Lecture 04

Course Title: Calculus with Analytic Geometry **Course Code:** MTH104

Objectives

The main aim of the lecture is to discuss:

- *Further on functions*
- *Composition of functions*
- *Inverse of functions*

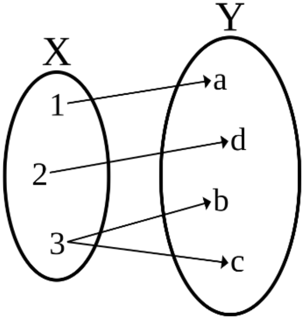
References:

- Earl W. Swokowski, Calculus with Analytic Geometry, PWS Publisher, Boston, 1988.
- James Stewart, Calculus Early Transcendental, 6th Ed., Thomson Brooks/Cole, 2008.

Further on functions

- Sum and difference of functions
- Production of functions
- Quotient of functions

In literature, there are such relations or expressions which provide two or more values for one input, these are known as "**multi-valued functions**" in the literature but actually, "**multi-valued function**" is not a "**function**".



Definition: A function f is a **polynomial function** (of degree *n*) if

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where the coefficients $a_0, a_1, \dots, a_n$ are real numbers and the exponents are nonnegative integers.

Further to read:

- Linear function (a polynomial of degree 1).
- Quadratic function (a polynomial of degree 2).
- Rational function (quotient of two polynomial functions).
    - Proper rational function
    - Improper rational function

Definition: A function f is called **algebraic** if it can be expressed in terms of sums, differences, products, quotients, or roots of polynomial functions. For example, if

$$f(x) = 5x^4 - 2\sqrt[3]{x} + \frac{x(x^2+5)}{\sqrt{x^3+\sqrt{x}}}$$

then f is an algebraic function.

Functions that are not algebraic are termed **transcendental**.

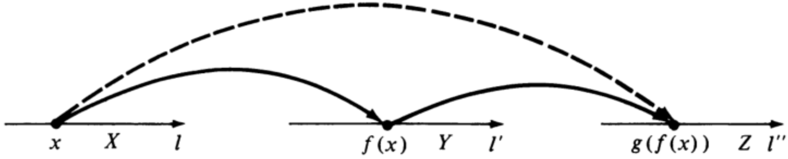
The trigonometric, exponential, and logarithmic functions are examples of transcendental functions.

Composition of functions:

If f is a function from X to Y and g is a function from Y to Z, then the **composite function** $g \circ f$ is the function from X to Z defined by

$$(g \circ f)(x) = g(f(x)),$$

for every x in X.



Example If $f(x) = x - 2$ and $g(x) = 5x + \sqrt{x}$, find $(g \circ f)(x)$.

Solution Using the definitions of $g \circ f$, f, and g,

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) = g(x - 2)\\ &= 5(x - 2) + \sqrt{x - 2}\\ &= 5x - 10 + \sqrt{x - 2}.\end{aligned}$$

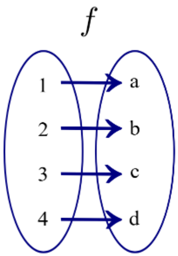
The domain X of f is the set of all real numbers; however, the last equality implies that $(g \circ f)(x)$ is a real number only if $x \geq 2$. Thus, when working with the composite function $g \circ f$ it is necessary to restrict x to the interval $[2, \infty)$. ■

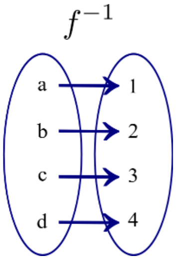
Solve Exercise 1.5

Inverse of functions:

The inverse function returns the original value for which a function gave the output.

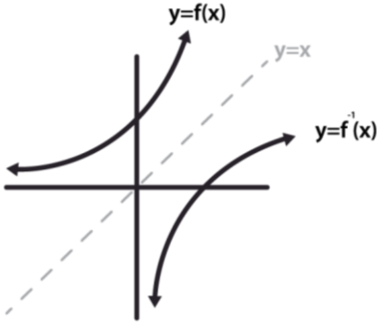
If f is a function its inverse is represented by f^{-1}.





What about domain and range of f^{-1}.

- It is not necessary that every function has inverse.
- If f is one-to-one and onto then it has inverse functions.



Thank you very much for your attention.