COMSATS University Islamabad
Attock Campus
Department of Mathematics
Quiz/Assignment # 04

Class: RMT & PMT   **Due Date:** 26-12-2024
Subject: Topology   **Course Code:** MTH631, MTH731
Instructor: Dr. Atiq ur Rehman   **Marks:** 20

Note: Every student must submit a handwritten assignment. (or email scanned copy at atiq@cuiatk.edu.pk with file name as you registration number e.g. SP24-RMT-057.pdf)

Question # 1: State Urysohn's lemma. Let $X = \mathbb{R}^2$ with usual topology. After considering two disjoint closed sets of X, construct a function which satisfy the Urysohn's lemma.

Question # 2: A topological space X is a T_1-space if and only if every singleton subset $\{p\}$ of X is closed. If $X = \{\{1\}, \{1,2\}\}$, then construct T_1-space using the above theorem.

Question # 3: Consider the metric spaces on $\mathbb{R}^2$ for $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$ as:
$$d_1(x,y) = |x_1 - y_1| + |x_2 - y_2|,$$
$$d_2(x,y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2},$$
$$d_\infty(x,y) = max\{|x_1 - y_1|, |x_2 - y_2|\}.$$
Draw open spheres and what are the induced topologies form by these metrics.

Academic Honesty Requirements:

You are encouraged to work with others in the completion of assignments, but it doesn't include copying. However, in the spirit of Academic Honesty, which includes crediting others for their contribution to your work, please include one of the following statements with every submitted assignment on title page:

1. **I worked alone on this assignment and write myself.**
2. **I worked with the following: List their full names. Include their relationship to you if they are not also a member of this class.**