# Lecture 01

**Course Title:** Calculus with Analytic Geometry **Course Code:** MTH104

## Objectives

The main aim of the lecture is to discuss:

- *Course content*
- *Number system*
- *Real line, inequalities, absolute value*
- *Extended real number system*

## References:


- Earl W. Swokowski, Calculus with Analytic Geometry, PWS Publisher, Boston, 1988.
- James Stewart, Calculus Early Transcendental, 6th Ed., Thomson Brooks/Cole, 2008.

**Course Contents:**

Inequalities, functions, shifting graphs, limits of function, continuity, derivative of a function, application of derivatives, integration, indefinite integrals, definite integrals, application of integral, area, arc-length, transcendental functions, L'Hôpital's rule, techniques of integration, improper integrals, infinite series, limit of sequences of numbers, convergence and divergence tests, alternating series test, absolute and conditional convergence, power series, Taylor's series and Maclaurin series, convergence of Taylor series, error estimates, applications of power series.

**Number System:**

We give some basic definitions and facts. These will help to learn and understand our main topic.

**Definition:** The set $\{1,2,3,...\}$, which is usually denoted by $\mathbb{N}$ is called set of natural numbers.

**Definition:** The set $\{...,-2,-1,0,1,2,...\}$, which is usually denoted by $\mathbb{Z}$ is called set of integers.

❖ **Remarks:**

a. A set $\mathbb{Z}=\{...,-2,-1,0,1,2,...\}$ can also be written as $\mathbb{Z}=\{0,\pm1,\pm2,\pm3,...\}$.

b. A set of positive integers is denoted by $\mathbb{Z}^{+}=\{1,2,3,...\}$ and set of negative integers is denoted by $\mathbb{Z}^{-}=\{-1,-2,-3,...\}$

c. $\mathbb{Z}=\mathbb{Z}^{-}\cup\{0\}\cup\mathbb{Z}^{+}$, that is, a number 0 is neither positive nor negative.

**Definition:** Given two integers $a,b\in\mathbb{Z}$, $a\neq 0$, we say $a$ divides $b$ if there exists some integer $q$ such that $b=a\cdot q$.

*Notation:* If $a$ divides $b$, then we write $a\mid b$ and if $a$ doesn't divides $b$, then we write $a\nmid b$.

*Examples:* (i) 2 divides 6, i.e. $2\mid 6$ because if $a=2$ and $b=6$, then $q=3$.

(ii) -2 divides 6, i.e. $-2\mid 6$ because if $a=-2$ and $b=6$, then $q=-3$.

(iii) -1, 1, -*a* and *a* divide every integer *a*.

(iv) Every non-zero integer divides 0.

**Definition:** An integer is called even if it is divisible by 2, otherwise it is called odd.

*Note:* A set $E:=\{0,\pm 2,\pm 4,\ldots\}$ represents set of all even integers and a set of odd integers is represented as $O:=\{\pm 1,\pm 3,\pm 5,\ldots\}$.

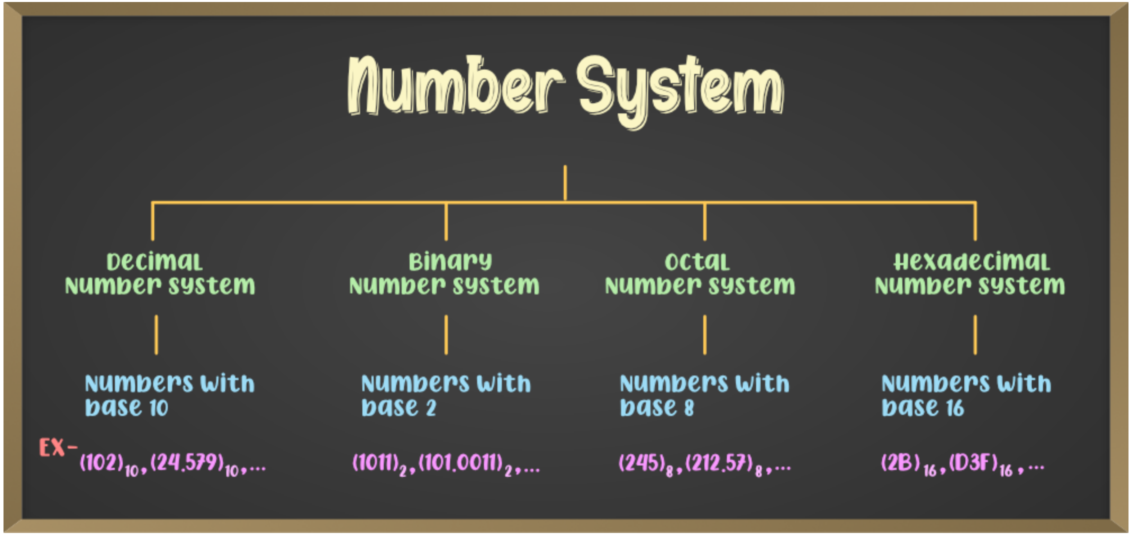
**Definition:** A positive integer *p* is called prime if it has exactly four divisors (or two positive divisors).

*Examples:* 2, 3, 11, 29 are prime numbers.

# Related to Computer Science:

## Hexadecimal Number System Table

| Decimal | Binary | Hexa-Decimal |
|---|---|---|
| 0 | 0000 | 0 |
| 1 | 0001 | 1 |
| 2 | 0010 | 2 |
| 3 | 0011 | 3 |
| 4 | 0100 | 4 |
| 5 | 0101 | 5 |
| 6 | 0110 | 6 |
| 7 | 0111 | 7 |
| 8 | 1000 | 8 |
| 9 | 1001 | 9 |
| 10 | 1010 | A |
| 11 | 1011 | B |
| 12 | 1100 | C |
| 13 | 1101 | D |
| 14 | 1110 | E |
| 15 | 1111 | F |




- Can you guess the number system used to measure time (i.e. used in clock)?
- See https://en.m.wikipedia.org/wiki/Decimal_time for more interesting facts.


**Our aim is to study the set of real numbers so let's change our direction.**

**Definition:** A set $\left\{\frac{p}{q} \mid p,q\in\mathbb{Z}\wedge q\neq 0\right\}$ is called set of rational numbers and it is denoted by $\mathbb{Q}$.

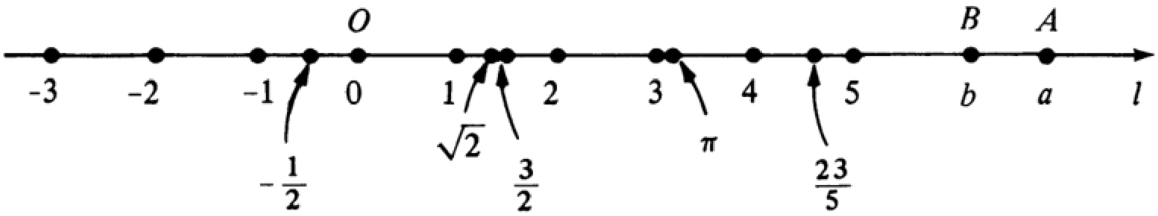
❖ **Remarks:**

a. All the integers are rational numbers but there are numbers which are rational but not integer.
b. One rational number can be written as infinitely many ways e.g. $\frac{1}{3}$ can be written as $0.333\ldots$ or $\frac{2}{6}$ or $\frac{-4}{-12}$.
c. Between any two rational numbers there exists a rational number, that is, there are infinity many rational between any two rational numbers.
d. There are operations of addition (+) and multiplication $(\cdot)$ on $\mathbb{N},\mathbb{Z}$ and $\mathbb{Q}$, which has nice properties.
e. The set of integers is exclusively the point of interest in Number Theory.

## Real Numbers:

**It is very difficult to define the set of real numbers.**

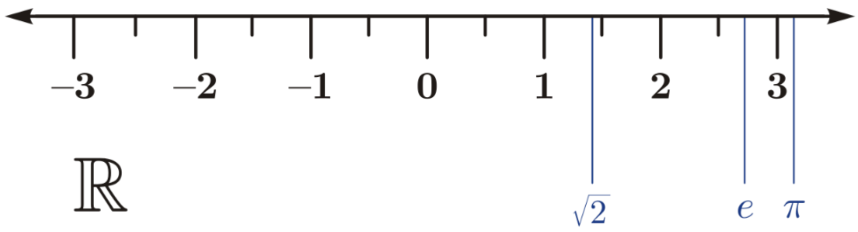
Read article 1.1 of our first recommended book.




The number a that is associated with *a* point *A* on *l* is called the coordinate of *A*. An assignment of coordinates to points on *l* is called a coordinate system for *l*, and *l* is called a ***coordinate line***, or a ***real line***. A direction can be assigned to *l* by taking the positive direction to the right and the negative direction to the left. The positive direction is noted by placing an arrowhead on *l* as shown in figure above.

The real numbers which correspond to points to the right of *O* in figure above are called ***positive real numbers***, whereas those which correspond to points to the left of *O* are ***negative real numbers***. The real ***number 0 is neither positive nor negative***.

The collection of positive real numbers is closed relative to addition and multiplication; that is, if *a* and *b* are positive, then so is the sum $a + b$ and the product $ab$.




Set of all real numbers is denoted by $\mathbb{R}$. Set of positive and negative real numbers is denoted by $\mathbb{R}^+$ and $\mathbb{R}^-$ respectively.

**Definition:** Those real numbers which are not rational are called *irrational numbers*.

**Inequalities:**

If $a$ and $b$ are real numbers, and $a - b$ is positive, we say that ***a* is greater than *b*** and write $a > b$.
An equivalent statement is ***b* is less than *a***, written $b < a$.
The symbols > or < are called inequality signs and expressions such as $a > b$ or $b < a$ are called ***inequalities***.

From the manner in which we constructed the coordinate line $l$ in shown in above figure, we see that if $A$ and $B$ are points with coordinates $a$ and $b$, respectively, then $a > b$ (or $b < a$) if and only if $A$ lies to the right of $B$.
Since $a - 0 = a$, it follows that $a > 0$ if and only if $a$ is positive. Similarly, $a < 0$ means that $a$ is negative.

The following properties of inequalities can be proved.

If $a > b$ and $b > c$, then $a > c$.

If $a > b$, then $a + c > b + c$.

If $a > b$ and $c > 0$, then $ac > bc$.

If $a > b$ and $c < 0$, then $ac < bc$.

Analogous properties for " ***less than*** " can also be established.

The symbol $a \geq b$, which is read ***a* is greater than or equal to *b***, means that either $a > b$ or $a = b$. The symbol $a < b < c$ means that $a < b$ and $b < c$, in which case we say that $b$ is between $a$ and $c$.
The notations $a \leq b$, $a < b \leq c$, $a \leq b < c$, $a \leq b \leq c$, and so on, have similar meanings.

## Absolute Value

The non-negative number | *a* | called the absolute value of *a* and is defined as follows:

$$|a| = \begin{cases} a & if\ a \geq 0, \\ -a & if\ a < 0. \end{cases}$$

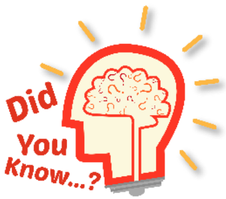
**Examples:**

(i) | 3 | = 3

(ii) | -3 | = 3

(iii) | 0 | = 0

(iv) $|\sqrt{2} - 2| = 2 - \sqrt{2}$




What is the square root of non-negative real number?

**Extended Real Number System:**

**Definition:** The extended real number system consists of $\mathbb{R}$ and two symbols $+\infty$ or $\infty$ and $-\infty$. We preserve the original order in $\mathbb{R}$ and define

$$-\infty < x < +\infty \quad \forall\, x \in \mathbb{R}.$$

**Remark:** Now the symbol $\infty$ (infinity) has been evolved in many ways. It is considered to be more difficult to understand. Its comprehension is challenging, not just for mathematicians but also for other intellectuals including engineers, scientists, philosophers, artist and directors.


**The $\infty$ (infinity) is a symbol representing some idea not a number. So please don't mix it with numbers.**

THANKS FOR YOUR ATTENTION