



COMSATS University Islamabad

Attock Campus

Department of Mathematics

Quiz/Assignment # 03

Class: RMT & PMT
Subject: Topology
Instructor: Dr. Atiq ur Rehman

Due Date: 18-11-2024
Course Code: MTH631, MTH731
Marks: 20

Note: Every student must submit a handwritten assignment.

Question # 1: Prove that the open interval $(-1,1)$ is homeomorphic to \mathbb{R} under usual topologies.

Question # 2: Consider a function $f: \mathbb{Z} \rightarrow \mathbb{N}$ define by

$$f(n) = \begin{cases} 2n & n \geq 0, \\ -2k + 1 & n < 0. \end{cases}$$

Under this function prove that \mathbb{Z} is homomorphic to \mathbb{N} under discrete topological spaces.

Question # 3: Let a real function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. Prove that f is not open.

Question # 4: Show that the function $f: (0, \infty) \rightarrow [-1,1]$ defined by $f(x) = \sin\left(\frac{1}{x}\right)$ is continuous but neither open nor closed, where $(0, \infty)$ and $[-1,1]$ have the relativized usual topologies.

Academic Honesty Requirements:

You are encouraged to work with others in the completion of assignments, but it doesn't include copying. However, in the spirit of Academic Honesty, which includes crediting others for their contribution to your work, please include one of the following statements with every submitted assignment on title page:

1. **I worked alone on this assignment and write myself.**
2. **I worked with the following: List their full names. Include their relationship to you if they are not also a member of this class.**