Lecture 02

Course Title: Calculus with Analytic Geometry **Course Code:** MTH104

Objectives

The main aim of the lecture is to discuss:

- *Intervals*
- *Functions*

References:

- Earl W. Swokowski, Calculus with Analytic Geometry, PWS Publisher, Boston, 1988.
- James Stewart, Calculus Early Transcendental, 6th Ed., Thomson Brooks/Cole, 2008.

Intervals:

Assume two real numbers *a* and *b* with *a* < *b*, then we define

Closed interval: $[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$

Open interval: $(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$

Half open intervals: $[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$

$(a, b] = \{x \in \mathbb{R} \mid a < x \leq b\}$

Infinite intervals: $[a, \infty) = \{x \in \mathbb{R} \mid a \leq x < \infty\} = \{x \in \mathbb{R} \mid x \geq a\}$

$(a, \infty) = \{x \in \mathbb{R} \mid a < x < \infty\} = \{x \in \mathbb{R} \mid x > a\}$

$(-\infty, a]$ and $(-\infty, a)$ can be defined in a similar way.

Also note $\mathbb{R} = (-\infty, \infty)$.

What about the intervals $\left(\frac{1}{2},\frac{1}{3}\right)$ and [-3,-4]?

Algebraic expressions

Inequality or equation involving one or more variables is called algebraic expressions.

e.g. $\sqrt{x+1}, x^2+1=0, |x-5| \leq 3,\ x+2y=24.$

For the values of variable for which algebraic expressions are valid are called solution of the expression.

e.g. $\sqrt{x}$ is valid for $x \geq 0$, thus $[0, \infty)$ is its solution.

Example Solve the inequality $4x + 3 > 2x - 5$.

Solution The following inequalities are equivalent:

$$\begin{aligned} 4x + 3 &> 2x - 5 \\ 4x &> 2x - 8 \\ 2x &> -8 \\ x &> -4 \end{aligned}$$

Hence the solutions consist of all real numbers greater than -4, that is, the numbers in the infinite interval $(-4, \infty)$. ■

Note: Properties (or law) of inequalities are discussed in the first chapter of FSc-I Mathematics.

Example 5 Solve $x^2 - 7x + 10 > 0$.

Solution Since the inequality may be written

$$(x - 5)(x - 2) > 0,$$

There are two cases

$x - 5 > 0$ and $x - 2 > 0$

$x > 5$ and $x > 2$.

$x - 5 < 0$ and $x - 2 < 0$

$x < 5$ and $x < 2$.

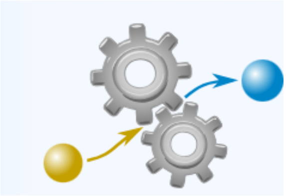
Sign of $x - 2$: − − − + + + + + + + + + +

Sign of $x - 5$: − − − − − − − − − + + + +

Hence the solution is $(-\infty, 2) \cup (5, \infty)$ $-\infty < x < +\infty \quad \forall\, x \in \mathbb{R}$.

Please solve all possible questions of Exercise 1.1 of our first reference.

Function:



It is like a machine that has input and an output.
And the output is related somehow to the input.

We will see many ways to think about functions, but there are always three main parts:

- The input (called domain)
- The relationship (pictorial or algebraic expressions or any other way)
- The output (called codomain or range)

Thank you for your attention.