Lecture 06

Course Title: Calculus with Analytic Geometry **Course Code:** MTH104

Objectives

The main aim of the lecture is to discuss:

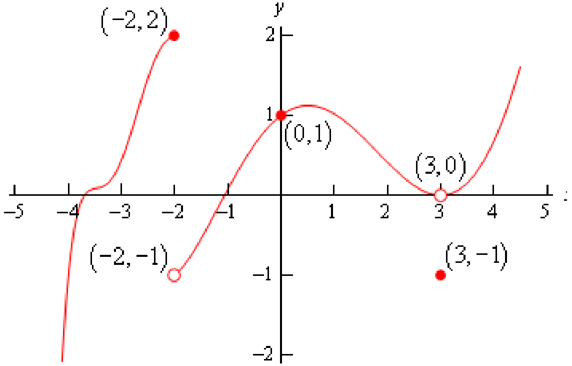
- *Continuous function*
- *Limit of the function*

References:

- Earl W. Swokowski, Calculus with Analytic Geometry, PWS Publisher, Boston, 1988.
- James Stewart, Calculus Early Transcendental, 6th Ed., Thomson Brooks/Cole, 2008.

Continuous functions

Many functions have the property that their graphs can be traced with a pencil without lifting the pencil from the page. Such functions are called *continuous*. Other functions have points at which a break in the graph occurs, but satisfy this property over intervals contained in their domains. They are continuous on these intervals and are said to have a *discontinuity* at a point where a break occurs.



In calculus, continuity is a local property not the global one.

We will give the formal defintion later.

Examples:

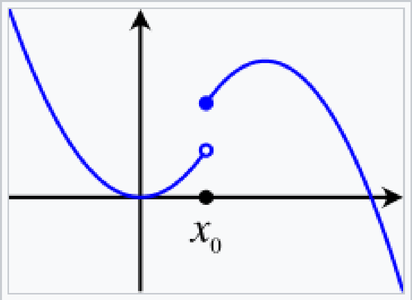
- All polynomials are continuous.
- Sine, cosine, exponetial and lograthmic functions are continuous functions
- Tangent is continuous in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.
- A funciton $f(x) = \begin{cases} \frac{x^2-9}{x-3} & if \;\; x \neq 3; \\ 10 & if \; x = 3, \end{cases}$ is not continous. (precisely it is not continuous at *x*=3.
- A function $h(x) = \begin{cases} 3 & if\ x\ is\ an\ integer \\ -1 & if\ x\ is\ not\ an\ integer \end{cases}$ is not continuous.
- A function $g(x) = \begin{cases} 1 & if\ x\ is\ rational \\ -1 & if\ x\ is\ irrational \end{cases}$ is discontinous at each point of $\mathbb{R}$.

Limit of the function:

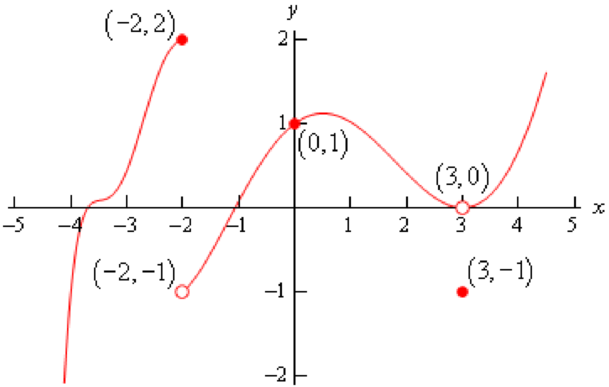
Limits are the building blocks of calculus

- You've already studied limits in your high school.
- We'll explore limits using graphs and the idea of continuity.
- The concept of limits comes before continuity in textbooks.

On real line, every point can be approached from two directions.



This can be mathematically written as $\lim_{x \to x_0^+} f(x)$ and $\lim_{x \to x_0^-} f(x)$, called right hand limit and left hand limit respectively.



What about left-hand limit and right-hand limit of functions at

x=3, *x*=0 and *x*=-2?

We say that limit of the function at *x*=*a* exists if

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x).$$

It is written as $\lim_{x\to a} f(x)$

The definition of the limit is much more difficult than its concept.

Limit of function: A number L is called the limit of f when x approaches to a if for all $\varepsilon > 0$, there exists $\delta > 0$ (depending upon ε) such that

$$|f(x) - L| < \varepsilon \text{ whenever } 0 < |x - a| < \delta.$$

Theorem:

A function f is continuous at $x = a$ iff

$$\lim_{x \to a} f(x) = f(a).$$

Remark:

Please note a function is continuous at it domain if it is continuous at each point of its domain.

See properties or theorems related to limit in your HSSC Book.

Thank you very much for your attention.