - 9 = 0$

$2x^2 - 13x + 15 = 0$

Now Factorizing

$2x^2 - 10x - 3x + 15 = 0$

$2x(x-5) - 3(x-5) = 0$

$(x-5)(2x-3)=0$

$Thus\ \ x-5=0\ or\ \ 2x-3=0$

$x=5\quad ,\quad x=\frac{3}{2}$

Solution Set = $\{5,\frac{3}{2}\}$

Now we will learn completing Square Technique to solve quadratic Equation.

Derivation of Quadratic Formula using completing square.

let's go

$ax^2+bx+c=0$ (1)

Step-1 Coefficient of x^2 should be 1. If it is not 1 make it 1 by dividing.

Dividing Above Equation by "a" ,We get

$\frac{ax^2}{a}+\frac{bx}{a}+\frac{c}{a}=\frac{0}{a}$

Step-2 Put Constant term Over the Other side (i.e. $\frac{c}{a}$.)

$x^2+\frac{bx}{a}=-\frac{c}{a}$

Step-3 Take half of Coefficient of x-term, and square it and add on both sides.

$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2$

Step-4 Convert the left-hand side to square forms and simplify on the right-hand side.

Step-4
Convert left-hand side in square form using this
$(x)^2+2(x)(y)+(y)^2=(x+y)^2$

$(x)^2+2(x)\left(\frac{b}{2a}\right)+\left(\frac{b}{2a}\right)^2=-\frac{c}{a}+\frac{b^2}{4a^2}$

$\left(x+\frac{b}{2a}\right)^2=\frac{-4ac+b^2}{4a^2}$

Step-5 Square-root both sides, remembering to put the "±" on the right.

$$\sqrt{\left(x+\frac{b}{2a}\right)^2} = \pm\sqrt{\frac{-4ac+b^2}{4a^2}}$$

$$x+\frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$$

Step-6 Solve for "x =", and simplify as necessary.

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a}, \quad or \quad x = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

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