Lecture 07

Course Title: Calculus with Analytic Geometry **Course Code:** MTH104

Objectives

The main aim of the lecture is to discuss:

- *Right continuous and left continuous*
- *Limit of the function*
    - *Limit at infinity or negative infinity*
    - *Limit as a infinity or negative infinity*
- *Theorem on limit*

References:

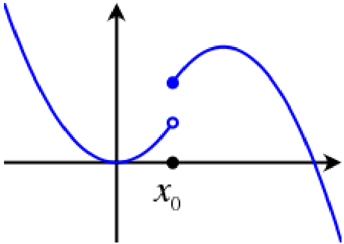
- Earl W. Swokowski, Calculus with Analytic Geometry, PWS Publisher, Boston, 1988.
- James Stewart, Calculus Early Transcendental, 6th Ed., Thomson Brooks/Cole, 2008.

Right Continuous and Left Continuous

Let f be a real valued function.

It is said to be right continuous at point a if $\lim_{x\to a+} f(x) = f(a)$ and

it is said to be left continuous at point a if $\lim_{x\to a-} f(x) = f(a)$.



Limit as a Infinity

Let f be a function defined on an interval that contains $x=a$, except possibly at $x=a$. Then we say that

$$\lim_{x\to a} f(x)=\infty$$

if for every number $M>0$, there is some number $\delta>0$ such that

$$f(x)>M \quad \text{whenever} \quad 0<|x-a|<\delta.$$

Limit as a negative infinity can be defined in a similar way.

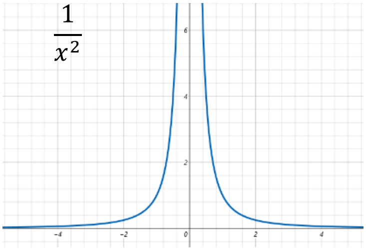
Examples:

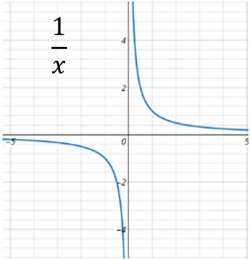
(i) $\lim_{x\to 0}\frac{1}{x^2}=\infty$

(ii) $\lim_{x\to 0}\frac{-1}{x^2}=-\infty$

(iii) $\lim_{x\to 0}\frac{1}{x}=not\ exist$

(iv) $\lim_{x\to 1}\frac{1}{1-x}=not\ exist$





Limit at Infinity

A function $f : X \to Y$ is said to tend to limit L as $x \to \infty$, if for a real number $\varepsilon > 0$ however small, there exists a positive number M which depends upon ε such that distance

$$|f(x) - L| < \varepsilon \quad \text{when} \quad x > M .$$

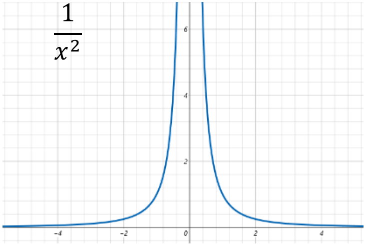
This is written as $\lim_{x\to\infty} f(x) = L$. The limit at $-\infty$ can be defined in similar way.

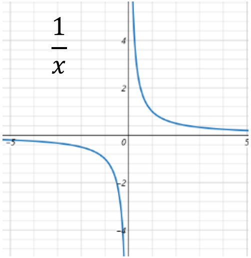
Examples:

(i) $\lim_{x\to\infty} \frac{1}{x^2} = 0$

(ii) $\lim_{x\to-\infty} \frac{1}{x^2} = 0$

(iii) $\lim_{x\to\infty} \frac{1}{x} = 0 \; and \; \lim_{x\to-\infty} \frac{1}{x} = 0$





Theorem on Limits:

(i) $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

(ii) $\lim_{x\to a} [f(x) \cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

(iii) $\lim_{x\to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, if $\lim_{x\to a} g(x) \neq 0$

(iv) $\lim_{x\to a} [cf(x)] = c\left[\lim_{x\to a} f(x)\right]$

(v) $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x).$

Example:

If $\lim_{x\to 3} f(x) = 10$ then

$$\lim_{x\to 3} f^2(x) = ?.$$

If $\lim_{x\to 2} f(x) = 5$ and $\lim_{x\to 2} g(x) = 3$, then

$$\lim_{x\to 2} \frac{f(x) + 3}{1 - g^2(x)} = ?$$

Let’s solve few questions.