Lecture 08

Course Title: Calculus with Analytic Geometry **Course Code:** MTH104

Objectives

The main aim of the lecture is to discuss:

- *Derivative of the function.*
- *Left derivative of the function.*
- *Right derivative of the function.*
- *L'Hôpital rule*

References:

- Earl W. Swokowski, Calculus with Analytic Geometry, PWS Publisher, Boston, 1988.
- James Stewart, Calculus Early Transcendental, 6th Ed., Thomson Brooks/Cole, 2008.

Derivative of the Function

Let f be defined on an open interval (a,b), and assume that $c \in (a,b)$. Then f is said to be differentiable at c whenever the limit $\lim_{x \to c} \dfrac{f(x) - f(c)}{x - c}$ exists.

This limit is denoted by $f'(c)$ (there are other notation also) and is called the derivative of f at point c.

If f is differentiable at each point of (a,b), then we say f is differentiable on (a,b).

- If $x - c = h$, then we have $f'(c) = \lim_{h \to 0} \dfrac{f(c+h) - f(c)}{h}$.
- As derivative is a limit, therefore concept of left derivative and right derivative exist.

There was lot of questions in HSSC to find derivative by definition (or first principle). Our aim is not to find derivative by definition.

Example:

What about the derivate of the constant function.

e.g $f(x)$= 3.

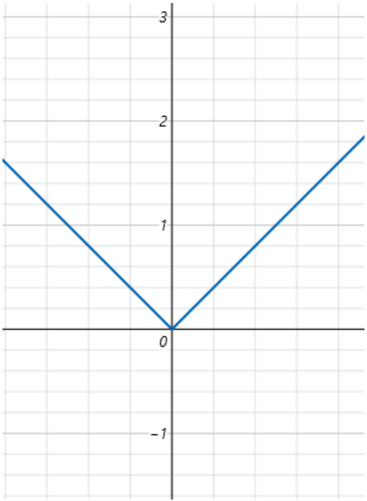
What about the derivative of $g(x) = x\ x$?

What about the derivative of $k(x) = |x|$?

What about the derivative of $h(x) = x^2$?

What about the derivative of $\sin(x)$?

What about the derivative of $\exp(x)$ or e^x?



Please remembers, where there is a rate of change, there is a derivative.

L'Hôpital Rule

Let f and g be two functions such that $f(c) = g(c) = 0$. Then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}.$$

Example:

$$\lim_{x \to 0} \frac{\sin(x)}{x} = \lim_{x \to 0} \frac{\cos(x)}{1} = \cos(0) = 1$$

Let's solve few questions.