OBJECTIVE

Mathematics

12

For Intermediate Students

Salient Features

- Summary
- Definitions
- MCQ'S
- Important S/Q
- Important L/Q

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Dedication

Dedicated to my Students

CONTENTS

UNIT 1

Functions and Limits

DEFINITIONS + SUMMARY

FUNCTION

Dependence of one quantity to another quantity is called ***Function.*** **OR** A function is a rule or correspondence, relating two sets in such a way that each element in the first set corresponds to one and only one element in the second set.

Example:- The area "*A*" of a square depends on one of its sides "*x*" by the formula $A = x^2$, so we say that *A* is a function of *x*.

DEFINITION (FUNCTION-DOMAIN-RANGE)

A ***Function*** *f* from a set *X* to a set *Y* is a rule or a correspondence that assigns to each element *x* in *X* a unique element *y* in *Y*. The set *X* is called the ***domain*** of ***f***. The set of corresponding elements *y* in *Y* is called the ***range*** of ***f***.

NOTATION AND VALUE OF A FUNCTION

If a variable *y* depends on a variable *x* in such a way that each value of *x* determines exactly one value of *y*, then we say that **"*y* is a function of *x*"**.
Swiss mathematician Euler (1707-1783) invented a symbolic way to write the statement "*y* is a function of *x*" as ***y* = *f*(*x*),** which is read as "*y* is equal to *f* of *x*".
The variable *x* is called the ***independent variable*** of *f*, and the variable *y* is called the ***dependent variable*** of *f*.

> **Note**
> Functions are often denoted by the letters such as *f, g, h, F, G, H* and so on.

TYPES OF FUNCTIONS

1. POLYNOMIAL FUNCTION

A function *P* of the form $P(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0$ for all *x*, where the coefficients $a_n, a_{n-1}, a_{n-2}, \ldots, a_2, a_1, a_0$ are real numbers and the exponents are non-negative integers, is called a ***polynomial function***. The domain and range of *P*(*x*) are, in general, subsets of real numbers.
Example:- $P(x) = 2x^4 - 3x^3 + 2x - 1$ is a **polynomial function** of degree 4.

2. LINEAR FUNCTION

If the degree of a polynomial function is 1, then it is called a ***linear function***. A linear function is of the form: $f(x) = ax + b\ (a \neq 0), a, b$ are real numbers.

Example:- $f(x) = 3x + 4$ or $y = 3x + 4$ is a **linear function**. Its domain and range are the set of real numbers.

3. IDENTITY FUNCTION

For any set *X*, a function $I : X \to X$ of the form $I(x) = x \ \forall\, x \in X$, is called an ***identity function***. Its domain and range is the set *X* itself.
Example:- if $X = R$, then $I(x) = x$, for all $x \in R$ is the **identity function**.

4. CONSTANT FUNCTION

Let X and Y be sets of real numbers. A function $C : X \to Y$ defined by $C(x) = a, \forall x \in X, a \in Y$ and fixed is called a ***constant function***.

Example:- $C : R \to R$ defined by $C(x) = 2, \forall x \in R$ is a **constant function**

5. RATIONAL FUNCTION

A function $R(x)$ of the form $\frac{P(x)}{Q(x)}$ where both $P(x)$ and $Q(x)$ are polynomial functions and $Q(x) \neq 0$, is called a ***rational function***. The domain of a rational function $R(x)$ is the set of all real numbers x for which $Q(x) \neq 0$.

Example:- $\frac{x^2-2}{x-1}, \frac{2x^2-3x-3}{x^2+9}$

6. TRIGONOMETRIC FUNCTION

The functions $y = \sin x . y = \cos x, y = \tan x, y = \operatorname{cosec} x, y = \sec x, y = \cot x$ are called ***Trigonometric Functions.***

DOMAIN & RANGE OF TRIGONOMETRIC FUNCTIONS

Function	Domain	Range
$y = \sin x$	Set of all Real Numbers **OR** $-\infty \le x \le \infty$	$-1 \le y \le 1$
$y = \cos x$	Set of all Real Numbers **OR** $-\infty \le x \le \infty$	$-1 \le y \le 1$
$y = \tan x$	$R - \left\{x \mid x = \left(\frac{2n+1}{2}\right)\pi, n \in z\right\}$ **OR** $-\infty \le x \le \infty, x \neq \left(\frac{2n+1}{2}\right)\pi, n \in z$	Set of all Real Numbers **OR** $-\infty \le y \le \infty$
$y = \cot x$	$R - \{x \mid x = n\pi, n \in z\}$ **OR** $-\infty \le x \le \infty, x \neq n\pi, n \in z$	Set of all Real Numbers **OR** $-\infty \le y \le \infty$
$y = \sec x$	$R - \left\{x \mid x = \left(\frac{2n+1}{2}\right)\pi, n \in z\right\}$ **OR** $-\infty \le x \le \infty, x \neq \left(\frac{2n+1}{2}\right)\pi, n \in z$	$y \ge 1 \text{ or } y \le -1$
$y = \operatorname{cosec} x$	$R - \{x \mid x = n\pi, n \in z\}$ **OR** $-\infty \le x \le \infty, x \neq n\pi, n \in z$	$y \ge 1 \text{ or } y \le -1$

7. INVERSE TRIGONOMETRIC FUNCTION

The functions $y = \sin^{-1} x . y = \cos^{-1} x, y = \tan^{-1} x, y = \operatorname{cosec}^{-1} x, y = \sec^{-1} x, y = \cot^{-1} x$ are called ***Inverse Trigonometric Functions.***

DOMAIN & RANGE OF INVERSE TRIGONOMETRIC FUNCTIONS

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \leq x \leq 1$	Set of all Real Numbers **OR** $-\infty \leq y \leq \infty$
$y = \cos^{-1} x$	$-1 \leq x \leq 1$	Set of all Real Numbers **OR** $-\infty \leq y \leq \infty$
$y = \tan^{-1} x$	Set of all Real Numbers **OR** $-\infty \leq x \leq \infty$	$R - \left\{y \mid y = \left(\frac{2n+1}{2}\right)\pi, n \in z\right\}$ **OR** $-\infty \leq y \leq \infty, y \neq \left(\frac{2n+1}{2}\right)\pi, n \in z$
$y = \cot^{-1} x$	Set of all Real Numbers **OR** $-\infty \leq x \leq \infty$	$R - \{y \mid y = n\pi, n \in z\}$ **OR** $-\infty \leq y \leq \infty, y \neq n\pi, n \in z$
$y = \sec^{-1} x$	$x \geq 1 \; or \; x \leq -1$	$R - \left\{y \mid y = \left(\frac{2n+1}{2}\right)\pi, n \in z\right\}$ **OR** $-\infty \leq y \leq \infty, y \neq \left(\frac{2n+1}{2}\right)\pi, n \in z$
$y = \operatorname{cosec}^{-1} x$	$x \geq 1 \; or \; x \leq -1$	$R - \{x \mid x = n\pi, n \in z\}$ **OR** $-\infty \leq x \leq \infty, x \neq n\pi, n \in z$

8. EXPONENTIAL FUNCTION

A function, in which the variable appears as exponent (power), is called an ***exponential function***.

Example:- $y = e^x, y = e^{ax}, y = 2^x, y = e^{x \ln 2}$ etc.

9. LOGARITHMIC FUNCTION

The functions $f(x) = log_a x$, where $a > 0, a \neq 1$ is called ***Logarithmic Function*** of x.

(i) If $a = 10$, then we have $log_{10}x$ (written as $\log x$) which is known as the ***common logarithm*** of x.

(ii) If $a = e$, then we have $log_e x$ (written as $In\, x$) which is known as the ***natural logarithm*** of x.

Note	$If\ x = a^y \Leftrightarrow y = log_a x$

10.HYPERBOLIC FUNCTION

$\sinh x = \dfrac{e^x - e^{-x}}{2}$	$\text{cosech}\, x = \dfrac{2}{e^x - e^{-x}}$
$\cosh x = \dfrac{e^x + e^{-x}}{2}$	$\text{sech}\, x = \dfrac{2}{e^x + e^{-x}}$
$\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$	$\coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

11.INVERSE HYPERBOLIC FUNCTIONS

The ***inverse hyperbolic functions*** are expressed in terms of natural logarithms.

$\sinh^{-1} x = ln\left(x + \sqrt{x^2+1}\right), \forall\, x$	$\text{cosech}^{-1} x = ln\left(\dfrac{1}{x} + \dfrac{\sqrt{1+x^2}}{\lvert x\rvert}\right), x \neq 0$
$\cosh^{-1} x = ln\left(x + \sqrt{x^2-1}\right), x \geq 1$	$\text{sech}^{-1} x = ln\left(\dfrac{1}{x} + \dfrac{\sqrt{1-x^2}}{x}\right), 0 < x \leq 1$
$\tanh^{-1} x = \dfrac{1}{2} ln\left(\dfrac{1+x}{1-x}\right), \lvert x\rvert < 1$	$\coth^{-1} x = \dfrac{1}{2} ln\left(\dfrac{x+1}{x-1}\right), \lvert x\rvert < 1$

12.EXPLICIT FUNCTION

If **"*y*"** is easily expressed in terms of the independent variable **"*x*",** then **"*y*"** is called an ***explicit function*** of **"*x*"**. Symbolically it can be written as $y = f(x)$.
Example:- $(i)\ y = x^2 + 2x - 1\ (ii)\ y = x - 1$ are explicit functions of *x*.

13.IMPLICIT FUNCTION

If ***x*** and ***y*** are so mixed up and *y* cannot be expressed in terms of the independent variable *x*, then *y* is called an ***implicit function*** of *x*. Symbolically it is written as $f(x, y) = 0$.

Example:- $(i)\ x^2 + xy + y^2 = 2\ \ (ii)\ \frac{xy^2 - y + 9}{xy} = 1$ are implicit functions of x and y.

14.PARAMETRIC FUNCTIONS

Sometimes a curve is described by expressing both *x* and *y* as function of a third variable "*t*" or "θ" which is called a parameter. The equations of the type $x = f(t)$ and $y = g(t)$ are called ***the parametric equations*** of the curve.
Example: - The functions of the form:
(i) $x = at^2, y = at$
(ii) $x = a\cos\theta, y = a\sin\theta$
are called **parametric functions**. Here the variable *t* or θ is called parameter.

15.EVEN FUNCTION

A function f is said to be even if $f(-x) = f(x)$, for every number x in the domain of f.

Example:- $f(x) = x^2$ and $f(x) = cos\ x$ are even functions of x.
Here $f(-x) = (-x)^2 = x^2 = f(x)$ and $f(-x) = cos\ (-x) = cos\ x = f(x)$

16.ODD FUNCTION

A function f is said to be odd if $f(-x) = -f(x)$, for every number x in the domain of f.

Example:- $f(x) = x^3$ and $f(x) = sin\ x$ are odd functions of x.
Here $f(-x) = (-x)^3 = -x^3 = -f(x)$ and $f(-x) = sin\ (-x) = -sin\ x = -f(x)$

COMPOSITION OF FUNCTION

Let f be a function from set X to set Y and g be a function from set Y to set Z. The composition of f and g is a function, denoted by gof, from X to Z and is defined by

$(gof)(x) = g(f(x)) = gf(x)$, for all $x \in X$.

Note **(i)** In general, $gf\ (x) \neq fg\ (x)$.

(ii) We usually write ff as f^2 and fff as f^3 and so on.

INVERSE OF A FUNCTION

Let f be a one-one function from X onto Y. The **inverse function** of f denoted by f^{-1}, is a function from Y onto X and is defined by: $x = f^{-1}(y), \forall\ y \in Y$ if and only if $y = f(x)$, $\forall\ x \in X$.

LIMIT OF A FUNCTION

Let a function $\boldsymbol{f(x)}$ be defined in an open interval near the number **"*a*"** (need not be at a).
If, as x approaches **"*a*"** from both left and right side of **"*a*"**, $\boldsymbol{f(x)}$ approaches a specific number **"*L*"** then **"*L*"**, is called the **limit of $\boldsymbol{f(x)}$** as x approaches a.
Symbolically it is written as:

$$\lim_{x \to a} f(x) = L \text{ read as "limit of } f(x), \text{as } x \to a \text{, is } L\text{".}$$

THEOREM ON LIMITS OF FUNCTION

Let f and g be two functions, for which $\lim_{x \to a} f(x) = L$ *and* $\lim_{x \to a} g(x) = M$, *then*

Theorem 1: - The limit of the sum of two functions is equal to the sum of their limits.

$$\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = L + M$$

Theorem 2: - The limit of the difference of two functions is equal to the difference of their limits.

$$\lim_{x \to a}[f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = L - M$$

Theorem 3: - If k is any real number, then

$$\lim_{x \to a} [kf(x)] = k \lim_{x \to a} f(x) = kL$$

Theorem 4: - The limit of the product of the functions is equal to the product of their limits.

$$\lim_{x\to a}[f(x)g(x)] = [\lim_{x\to a} f(x)][\lim_{x\to a} g(x)] = LM$$

Theorem 5: - The limit of the quotient of the functions is equal to the quotient of their limits provided the limit of the denominator is non-zero.

$$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L}{M}$$

Theorem 6: - Limit of $[f(x)]^n$ where n is an integer.

$$\lim_{x\to a}[f(x)]^n = \left(\lim_{x\to a} f(x)\right)^n$$

LIMITS OF IMPORTANT FUNCTION

1. $\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$. where n is an integer $a > 0$
2. $\lim_{x\to 0}\frac{\sqrt{x+a}-\sqrt{a}}{x} = \frac{1}{2\sqrt{a}}$
3. $(i)\ \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$ $(ii)\ \lim_{x\to 0}(1+x)^{1/x} = e$
4. $(i)\ \text{Lim}_{x\to 0}\frac{a^x-1}{x} = \log_e a$ $(ii)\ \text{Lim}_{x\to 0}\frac{e^x-1}{x} = \log_e e = 1$
5. $\text{Lim}_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$ (Where θ is measured in radian)

THE SANDWICH THEOREM

Let f, g and h be three functions such that $f(x) \leq g(x) \leq h(x)$ for all numbers x in some open interval $(\forall x \in (a,b))$ containing "c", except possibly at c itself.

If $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = L$, then $\lim_{x\to c} h(x) = L$

CONTINUITY OF A FUNCTION

1. CONTINUOUS FUNCTION

A function f is said to be ***continuous*** at a number "c" if and only if the following three conditions are satisfied:

(i) $f(c)$ is defined. (ii) $\lim_{x\to c} f(x)$ exists (iii) $\lim_{x\to c} f(x) = f(c)$

2. DISCONTINUOUS FUNCTION

If one or more of these three conditions fail to hold at "c", then the function f is said to be ***discontinuous*** at "c".

MCQ's

Choose the correct answer.

1	The term which are used to explain the relationship between the variables or quantities are called						
a	Domain	b	Range	c	Function	d	Formula
2	The term function was recognized by a German Mathematician						
a	Leibnitz	b	Newton	c	Euler	d	Cauchy
3	The area A of a square depends on its						
a	sides	b	diagonals	c	radius	d	none
4	A _____ is a rule or correspondence, relating two sets in such a way that each element in the first set corresponds to one and only one element in the second set.						
a	Domain	b	Range	c	Function	d	Formula
5	If y is the function of x, the mathematically it can be written as						
a	$x = y$	b	$x = y(x)$	c	$y = f(x)$	d	$y = x^{-1}$
6	Which mathematician invented a symbolic way to write the statement " y is the function of x"						
a	Leibnitz	b	Newton	c	Euler	d	Cauchy
7	If y is the function of x i.e. $y = f(x)$ then x is called						
a	Dependent variable	b	Independent variable	c	constants	d	Both a & b
8	If y is the function of x i.e. $y = f(x)$ then y is called						
a	Dependent variable	b	Independent variable	c	constants	d	Both a & b
9	If y is the function of x, the mathematically it can be written as						
a	$y = f(x)$	b	$f: x \to y$	c	$x = y(x)$	d	Both a & b
10	If $f: x \to y$ be a function then x is called						
a	Domain	b	Range	c	Co-domain	d	Both a and c
11	If $f: x \to y$ be a function then y is called						
a	Domain	b	Range	c	Co-domain	d	None
12	The area **A** of a circle as a function of its circumference **C** is						
a	πr^2	b	$2\pi r$	c	$\frac{C^2}{4\pi}$	d	$\frac{C}{2\pi}$

13	The volume **V** of a cube as a function of its base						
a	$A^{2/3}$	b	$2A$	c	$A^{3/2}$	d	$4A$
14	The parameter P of a square as a function of its area A is						
a	$\sqrt{A}$	b	$2\sqrt{A}$	c	$3\sqrt{A}$	d	$4\sqrt{A}$
15	If a function is define on R (set of real numbers) then it is called						
a	Complex valued	b	Real Valued	c	Linear	d	none
16	A function which is defined by algebraic expressions are called _____ functions.						
a	Trigonometric	b	Hyperbolic	c	Inverse hyperbolic	d	Algebraic
17	A function in which variable appear as an exponent is called ____ function.						
a	Hyperbolic	b	Exponential	c	Rational	d	None of these
18	A function of the form $I(x) = x, \forall\, x \in X$ (X be any set) is called _____ function.						
a	Identity	b	Constant	c	Linear	d	Rational
19	If $C: R \rightarrow R$ define by $C(x) = 2, \forall\, x \in R$ is called						
a	Identity	b	Constant	c	Linear	d	Rational
20	If the degree of polynomial function is 1 then it is called _____ function.						
a	Identity	b	Constant	c	Linear	d	None of these
21	$f(x) = 2x^4 - 3x^3 + 2x - 1$ is a polynomial function of degree						
a	4	b	3	c	2	d	1
22	The domain & range of the polynomial function in general is						
a	Natural numbers	b	Real numbers	c	Non-negative real numbers	d	Positive real numbers
23	The domain and range of identity function is						
a	Natural numbers	b	Real numbers	c	Non-negative real numbers	d	Positive real numbers
24	Which one is constant function?						
a	$f(x) = x$	b	$f(x) = cotx$	c	$f(x) = 5$	d	None of these
25	If $f(x) = x^2$, then domain of f is						
a	Set of all real numbers	b	Set of all non-negative numbers	c	Set of natural numbers	d	None of these
26	If $f(x) = x^2$, then range of f is						
a	Set of all real numbers	b	Set of all non-negative numbers	c	Set of natural numbers	d	None of these

27	If $f(x)=\frac{x}{x^2-4}$, then domain of f is						
a	Set of all real numbers	b	Set of all non-negative numbers	c	Set of natural numbers	d	Set of all real numbers except -2 & 2
28	If $f(x)=\frac{x}{x^2-4}$, then range of f is						
a	Set of all real numbers	b	Set of all non-negative numbers	c	Set of natural numbers	d	Set of all real numbers except -2 & 2
29	Let $f(x)=\sqrt{x^2-9}$ then domain of f is						
a	All real numbers	b	$[3,+\infty)$	c	$(-\infty,-3]\cup[3,+\infty)$	d	$(-\infty,-3]$
30	If $f(x)=x^2+1$, then domain of f is						
a	Set of all real numbers	b	Set of all non-negative numbers	c	Set of natural numbers	d	Set of all real numbers except -2 & 2
31	If $f(x)=\sqrt{x+1}$ then domain of f is						
a	$[-1,\infty)$	b	$(-\infty,\infty)$	c	$[0,\infty)$	d	$[-1,1]$
32	If $f(x)=\sqrt{x+1}$ then range of f is						
a	R	b	$(-\infty,\infty)$	c	$[0,\infty)$	d	$[-1,1]$
33	The range of $f(x)=2+\sqrt{x-1}$						
a	$(-1,\infty)$	b	$[0,\infty)$	c	$[2,\infty)$	d	$[-2,\infty)$
34	Domain of sine function is						
a	All Real Numbers	b	$R-\left\{x\mid x=\left(\frac{2n+1}{2}\right)\pi, n\in z\right\}$	c	$R-\{x\mid x=n\pi, n\in z\}$	d	All natural Numbers
35	Domain of cosine function is						
a	All Real Numbers	b	$R-\left\{x\mid x=\left(\frac{2n+1}{2}\right)\pi, n\in z\right\}$	c	$R-\{x\mid x=n\pi, n\in z\}$	d	All natural Numbers
36	Domain of tangent function is						
a	All Real Numbers	b	$R-\left\{x\mid x=\left(\frac{2n+1}{2}\right)\pi, n\in z\right\}$	c	$R-\{x\mid x=n\pi, n\in z\}$	d	All natural Numbers
37	Domain of cotangent function is						

a	All Real Numbers	b	$R-\{x\mid x=\left(\frac{2n+1}{2}\right)\pi, n\in z\}$	c	$R-\{x\mid x=n\pi, n\in z\}$	d	All natural Numbers
38	Domain of secant function is						
a	All Real Numbers	b	$R-\{x\mid x=\left(\frac{2n+1}{2}\right)\pi, n\in z\}$	c	$R-\{x\mid x=n\pi, n\in z\}$	d	All natural Numbers
39	Domain of cosecant function is						
a	All Real Numbers	b	$R-\{x\mid x=\left(\frac{2n+1}{2}\right)\pi, n\in z\}$	c	$R-\{x\mid x=n\pi, n\in z\}$	d	All natural Numbers
40	The domain of $y=\sin^{-1}x$ is						
a	$-1\le x\le 1$	b	All real numbers	c	$x\ge 1$ or $x\le -1$	d	$0<x<\pi$
41	The domain of $y=\cos^{-1}x$ is						
a	$-1\le x\le 1$	b	All real numbers	c	$x\ge 1$ or $x\le -1$	d	$0<x<\pi$
42	The domain of $y=\tan^{-1}x$ is						
a	$-1\le x\le 1$	b	All real numbers	c	$x\ge 1$ or $x\le -1$	d	$0<x<\pi$
43	The domain of $y=\cot^{-1}x$ is						
a	$-1\le x\le 1$	b	All real numbers	c	$x\ge 1$ or $x\le -1$	d	$0<x<\pi$
44	The domain of $y=\sec^{-1}x$ is						
a	$-1\le x\le 1$	b	All real numbers	c	$x\ge 1$ or $x\le -1$	d	$0<x<\pi$
45	The domain of $y=\text{cosec}^{-1}x$ is						
a	$-1\le x\le 1$	b	All real numbers	c	$x\ge 1$ or $x\le -1$	d	$0<x<\pi$
46	$\sinh x=$						
a	$\frac{e^x-e^{-x}}{2}$	b	$\frac{e^x+e^{-x}}{2}$	c	$\frac{e^x-e^{-x}}{e^x+e^{-x}}$	d	$\frac{2}{e^x+e^{-x}}$
47	$\cosh x=$						
a	$\frac{e^x-e^{-x}}{2}$	b	$\frac{e^x+e^{-x}}{2}$	c	$\frac{e^x-e^{-x}}{e^x+e^{-x}}$	d	$\frac{2}{e^x+e^{-x}}$
48	$\tanh x=$						
a	$\frac{e^x-e^{-x}}{2}$	b	$\frac{e^x+e^{-x}}{2}$	c	$\frac{e^x-e^{-x}}{e^x+e^{-x}}$	d	$\frac{2}{e^x+e^{-x}}$
49	If $y=\log_a x$ and $a=10$ then y is known as						
a	Common logarithm	b	Natural logarithm	c	Exponential function	d	None of these

50	If $y = log_a x$ and $a = e$ then y is known as						
a	Common logarithm	b	Natural logarithm	c	Exponential function	d	None of these
51	If y is easily expressed in terms of independent variable x, then y is called ___function.						
a	Implicit	b	Explicit	c	Parametric	d	Even
52	Symbolically explicit function written as						
a	$y = f(x)$	b	$f(x, y) = 0$	c	$y = f(-x)$	d	None of these
53	$y = x^2 + 2x - 1$ is example of _____ function						
a	Implicit	b	Explicit	c	Parametric	d	Even
54	If $\boldsymbol{x}$ and $\boldsymbol{y}$ are so mixed up and y cannot be expressed in terms of the independent variable x, then y is called a/an ____ function.						
a	Implicit	b	Explicit	c	Parametric	d	Even
55	Symbolically implicit function written as						
a	$y = f(x)$	b	$f(x, y) = 0$	c	$y = f(-x)$	d	None of these
56	$x^2 + xy + y^2 = 2$ is example of _____ function.						
a	Implicit	b	Explicit	c	Parametric	d	Even
57	The functions of the form $x = at^2, y = at$ is known as ____ function.						
a	Implicit	b	Explicit	c	Parametric	d	Even
58	The equations $x = acost, y = asint$ represents						
a	Circle	b	Line	c	Parabola	d	Hyperbola
59	The equations $x = at^2, y = 2at$ represents						
a	Circle	b	Line	c	Parabola	d	Hyperbola
60	$\cosh^2 - \sinh^2 x =$ _____						
a	$\sinh^2 x$	b	0	c	1	d	2
61	$\sinh^{-1} x =$						
a	$ln\left(x+\sqrt{x^2+1}\right)$	b	$ln\left(x+\sqrt{x^2-1}\right)$	c	$\frac{1}{2}ln\left(\frac{1+x}{1-x}\right)$	d	$\frac{1}{2}ln\left(\frac{x+1}{x-1}\right)$
62	$\cosh^{-1} x$						
a	$ln\left(x+\sqrt{x^2+1}\right)$	b	$ln\left(x+\sqrt{x^2-1}\right)$	c	$\frac{1}{2}ln\left(\frac{1+x}{1-x}\right)$	d	$\frac{1}{2}ln\left(\frac{x+1}{x-1}\right)$
63	$\tanh^{-1} x$						
a	$ln\left(x+\sqrt{x^2+1}\right)$	b	$ln\left(x+\sqrt{x^2-1}\right)$	c	$\frac{1}{2}ln\left(\frac{1+x}{1-x}\right)$	d	$\frac{1}{2}ln\left(\frac{x+1}{x-1}\right)$

64	$\coth^{-1} x$						
a	$ln\left(x+\sqrt{x^2+1}\right)$	b	$ln\left(x+\sqrt{x^2-1}\right)$	c	$\frac{1}{2}ln\left(\frac{1+x}{1-x}\right)$	d	$\frac{1}{2}ln\left(\frac{x+1}{x-1}\right)$
65	$\text{sech}^{-1} x$						
a	$ln\left(\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right)$	b	$ln\left(\frac{1}{x}+\frac{\sqrt{1+x^2}}{\lvert x\rvert}\right)$	c	$\frac{1}{2}ln\left(\frac{1+x}{1-x}\right)$	d	$\frac{1}{2}ln\left(\frac{x+1}{x-1}\right)$
66	$\text{cosech}^{-1} x$						
a	$ln\left(\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right)$	b	$ln\left(\frac{1}{x}+\frac{\sqrt{1+x^2}}{\lvert x\rvert}\right)$	c	$\frac{1}{2}ln\left(\frac{1+x}{1-x}\right)$	d	$\frac{1}{2}ln\left(\frac{x+1}{x-1}\right)$
67	If $f(x)=x^3-2x^2+4x+1$ then $f(-2)=$ __						
a	-1	b	2	c	-25	d	-23
68	If $f(x)=2^x-x$, then $f(0)=$ __						
a	1	b	0	c	-1	d	3
69	If $f(x)=\sqrt{x-12}$ then $f(16)=$ ___						
a	16	b	12	c	28	d	2
70	If $f(x)=\sqrt{x+4}$ then $f(x^2+4)=$ ___						
a	$\sqrt{x^2+4}$	b	$\sqrt{x^2+8}$	c	$\sqrt{x^2-8}$	d	$x-8$
71	If $f(x)=x^{2/3}+6$ then $f(0)=$ ___						
a	1	b	4	c	6	d	0
72	If $f(x)=x^2-x$ then $f(-2)=$ ___						
a	2	b	0	c	6	d	-6
73	If $f(x)=2x+5$ then $f(2)=$ ___						
a	1	b	9	c	-9	d	10
74	If $f(x)=2x^2+4x+2$ then $f(-2)=$ ____						
a	0	b	1	c	2	d	-2
75	The parametric equation $x=a\cos t, y=a\sin t$ represents the equation						
a	$x^2+y^2=a^2$	b	$y^2=4ax$	c	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	d	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
76	The parametric equation $x=at^2, y=2at$ represents parametric equation						
a	$x^2+y^2=a^2$	b	$y^2=4ax$	c	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	d	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
77	The parametric equation $x=a\cos\theta, y=b\sin\theta$ represents the equation						

a	$x^2 + y^2 = a^2$	b	$y^2 = 4ax$	c	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	d	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
78	The parametric equation $x = a\sec\theta, y = b\tan\theta$ represents the equation						
a	$x^2 + y^2 = a^2$	b	$y^2 = 4ax$	c	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	d	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
79	The function $y = a^x$ is called ____ function						
a	Algebraic	b	Trigonometric	c	Exponential	d	Identity
80	A function f is said to be ____ if $f(-x) = f(x)$.						
a	Implicit	b	Explicit	c	Parametric	d	Even
81	A function f is said to be ____ if $f(-x) = -f(x)$.						
a	Implicit	b	Explicit	c	Odd	d	Even
82	If $f(x) = sin\, x$ then $f(x)$ is						
a	Even function	b	Odd function	c	Rational function	d	None of these
83	If $f(x) = cos\, x$ then $f(x)$ is						
a	Even function	b	Odd function	c	Rational function	d	None of these
84	The function $f(x) = (x+2)^2$ is						
a	Even function	b	Odd function	c	Both even & odd	d	Neither even nor odd
85	If $f(x) = x^3 - sinx$ then $f(x)$ is ___						
a	Constant function	b	Even function	c	Odd function	d	Neither even nor odd
86	If $f(x) = \frac{3x}{x^2+1}$ then $f(x)$ is ___						
a	Constant function	b	Even function	c	Odd function	d	Neither even nor odd
87	$f(x) = sinx + cosx$ is ____ function.						
a	Odd	b	Even	c	Both even & odd	d	Neither even nor odd
88	$f(x) = x^{2/3}$ is ____ function						
a	Odd	b	Even	c	Both even & odd	d	Neither even nor odd
89	If $f(x) = xcotx$ is ____ function.						
a	Constant	b	Quadratic	c	Even	d	Odd

90	If $f(x) = x^2 + cosx$ then $f(x)$ is ___						
a	Constant function	b	Even function	c	Odd function	d	Neither even nor odd
91	If $f(x) = sinx + cosx$ then $f(x) + f(-x) =$ ____						
a	$2sinx$	b	$2cosx$	c	$-2cosx$	d	0
92	If $f: x \rightarrow y$ be a function then inverse of f is define as						
a	$f^{-1}(y) = x$	b	$y = f(x)$	c	$f^{-1}(x) = y$	d	None of these
93	Only ____ function will have its inverse.						
a	On-to	b	In-to	c	Bijective	d	None of these
94	If $f(x) = -2x + 6$, then $f^{-1}(x) =$ _____						
a	$\frac{2-x}{6}$	b	$\frac{2}{6-x}$	c	$\frac{6-x}{2}$	d	$2x - 6$
95	If $f(x) = -2x + 8$, then $f^{-1}(x) =$ _____						
a	$\frac{x+8}{2}$	b	$\frac{2}{8-x}$	c	$\frac{8-x}{2}$	d	$\frac{x-8}{2}$
96	$fof^{-1}(x)$ is _____ function.						
a	Constant	b	Identity	c	Even	d	Exponential
97	If "f" be a bijective function then $f(f^{-1}(x))$ equal to						
a	x	b	$f(x)$	c	$f^{-1}(x)$	d	None of these
98	A rule that assigns to each element $x \in X$ a unique element $y \in Y$ is called a function from.						
a	X to X	b	Y to Y	c	X to Y	d	Y to X
99	If y is image of x under the function f, we write it as						
a	$x = f(x)$	b	$y \neq f(x)$	c	$y = f(x)$	d	$y = x$
100	The composition of two functions f and g is denoted by						
a	$gf(x)$	b	$(gof)x$	c	$g \times f$	d	Both a and b
101	If $f(x) = sinx$ and $g(x) = sin^{-1}x$ then $gof(x)$ is						
a	$sinx$	b	$sin^{-1}x$	c	x	d	None of these
102	If $f(x) = 2x + 1$ and $g(x) = x^2 - 1$ then $fg(x)$ is						
a	$x^4 - 2x^2$	b	$4x + 3$	c	$4x^2 + 4x$	d	$2x^2 - 1$
103	If $f(x) = 2x + 1$ and $g(x) = x^2 - 1$ then $g^2(x)$ is						
a	$x^4 - 2x^2$	b	$4x + 3$	c	$4x^2 + 4x$	d	$2x^2 - 1$

104	If $f(x) = 2x + 1$ and $g(x) = x^2 - 1$ then $gf(x)$ is						
a	$x^4 - 2x^2$	b	$4x + 3$	c	$4x^2 + 4x$	d	$2x^2 - 1$
105	If $f(x) = 2x + 1$ and $g(x) = x^2 - 1$ then $f^2(x)$ is						
a	$x^4 - 2x^2$	b	$4x + 3$	c	$4x^2 + 4x$	d	$2x^2 - 1$
106	If $f(x) = \frac{1}{x^2}$ then $fof =$ ___						
a	x^2	b	x^4	c	$\frac{1}{x^4}$	d	1
107	If $P(x)$ be a polynomial function then $\lim_{x\to c} P(x) =$ ____						
a	$P(x)$	b	$P(c)$	c	$cP(x)$	d	None of these
108	If x approaches **"a"** from both left and right side of **"a", $f(x)$** approaches a specific number **"L"** then **"L"**, is called						
a	Inverse of $f(x)$	b	Domain of $f(x)$	c	Range of $f(x)$	d	Limit of $f(x)$
109	If p be a positive rational number and x^p is defined, then $\lim_{x\to\infty} \frac{a}{x^p}$ is equal to						
a	p	b	x	c	∞	d	0
110	$\lim_{x\to a} \frac{x^n - a^n}{x - a} =$ _____						
a	a^{n-1}	b	na^{n-1}	c	na	d	1
111	$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n =$ _______						
a	1	b	e	c	n	d	∞
112	$\lim_{x\to 0} (1 + x)^{1/x} =$ ___						
a	1	b	e	c	n	d	∞
113	$\text{Lim}_{x\to 0} \frac{a^x - 1}{x} =$ ________						
a	$\log a$	b	$\log_a e$	c	$\log_e a$	d	1
114	$\text{Lim}_{x\to 0} \frac{e^x - 1}{x} =$ _________						
a	$\log a$	b	$\log_a e$	c	$\log_e a$	d	1
115	$\text{Lim}_{\theta\to 0} \frac{\sin\theta}{\theta} =$ ______ (where θ is measured in radian)						
a	e	b	-1	c	1	d	0
116	$\lim_{x\to\infty} (e^x) =$ ____						

a	1	b	e	c	n	d	∞
117	$\lim_{x\to\infty}\frac{1}{x} =$ ______						
a	∞	b	1	c	0	d	-1
118	$\lim_{h\to 0}(1+2h)^{1/h} =$ ____						
a	e^2	b	e	c	0	d	1
119	$\mathrm{Lim}_{\theta\to 0}\frac{\sin 7\theta}{\theta} =$ ______						
a	1	b	7	c	0	d	e
120	$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n =$ ______						
a	1	b	e	c	n	d	e^{-1}
121	$\lim_{n\to\infty}\left(1+\frac{1}{3n}\right)^n =$ ______						
a	e^2	b	e^3	c	$e^{1/3}$	d	$\frac{1}{e^3}$
122	$\lim_{x\to 1}\frac{x^3-x}{x-1} =$ ______						
a	3	b	2	c	1	d	0
123	$\mathrm{Lim}_{\theta\to 0}\frac{1-\cos p\theta}{1-\cos q\theta} =$						
a	$\frac{p^2}{q^2}$	b	$\frac{p}{q}$	c	0	d	1
124	$\mathrm{Lim}_{x\to 0}\frac{sinax}{sinbx} =$						
a	$\frac{b}{a}$	b	$\frac{a}{b}$	c	$\frac{a^2}{b^2}$	d	$\frac{b^2}{a^2}$
125	$\lim_{n\to 2}\frac{\sqrt{x}-\sqrt{2}}{x-2} =$ ______						
a	$2\sqrt{2}$	b	$\frac{1}{2\sqrt{2}}$	c	$\sqrt{2}$	d	∞
126	$\lim_{n\to 2}\frac{x-2}{\sqrt{x}-\sqrt{2}} =$ ______						
a	$2\sqrt{2}$	b	$\frac{1}{2\sqrt{2}}$	c	$\sqrt{2}$	d	∞

127	$\lim_{\theta\to 0}\frac{sin^2\theta}{\theta} =$ ______						
a	∞	b	0	c	1	d	e
128	$\lim_{x\to a}\frac{x^3-a^3}{x-a} =$ _____						
a	∞	b	$3a^2$	c	$2a^2$	d	$2a$
129	$\lim_{x\to a}\frac{x^2-a^2}{x-a} =$ _____						
a	∞	b	$3a^2$	c	$2a^2$	d	$2a$
130	$\lim_{x\to 0}\frac{x}{sin2x} =$ _____						
a	2	b	$\frac{1}{2}$	c	$-\frac{1}{2}$	d	-2
131	$\lim_{x\to 0}\frac{x}{tanx} =$ _____						
a	∞	b	0	c	1	d	a
132	A function f is said to be ***continuous*** at a number "c" if						
a	$f(c)$ is defined.	b	$\lim_{x\to c} f(x)$ exists	c	$\lim_{x\to c} f(x) = f(c)$	d	All of these
133	A function f is said to be ***discontinuous*** at a number "c" if						
a	$f(c)$ is defined.	b	$\lim_{x\to c} f(x)$ exists	c	$\lim_{x\to c} f(x) \neq f(c)$	d	None of these
134	$f(x) = 3x^2 - 5x + 5$ is continuous at						
a	2	b	3	c	-2	d	Set of real numbers
135	The function $f(x) = \frac{x}{x^2-4}$ is discontinuous at :						
a	0	b	1	c	± 2	d	± 1
136	Let f, g and h be three functions such that $f(x) \leq g(x) \leq h(x)$ for all numbers x in some open interval $(\forall x \in (a,b))$ containing "c", except possibly at c itself is called						
a	Quotient Theorem	b	Sandwich Theorem	c	Limit Theorem	d	None of these
137	If $f(x) = \frac{x^2-9}{x-3}$ then $\lim_{x\to -3} f(x) =$						
a	-3	b	∞	c	0	d	3

138	If $f(x) = \|x-5\|$ then $\lim_{x\to 5} f(x) =$						
a	0	b	∞	c	1	d	-1
139	If $f(x) = \begin{cases} 2x+1 & if\ 0 \le x \le 2 \\ 7-x & if\ 2 < x < 4 \end{cases}$ then $f(2) =$						
a	2	b	3	c	4	d	5
140	If $f(x) = \begin{cases} x+2 & if\ x \le -1 \\ c+2 & if\ x > -1 \end{cases}$ and $\lim_{x\to -1} f(x)$ exists then $c =$ ____						
a	2	b	-2	c	1	d	-1

MCQ's Answers Key

1	2	3	4	5	6	7	8	9	10
c	a	a	c	c	c	b	a	d	a
11	12	13	14	15	16	17	18	19	20
b	c	c	d	b	d	b	a	b	c
21	22	23	24	25	26	27	28	29	30
a	b	b	c	a	b	d	a	c	a
31	32	33	34	35	36	37	38	39	40
a	c	c	a	a	b	c	b	c	a
41	42	43	44	45	46	47	48	49	50
a	b	b	c	c	a	b	c	a	b
51	52	53	54	55	56	57	58	59	60
b	a	b	a	b	a	c	a	c	c
61	62	63	64	65	66	67	68	69	70
a	b	c	d	a	b	d	b	d	b
71	72	73	74	75	76	77	78	79	80
c	c	b	c	a	b	c	d	c	d
81	82	83	84	85	86	87	88	89	90
c	b	a	d	c	c	d	b	c	b
91	92	93	94	95	96	97	98	99	100
b	a	c	c	c	b	a	d	c	d
101	102	103	104	105	106	107	108	109	110
c	d	a	c	b	b	b	d	d	b
111	112	113	114	115	116	117	118	119	120
b	b	b	d	c	d	c	a	b	d
121	122	123	124	125	126	127	128	129	130
c	b	a	b	b	a	b	b	d	b
131	132	133	134	135	136	137	138	139	140
c	d	c	d	c	b	c	a	d	d

IMPORTANT SHORT QUESTIONS

1. What is function?
2. If $f(x) = x^3 - 2x^2 + 4x - 1$ then find (i) $f\left(\frac{1}{x}\right)$ $(ii)\ f(1)$ $(iii)\ f(1+x)$
3. Find domain and range of $f(x) = x^2$
4. Find domain and range of $f(x) = \frac{x}{x^2-4}$
5. Find domain and range of $f(x) = \sqrt{x^2 - 9}$
6. Define a polynomial function of degree n.
7. Define Linear Function. Give example.
8. Define Identity Function with example.
9. Define Constant Function. Give one example.
10. Define Rational Function. Give example.
11. Define Exponential Function. Give example.
12. What is Implicit Function? Give example.
13. What is Explicit Function? Give example.
14. Define Even Function. Give one example.
15. Define Odd Function. Give example.
16. Show that the parametric equation $x = a\cos t, y = a\sin t$ represents the equation of circle $x^2 + y^2 = a^2$
17. Prove the identity $\cosh^2 x - \sinh^2 x = 1$
18. Prove the identity $\cosh^2 x + \sinh^2 x = cosh2x$
19. Determine whether the given function $f(x) = \frac{3x}{x^2+1}$ is even or odd.
20. Determine whether the given function $f(x) = sinx + cosx$ is even or odd.
21. If $f(x) = x^2 - x$ then find $(i)\ f(x-1)$ $(ii)\ f(-2)$
22. For the function $f(x) = \sqrt{x+4}$, find $(i)\ f(x-1)$ $(ii)\ f(x^2+4)$
23. Find $\frac{f(a+h)-f(a)}{h}$ and simplify where $f(x) = 6x - 9$
24. Find $\frac{f(a+h)-f(a)}{h}$ and simplify where $f(x) = sinx$
25. Find $\frac{f(a+h)-f(a)}{h}$ and simplify where $f(x) = cosx$
26. Express perimeter **P** of a square as a function of its area **A**.
27. Express the area **A** of a circle as a function of its circumference **C**.
28. Express the volume **V** of a cube as a function of its base.
29. Find domain and range of $f(x) = \sqrt{x^2 - 4}$
30. Find domain and range of $f(x) = \sqrt{x+1}$
31. Find domain and range of $f(x) = |x - 3|$
32. Given $f(x) = x^3 - ax^2 + bx + 1$, if $f(2) = -3$ and $f(-1) = 0$. Find the values of a and b.
33. Show that $x = at^2, y = 2at$ represents parametric equation of parabola $y^2 = 4ax$.
34. Show that the parametric equation $x = a\cos\theta, y = b\sin\theta$ represents the equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

35. Show that the parametric equation $x = a\sec\theta, y = b\tan\theta$ represents the equation of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

36. Prove the identity $sinh2x = 2sinhx\, coshx$

37. Prove the identity $\text{sech}^2 x = 1 - \tanh^2 x$

38. Determine whether the given function $f(x) = x^3 + x$ is even or odd.

39. Determine whether the given function $f(x) = x\sqrt{x^2 + 5}$ is even or odd.

40. Check whether the given function $f(x) = x^{2/3} + 6$ is even or odd.

41. Determine whether the given function $f(x) = \frac{x^3 - x}{x^2 + 1}$ is even or odd.

42. For any real valued function 'f' and 'g' defined by $f(x) = 2x + 1$, $g(x) = x^2 - 1$. Find $(i)\ fog(x)\ (ii)\ gof(x)\ (iii)\ f^2(x)$

43. If $f(x) = 2x + 1$ then find $f^{-1}(x)$.

44. Without finding inverse, state domain & range of f^{-1} when $f(x) = 2 + \sqrt{x - 1}$

45. If $f(x) = 2x + 1, g(x) = \frac{3}{x-1},\ find\ fog(x)$.

46. For the function $f(x) = \sqrt{x + 1}$, $g(x) = \frac{1}{x^2}$ find $(i)\ fog(x)\ (ii)\ gof(x)$

47. For the function $f(x) = \frac{1}{\sqrt{x-1}}$, $g(x) = \frac{1}{x^2}$ find $(i)\ fog(x)\ (ii)\ gof(x)$

48. For the function $f(x) = 3x^4 - 2x^2, g(x) = \frac{2}{\sqrt{x}}$ find $(i)\ fog(x)\ (ii)\ gof(x)$

49. For the function $f(x) = \sqrt{x - 1}$, find $fof(x)$.

50. If $f(x) = 2x + 1, g(x) = x^2 - 1, find\ gof(x)$.

51. If $f(x) = \frac{1}{\sqrt{x-1}}, g(x) = (x^2 + 1)^2$ find $(i)\ fog(x)\ (ii)\ gof(x)\ (iii)\ fof(x)$

52. For any real valued function of $g(x) = \frac{1}{x^2}$, find $gog(x)$

53. For real valued function of $f(x) = 3x^3 + 7$, find $f^{-1}(x)$.

54. If $f(x) = -2x + 8$ then find $f^{-1}(x)$ and $f^{-1}(-1)$

55. If $f(x) = (-x + 9)^3$ then find $f^{-1}(x)$

56. Without finding inverse, state domain & range of f^{-1} when $f(x) = \frac{1}{x+3}$

57. Without finding inverse, state domain & range of f^{-1} when $f(x) = \frac{x-1}{x-4}, x \neq 4$.

58. Without finding inverse, state domain & range of f^{-1} when $f(x) = (x - 5)^2, x \geq 5$.

59. Prove that $\lim\limits_{x\to 0} \frac{\sqrt{x+a} - \sqrt{a}}{x} = \frac{1}{2\sqrt{a}}$

60. Evaluate $\lim\limits_{x\to 3} \frac{x-3}{\sqrt{x} - \sqrt{3}}$

61. Evaluate $\lim\limits_{x\to\infty} \frac{5x^4 - 10x^2 + 1}{-3x^3 + 10x^2 + 50}$

62. Evaluate $\lim\limits_{x\to+\infty} \frac{2-3x}{\sqrt{3+4x^2}}$

63. Evaluate $\lim\limits_{n\to\infty} \left(1 + \frac{3}{n}\right)^{2n}$

64. State Sandwich Theorem.

65. Evaluate $\lim\limits_{\theta\to 0} \frac{1 - cos\theta}{\theta}$

66. Evaluate $\lim\limits_{x\to -2} \frac{2x^3 + 5x}{3x - 2}$

67. Evaluate $\lim_{x\to -1}\frac{x^3-x}{x+1}$

68. Evaluate $\lim_{x\to 2}\frac{x^3-8}{x^2+x-6}$

69. Evaluate $\lim_{x\to 1}\frac{x^3-3x^2+3x-1}{x^3-x}$

70. Evaluate $\lim_{x\to a}\frac{x^n-a^n}{x^m-a^m}$

71. Evaluate $\lim_{x\to 0}\frac{\sin x^\circ}{x}$

72. Evaluate $\lim_{\theta\to 0}\frac{1-\cos\theta}{\sin\theta}$

73. Evaluate $\lim_{x\to \pi}\frac{\sin x}{\pi-x}$

74. Evaluate $\lim_{x\to 0}\frac{\sin ax}{\sin bx}$

75. Evaluate $\lim_{x\to 0}\frac{x}{\tan x}$

76. Evaluate $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$

77. Evaluate $\lim_{x\to 0}\frac{1-\cos x}{\sin^2 x}$

78. Express the limit $\lim_{x\to 0}(1+3x)^{2/x}$ in terms of e.

79. Express the limit $\lim_{x\to 0}(1+2x^2)^{1/x^2}$ in terms of e.

80. Evaluate $\lim_{h\to 0}(1-2h)^{1/h}$.

81. Evaluate $\lim_{x\to \infty}\left(\frac{x}{1+x}\right)^x$

82. Define Left Hand Limit & Right Hand Limit.

83. Give three conditions for a function $f(x)$ to be continuous at a number 'c'.

84. What is Discontinuous Function? Give any example and sketch graphically

85. Discuss continuity of $g(x)=\frac{x^2-9}{x-3}$ at $x=3$.

86. If $f(x)=2x^2+x-5$ then find left hand and right hand limit of $f(x)\ at\ x=1$

87. If $f(x)=|x-5|$ then find left hand and right hand limit of $f(x)\ at\ x=5$.

88. Discuss continuity of $f(x)\ at\ x=2$ when $f(x)=\begin{cases}2x+5 & if\ x\le 2\\ 4x+1 & if\ x>2\end{cases}$

89. If $f(x)=\begin{cases}x+2 & if\ x\le -1\\ c+2 & if\ x>-1\end{cases}$ find c so that $\lim_{x\to -1}f(x)$ exist.

90. Find the value of m, such that function is continuous at $x=3$ if $f(x)=\begin{cases}mx & if\ x<3\\ x^2 & if\ x\ge 3\end{cases}$

IMPORTANT LONG QUESTIONS

1. For the real valued function If $f(x) = (-x+9)^3$, find $(i)\ f^{-1}(x)$ $(ii)\ f^{-1}(-1)$ and verify that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

2. Evaluate $\lim\limits_{x\to 0}\frac{a^x-1}{x} = \log_e a$

3. Prove that $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

4. If θ is measured in radian then prove that $\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$

5. Evaluate $\lim\limits_{x\to 0}\frac{\sec x-\cos x}{x}$

6. Evaluate $\lim\limits_{\theta\to 0}\frac{1-\cos p\theta}{1-\cos q\theta}$

7. Evaluate $\lim\limits_{\theta\to 0}\frac{\tan\theta-\sin\theta}{\sin^3\theta}$

8. Express the limit in terms of e; $\lim\limits_{x\to 0}\frac{e^{1/x}-1}{e^{1/x}+1}$; $x<0$

9. Express the limit in terms of e; $\lim\limits_{x\to 0}\frac{e^{1/x}-1}{e^{1/x}+1}$; $x>0$

10. Discuss continuity of $f(x)$ at x=3 when $f(x) = \begin{cases} x-1 & if\ x<3 \\ 2x+1 & if\ x\geq 3\end{cases}$

11. Discuss the continuity of $f(x)$ at $x=1$;

If $f(x) = \begin{cases} 3x-1 & if\ x<1 \\ 4 & if\ x=1 \\ 2x & if\ x>1\end{cases}$

12. If $f(x) = \begin{cases} 3x & if\ x\leq -2 \\ x^2-1 & if -2<x<2 \\ 3 & if\ x\geq 2\end{cases}$

Discuss the continuity at $x=2$ and $x=-2$

13. Find the values of m and n, so that the given function is continuous at $x=3$;

$f(x) = \begin{cases} mx & if\ x<3 \\ n & if\ x=3 \\ -2x+9 & if\ x>3\end{cases}$

14. If $f(x) = \begin{cases} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} & ,x\neq 2 \\ k & ,x=2\end{cases}$

Find value of k so that $f(x)$ is continuous at $x=2$

UNIT

2

Differentiation

DEFINITIONS + SUMMARY

INCREMENT

In mathematics increment means "the difference between two values of the variables". If y is a function of x. A small change in the value of x is called an increment in x and it is denoted by δx.

$\delta x = (\ x\ of\ terminal\ point\ ) - \ (\ x\ of\ initial\ point\ )$

$\delta y = (\ y\ of\ terminal\ point\ ) - \ (\ y\ of\ initial\ point\ )$

Note

If $y = f(x)$ where $x \in D_f$ (Domain of f)

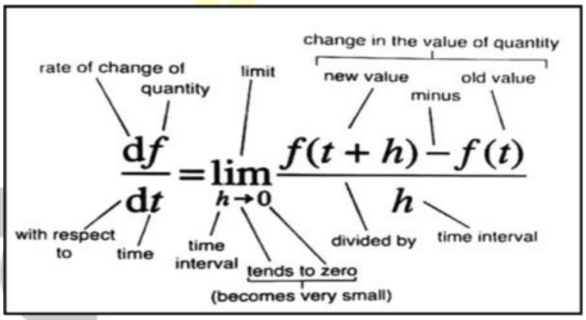
$\Rightarrow x$ is called independent variable and y is called dependent variable.

AVERAGE RATE OF CHANGE

Suppose a particle is moving in straight line and its positions after " t " and " t_1" are given by $s(t)$ and $s(t_1)$ then the quotient $\frac{s(t_1)-s(t)}{t_1-t}$ represents the average rate of change.

DERIVATIVE

If y be the function of x, then $\lim\limits_{\delta x \to 0} \frac{f(x+\delta x)-f(x)}{\delta x}$ is called derivative of $f(x)$ w.r.t x and is denoted by $f(x)$ or Dy or $\frac{dy}{dx}$.



NOTATION FOR DERIVATIVE

In a table the notations for the derivative of $y = f(x)$ used by different mathematicians:

Name of Mathematician	Leibnitz	Newton	Lagrange	Cauchy
Notation used for derivative	$\frac{dy}{dx}$ or $\frac{df}{dx}$	$\dot{f}(x)$	$f'(x)$	$Df(x)$

FINDING $f'(x)$ FROM DEFINITION OF DERIVATIVE

Given a function $y = f(x)$, $f'(x)$ if it exists, can be found by the following four steps:

Step I Find $f(x+\delta x)$

Step II Simplify $f(x+\delta x) - f(x)$

Step III Dividing $f(x+\delta x) - f(x)$ by δx to get $\frac{f(x+\delta x)-f(x)}{\delta x}$ and simplify it

Step IV Find $\lim\limits_{\delta x \to 0} \frac{f(x+\delta x)-f(x)}{\delta x}$

The method of finding derivatives by this process is called **differentiation** by **definition** or by **ab-initio** or from **first principle.**

THEOREM ON DIFFERENTIATION

1. POWER RULE

$\frac{d}{dx}(x^n) = nx^{n-1}$, where n is rational number.

2. DERIVATIVE OF CONSTANT

$\frac{d}{dx}(c) = 0$, derivative of a constant is zero.

3. SUM OR DIFFERENCE THEOREM

If "f" and "g" are differentiable at x, then

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x) = f'(x) \pm g'(x)$$

4. PRODUCT THEOREM

If "f" and "g" are differentiable at x, then

$$\frac{d}{dx}[f(x).g(x)] = \frac{d}{dx}[f(x)]g(x) + f(x)\frac{d}{dx}[g(x)] = f'(x)g(x) + f(x)g'(x)$$

5. QUOTIENT THEOREM

If "f" and "g" are differentiable at x, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

THE CHAIN RULE (DIFFERENTIATION OF COMPOSITE FUNCTION)

If $y = f(u)$ and $u = g(x)$ are two differentiable functions, then the derivative of the composition function $y = f(g(x))$ is given by

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

DIFFERENTIATION OF IMPLICIT FUNCTIONS

A function which contains two or more variables that are not independent of each other is called an ***implicit function***.

Example:- $y^3 + 3xy + x^3 = 5$

The General Power Rule is given by

$$\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx}$$

DIFFERENTIATION OF PARAMETRIC FUNCTIONS

Sometimes, the dependent variable "y" is not given in terms of the independent variable "x" rather both variable are given as a function of another variable say "t" , is called ***parameter***.

Example:- $x = f(t), y = g(t)$ we find $\frac{dy}{dx}$ as fellows:

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \text{ (By Chain Rule)}$$

DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS

$\frac{d}{dx}(sinx) = cosx$	$\frac{d}{dx}(cosecx) = -cosecx\ cotx$
$\frac{d}{dx}(cosx) = -sinx$	$\frac{d}{dx}(secx) = secx\ tanx$
$\frac{d}{dx}(tanx) = sec^2x$	$\frac{d}{dx}(cotx) = -cosec^2x$

DIFFERENTIATION OF INVERSE TRIGONOMETRIC FUNCTIONS

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\text{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$
$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\frac{d}{dx}(cot^{-1} x) = -\frac{1}{1+x^2}$

DIFFERENTIATION OF LOGARITHMIC FUNCTIONS

1. $\frac{d}{dx}(lnx) = \frac{1}{x}$
2. $\frac{d}{dx}(\log_a x) = \frac{1}{xlna}$

DIFFERENTIATION OF EXPONENTIAL FUNCTIONS

1. $\frac{d}{dx}(e^x) = e^x$
2. $\frac{d}{dx}(a^x) = a^x lna$

DIFFERENTIATION OF HYPERBOLIC FUNCTIONS

$\frac{d}{dx}(sinhx) = coshx$	$\frac{d}{dx}(cosechx) = -cosechx\ cothx$
$\frac{d}{dx}(coshx) = sinhx$	$\frac{d}{dx}(sechx) = -sechx\ tanhx$
$\frac{d}{dx}(tanhx) = sech^2x$	$\frac{d}{dx}(cothx) = -cosech^2x$

DIFFERENTIATION OF INVERSE HYPERBOLIC FUNCTIONS

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$	$\frac{d}{dx}(\text{cosech}^{-1} x) = -\frac{1}{x\sqrt{1+x^2}}$
$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$	$\frac{d}{dx}(\text{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$
$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$	$\frac{d}{dx}(coth^{-1} x) = \frac{1}{1-x^2} = -\frac{1}{x^2-1}$

POWER SERIES

A series of the form $a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \cdots + a_nx^n + \cdots$ is called ***Power Series*** expansion of a function $f(x)$. where $a_0, a_1, a_2, a_3, a_4 \ldots a_n \ldots$ are constants and x s variable.

MACLAURIN SERIES OR MACLAURIN'S THEOREM

If $f(x)$ is expanded in ascending powers of x as an infinite series, then

$$f\ (x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots + \frac{x^n}{n!}f^n(0) + \cdots$$

TAYLOR SERIES OR TAYLOR'S THEOREM

If $\boldsymbol{f}$ is defined in the interval containing $\boldsymbol{'a'}$ and its derivatives of all orders exist at $\boldsymbol{x = a}$, then we can expand $\boldsymbol{f\ (x)}$ as

$$f\ (x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^n(a)}{n!}(x-a)^n + \cdots$$

If $a = 0$ then above expansion becomes Maclaurin Series.

Taylor's Theorem can be stated as: If $\boldsymbol{x}$ and $\boldsymbol{h}$ are two independent quantities and $\boldsymbol{f\ (x + h)}$ can be expanded in ascending power of $\boldsymbol{h}$ as an infinite series, then

$$f\ (x+h) = f(x) + f'(x)h + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \cdots + \frac{f^n(x)}{n!}h^n + \cdots$$

INCREASING FUNCTION

Let $f(x)$ be defined on an interval (a, b) and $x_1, x_2 \in (\mathrm{a}, \mathrm{b})$ such that

$f(x_1) < f(x_2)$, for all $x_1 < x_2$ then $f(x)$ is called **increasing** on the interval (a, b).

DECREASING FUNCTION

Let $f(x)$ be defined on an interval (a, b) and $x_1, x_2 \in (\mathrm{a}, \mathrm{b})$ such that

$f(x_1) > f(x_2)$, for all $x_1 < x_2$ then $f(x)$ is called **decreasing** on the interval (a, b).

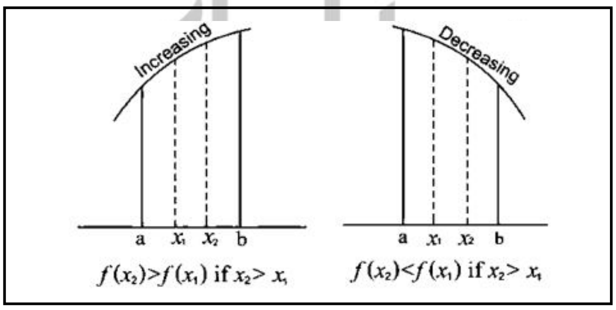


FIGURE 1: INCREASING & DECREASING FUNCTION

Note

Let $f(x)$ be the differentiable function on the open interval (a, b) then
(1) $f(x)$ is increasing on (a, b) if $f'(x) > 0$ for each $x \in (\mathrm{a}, \mathrm{b})$
(2) $f(x)$ is decreasing on (a, b) if $f'(x) < 0$ for each $x \in (\mathrm{a}, \mathrm{b})$
(3) $f(x)$ is neither increasing nor decreasing on (a, b) if $f'(x) = 0$ for each $x \in (\mathrm{a}, \mathrm{b})$

STATIONARY POINT

Any point where f is neither increasing nor decreasing is called ***Stationary Point***. At stationary point $f'(x) = 0$.

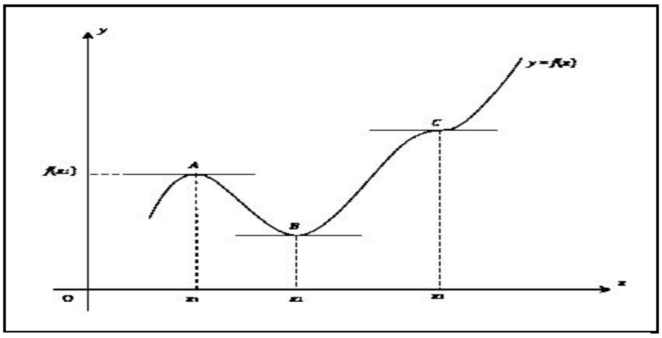


FIGURE 2: A,B,C ARE STATIONARY POINT

RELATIVE MAXIMA

A function "f" is said to have relative maxima/maximum at $x = c \in [a, b]$ if

(i) There exists interval $(a, c]$ in which f increases and
(ii) There exists interval $[c, b)$ in which f decreases

RELATIVE MINIMA

A function "f" is said to have relative minima/minimum at $x = c \in [a, b]$ if

(i) There exists interval $(a, c]$ in which "f" decreases and
(ii) There exists interval $[c, b)$ in which "f" increases

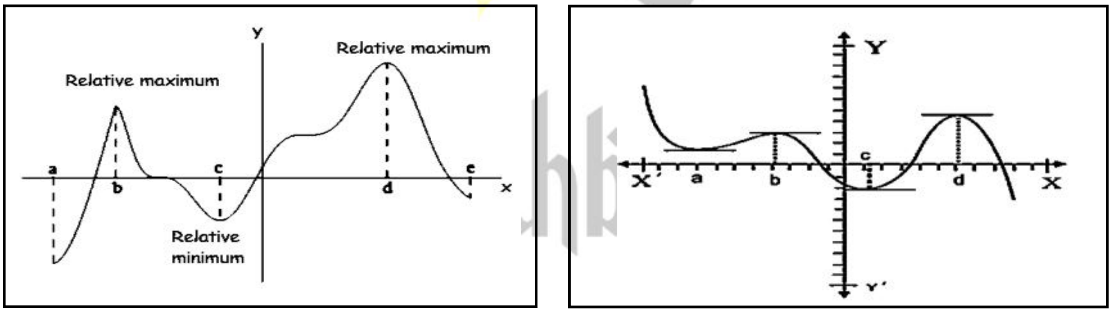


FIGURE 3: RELATIVE EXTREMA

> ***Note***
> Both Relative Maxima & Relative Minima are called in general Relative Extrema.

CRITICAL VALUE & CRITICAL POINT

If $c \in D_f$ and $f'(c) = 0$ or $f'(c)$ does not exist, then the number c is called ***critical value*** for f while the point $(c, f(c))$ on the graph of f is named as a ***critical point***.

FIRST DERIVATIVE RULE:

Let f be differentiable in neighborhood of c where $f'(c) = 0$.
1. If $f'(x)$ changes sign from positive to negative as x increases through c, then $f(c)$ the relative maxima of f.
2. If $f'(x)$ changes sign from negative to positive as x increases through c, then $f(c)$ is the relative minima of f.

SECOND DERIVATIVE RULE:

Let f be differentiable function in a neighborhood of c where $f'(c) = 0$ Then
1. f has relative minima at c if $f''(c) > 0$.
2. f has relative maxima at c if $f''(c) < 0$.

2ND DERIVATIVE TEST FOR EXTREME VALUES OF A FUNCTION

Let $f(x)$ be a given function.

Step I: Find $f'(x)$ and $f''(x)$

Step II: Put $f'(x) = 0$ and solve for x , let $x = a$

Step III: $f''(x)/_{x=a} > 0$ $(Positive)$, then $f(x)$ is minimum at $x = a$

Step IV: $f''(x)/_{x=a} < 0$ $(Negative)$, then $f(x)$ is maximum at $x = a$

Note

(1) A stationary point is called a **turning point** if it is either a maximum point or minimum point.

(2) If $f'(x) > 0$ before the point $x = a, f'(x) = 0$ at $x = a$ and if $f'(x) > 0$ after $x = a$, then f does not have a relative maxima.
See the graph of $f(x) = x^3$. In this case, we have
$f'(x) = 3x^2$, that is,
$f'(0 - \varepsilon) = 3(-\varepsilon)^2 = 3\varepsilon^2 > 0$
And $f'(0 + \varepsilon) = 3(\varepsilon)^2 = 3\varepsilon^2 > 0$
The function f is increasing before $x = 0$ and also, it is increasing after $x = 0$.
Such a point of the function is called the **point of inflexion or inflection.**

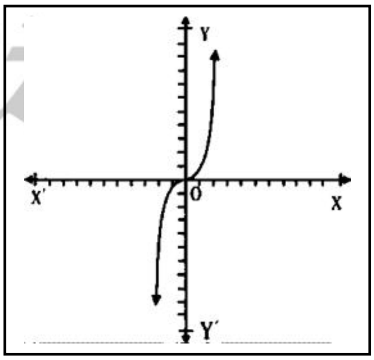


FIGURE 4: GRAPH OF x^3

MCQ's

Choose the correct answer.

1	$\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ exist, is called						
a	Derivative at x	b	Derivative at a	c	Derivative at h	d	Derivative at 0
2	$\lim\limits_{x\to 0}\frac{f(x)-f(a)}{x-a}$ exist, is called						
a	Derivative at x	b	Derivative at a	c	Derivative at h	d	Derivative at 0
3	$\lim\limits_{\delta x\to 0}\frac{f(x+\delta x)-f(x)}{\delta x}=$ ___						
a	$\frac{dy}{da}$	a	$f'(a)$	a	$\frac{fy}{dx}$	a	$\frac{dy}{dx}$
4	The process of finding the derivative of a function f at "x"						
a	Integration of x	a	Derivative of $f(x)$ w.r.t x	a	Derivative of $f(x)$ w.r.t y	a	None of these
5	The notation $\frac{dy}{dx}$ for the derivative of $y = f(x)$ used by						
a	Leibnitz	a	Newton	a	Lagrange	a	Cauchy
6	The notation $f'(x)$for the derivative of $y = f(x)$ used by						
a	Leibnitz	a	Newton	a	Lagrange	a	Cauchy
7	The notation $\dot{f}(x)$ for the derivative of $y = f(x)$ used by						
a	Leibnitz	a	Newton	a	Lagrange	a	Cauchy
8	The notation $Df(x)$ for the derivative of $y = f(x)$ used by						
a	Leibnitz	a	Newton	a	Lagrange	a	Cauchy
9	Leibniz used _____ notation for derivative.						
a	$\frac{dy}{dx}$	b	$\dot{f}(x)$	c	$f'(x)$	d	$Df(x)$
10	Newton used _____ notation for derivative.						
a	$\frac{dy}{dx}$	b	$\dot{f}(x)$	c	$f'(x)$	d	$Df(x)$
11	Lagrange used _____ notation for derivative.						
a	$\frac{dy}{dx}$	b	$\dot{f}(x)$	c	$f'(x)$	d	$Df(x)$
12	Cauchy used _____ notation for derivative.						
a	$\frac{dy}{dx}$	b	$\dot{f}(x)$	c	$f'(x)$	d	$Df(x)$

13	$\lim_{x\to 0}\frac{\delta y}{\delta x}$ is equal to						
a	$\frac{dy}{da}$	b	$f'(a)$	c	$\frac{fy}{dx}$	d	$\frac{dy}{dx}$
14	Derivative of a constant function is						
a	1	b	-1	c	0	d	x
15	Derivative of $\frac{1}{x}$ is equal to						
a	$-x^2$	b	x^{-2}	c	$-x^{-2}$	d	x
16	If $n=0$ then $\frac{d}{dx}(x^n)$ is equal to						
a	0	b	1	c	-1	d	2
17	$\frac{d}{dx}(x^n)=nx^{n-1}$ is known as _____ rule.						
a	Quotient	b	Product	c	Sum	d	Power
18	Derivative of $\sqrt{x}$ at $x=a$ is equal to						
a	$2\sqrt{x}$	b	$\frac{1}{2\sqrt{a}}$	c	$\frac{1}{\sqrt{a}}$	d	$\frac{2}{\sqrt{a}}$
19	If $y=\frac{1}{x^2}$ then $\frac{dy}{dx}$ at $x=-1$ is equal to						
a	0	b	2	c	-1	d	None of these
20	$\frac{d}{dx}\left(\frac{1}{ax+b}\right)=$ _______						
a	$\frac{1}{(ax+b)^2}$	b	$-\frac{1}{(ax+b)^2}$	c	$-\frac{a}{(ax+b)^2}$	d	$\frac{a}{(ax+b)^2}$
21	$\frac{d}{dx}\left(\frac{1}{(ax+b)^n}\right)=$ _______						
a	$\frac{a}{(ax+b)^{n+1}}$	b	$\frac{na}{(ax+b)^{-n+1}}$	c	$\frac{-na}{(ax+b)^{n+1}}$	d	$\frac{na}{(ax+b)^n}$
22	$\frac{d}{dx}(ax+b)^n=$ _____						
a	$n(ax^{n-1}+b)$	b	$n(ax+b)$	c	nax^{n-1}	d	$na(ax+b)^{n-1}$
23	If $y=x-\frac{1}{x}$ then $\frac{dy}{dx}=?$						
a	$1+\frac{1}{x^2}$	b	$1-\frac{1}{x^2}$	c	$1+\frac{1}{x}$	d	$1-\frac{1}{x}$
24	$\frac{d}{dx}\left(3x^{\frac{4}{3}}\right)=$ _____						
a	$4x^{\frac{1}{3}}$	b	$\frac{3}{4}x^{\frac{1}{3}}$	c	$\frac{4}{7}x^{\frac{1}{3}}$	d	$x^{\frac{1}{3}}$

25	$\frac{d}{dx}\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2$ =?						
a	$1+\frac{1}{x^2}$	b	$1-\frac{1}{x^2}$	c	$1-\frac{1}{2x}$	d	0
26	$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)=$ _____						
a	$\frac{1}{2x\sqrt{x}}$	b	$\frac{-1}{2x\sqrt{x}}$	c	$\frac{1}{2x}(x\sqrt{x})$	d	None of these
27	$\frac{d}{dx}\left(\frac{x}{a}\right)=$ _______						
a	$\frac{x}{a^2}$	b	$\frac{1}{a}$	c	$\frac{1}{a^2}$	d	$\frac{x^2}{a^2}$
28	$\frac{d}{dx}\left(\frac{a}{x}\right)$ =?						
a	$\frac{1}{x}$	b	$\frac{-a}{x}$	c	$\frac{a}{x^2}$	d	$-\frac{a}{x^2}$
29	$\frac{d}{dx}(x^2+1)^2=$ ____						
a	$2(x^2+1)$	b	$(x^2+1)^2$	c	$2x(x^2+1)$	d	$4x(x^2+1)$
30	$\frac{d}{dx}(x-5)(3-x)$ =?						
a	$2x+8$	b	$-2x+8$	c	$2x-8$	d	$x+8$
31	The derivative of $\frac{x^2-4}{x+2}$ is equal to:						
a	$2x$	b	-2	c	1	d	2
32	The derivative of $\frac{x^3+2x^2}{x^3}$ is equal to:						
a	$\frac{2}{x^2}$	b	$\frac{-2}{x^2}$	c	$\frac{1}{2x^2}$	d	$\frac{-1}{2x^2}$
33	$\frac{d}{dx}\left(\frac{1}{g(x)}\right)=$ _______						
a	$(-1)[g(x)]^{-2}g'(x)$	b	$(-1)[g(x)]g'(x)$	c	$[g(x)]^{-2}g'(x)$	d	None of these
34	Derivative of $(x^3+1)^9$ is equal to						
a	$x^2(x^3+1)^8$	b	$27x^2(x^3+1)^8$	c	$x^2(x^3+1)^{-8}$	d	$-x^2(x^3+1)^8$
35	$\frac{d}{dx}\left(x^2+\frac{1}{x^2}\right)=$ ______						
a	$2\left(x-\frac{1}{x^3}\right)$	b	$2\left(x-\frac{1}{x^2}\right)$	c	$2\left(x+\frac{1}{x^2}\right)$	d	$2\left(x+\frac{1}{x^3}\right)$
36	$\frac{d}{dx}(sinx)=$ _____						
a	$cosx$	b	$-sinx$	c	sec^2x	d	$-cosec^2x$

37	$\frac{d}{dx}(cosx) = \_\_\_\_\_$						
a	$cosx$	b	$-sinx$	c	sec^2x	d	$-cosec^2x$
38	$\frac{d}{dx}(tanx) = \_\_\_\_\_$						
a	$cosx$	b	$-sinx$	c	sec^2x	d	$-cosec^2x$
39	$\frac{d}{dx}(cotx) = \_\_\_\_\_$						
a	$cosx$	b	$-sinx$	c	sec^2x	d	$-cosec^2x$
40	$\frac{d}{dx}(secx) = \_\_\_\_\_$						
a	$secx\ tanx$	b	$-sinx$	c	sec^2x	d	$-cosecx\ cotx$
41	$\frac{d}{dx}(cosecx) = \_\_\_\_\_$						
a	$secx\ tanx$	b	$-sinx$	c	sec^2x	d	$-cosecx\ cotx$
42	$\frac{d}{dx}(sinhx) = \_\_\_\_\_$						
a	$coshx$	b	$sinhx$	c	$sech^2x$	d	$-cosech^2x$
43	$\frac{d}{dx}(coshx) = \_\_\_\_\_$						
a	$coshx$	b	$sinhx$	c	$sech^2x$	d	$-cosech^2x$
44	$\frac{d}{dx}(tanhx) = \_\_\_\_\_$						
a	$coshx$	b	$sinhx$	c	$sech^2x$	d	$-cosech^2x$
45	$\frac{d}{dx}(cothx) = \_\_\_\_\_$						
a	$coshx$	b	$sinhx$	c	$sech^2x$	d	$-cosech^2x$
46	$\frac{d}{dx}(cosechx) = \_\_\_\_\_$						
a	$coshx$	b	$-sechx\ tanhx$	c	$-cosechx\ cothx$	d	$-cosech^2x$
47	$\frac{d}{dx}(sechx) = \_\_\_\_\_$						
a	$coshx$	b	$-sechx\ tanhx$	c	$-cosechx\ cothx$	d	$-cosech^2x$
48	$\frac{d}{dx}(\sin^{-1}x) = \_\_\_\_\_\_$						
a	$\frac{1}{\sqrt{1-x^2}}$	b	$-\frac{1}{\sqrt{1-x^2}}$	c	$\frac{1}{1+x^2}$	d	$\frac{1}{x\sqrt{x^2-1}}$

49	$\frac{d}{dx}(\cos^{-1}x) =$ _______						
a	$\frac{1}{\sqrt{1-x^2}}$	b	$-\frac{1}{\sqrt{1-x^2}}$	c	$\frac{1}{1+x^2}$	d	$\frac{1}{x\sqrt{x^2-1}}$
50	$\frac{d}{dx}(\tan^{-1}x) =$ _______						
a	$\frac{1}{\sqrt{1-x^2}}$	b	$-\frac{1}{\sqrt{1-x^2}}$	c	$\frac{1}{1+x^2}$	d	$\frac{1}{x\sqrt{x^2-1}}$
51	$\frac{d}{dx}(\cot^{-1}x) =$ _______						
a	$\frac{1}{\sqrt{1-x^2}}$	b	$-\frac{1}{1+x^2}$	c	$\frac{1}{1+x^2}$	d	$\frac{1}{x\sqrt{x^2-1}}$
52	$\frac{d}{dx}(\sec^{-1}x) =$ _______						
a	$-\frac{1}{x\sqrt{1-x^2}}$	b	$-\frac{1}{\sqrt{1-x^2}}$	c	$\frac{1}{1+x^2}$	d	$\frac{1}{x\sqrt{x^2-1}}$
53	$\frac{d}{dx}(\text{cosec}^{-1}x) =$ _______						
a	$-\frac{1}{x\sqrt{x^2-1}}$	b	$-\frac{1}{\sqrt{1-x^2}}$	c	$\frac{1}{1+x^2}$	d	$\frac{1}{x\sqrt{x^2-1}}$
54	$\frac{d}{dx}(\sinh^{-1}x) =$ _______						
a	$\frac{1}{\sqrt{x^2-1}}$	b	$\frac{1}{1-x^2}$	c	$\frac{1}{\sqrt{1+x^2}}$	d	$-\frac{1}{x\sqrt{1+x^2}}$
55	$\frac{d}{dx}(\cosh^{-1}x) =$ _______						
a	$\frac{1}{\sqrt{x^2-1}}$	b	$\frac{1}{1-x^2}$	c	$\frac{1}{\sqrt{1+x^2}}$	d	$-\frac{1}{x\sqrt{1+x^2}}$
56	$\frac{d}{dx}(\tanh^{-1}x) =$ _______						
a	$\frac{1}{\sqrt{x^2-1}}$	b	$\frac{1}{1-x^2}$	c	$\frac{1}{\sqrt{1+x^2}}$	d	$-\frac{1}{x\sqrt{1+x^2}}$
57	$\frac{d}{dx}(\coth^{-1}x) =$ _______						
a	$\frac{1}{\sqrt{x^2-1}}$	b	$\frac{1}{1-x^2}$	c	$\frac{1}{\sqrt{1+x^2}}$	d	$-\frac{1}{x\sqrt{1+x^2}}$
58	$\frac{d}{dx}(\text{sech}^{-1}x) =$ _______						

a	$\frac{1}{\sqrt{x^2-1}}$	b	$\frac{1}{1-x^2}$	c	$-\frac{1}{x\sqrt{1-x^2}}$	d	$-\frac{1}{x\sqrt{1+x^2}}$
59	$\frac{d}{dx}(\operatorname{cosech}^{-1}x) =$ _______						
a	$\frac{1}{\sqrt{x^2-1}}$	b	$\frac{1}{1-x^2}$	c	$\frac{1}{\sqrt{1+x^2}}$	d	$-\frac{1}{x\sqrt{1+x^2}}$
60	$\frac{d}{dx}(a^x) =$ ______						
a	a^x	b	$a^x lnx$	c	lnx	d	$a^x lna$
61	$\frac{d}{dx}(e^{f(x)}) =$ ______						
a	$e^{f(x)}$	b	$e\,f'(x)$	c	$e^{f(x)}f'(x)$	d	$e^{f(x)}f(x)$
62	$\frac{d}{dx}(\log_a x) =$ _____						
a	$\frac{1}{x}$	b	$\frac{x}{lna}$	c	$\frac{1}{xlna}$	d	$\frac{1}{x}lna$
63	$\frac{d}{dx}(\cos 7x) =$ __						
a	$7\cos 7x$	b	$-7\cos 7x$	c	$7\sin 7x$	d	$-7\sin 7x$
64	$\frac{d}{dx}(\ln\ e^x) =$ ______						
a	1	b	$\frac{1}{e^x}$	c	$\frac{1}{x}$	d	x
65	$\frac{d}{dx}(\sinh x) =$ ______						
a	$\frac{e^x-e^{-x}}{2}$	b	$\frac{e^x+e^{-x}}{2}$	c	$\frac{e^x-e^{-x}}{e^x+e^{-x}}$	d	$\frac{2}{e^x+e^{-x}}$
66	$\frac{d}{dx}(cosx^2) =$ ______						
a	$-2\ sinx^2$	b	$2x\ sinx^2$	c	$-x\ sinx^2$	d	$-2x\ sinx^2$
67	$\frac{d}{dx}(\ln(lnx)) =$ ______						
a	$\frac{1}{lnx}$	b	$\frac{1}{xlnx}$	c	$\frac{1}{(lnx)^2}$	d	lnx
68	$\frac{d}{dx}(\ln(sinx)) =$ ______						
a	$\frac{1}{sinx}$	b	$tanx$	c	$cotx$	d	None of these
69	Derivative of $ln(ax^2+b)$ is						

a	$\frac{a}{ax^2+b}$	b	$\frac{2ax}{ax^2+b}$	c	$\frac{2a}{ax^2+b}$	d	None of these
70	$\frac{d}{dx}(\sqrt{tanx}) =$ ______						
a	$\frac{1}{2}\sqrt{tanx}\, sec^2x$	b	$\sqrt{tanx}\, sec^2x$	c	$\frac{1}{2\sqrt{tanx}}\, sec^2x$	d	None of these
71	$\frac{d}{dx}(cot^2x) =$ _____						
a	$cotx\ cosecx$	b	$-2cotx\ cosec^2x$	c	sec^2x	d	$-cosecx\ cotx$
72	$\frac{d}{dx}(f(x)\ sinx) =$ ______						
a	$f'(x)sinx + f(x)\ cosx$	b	$f'(x)sinx - f(x)\ cosx$	c	$f'(x)cosx + f(x)\ sinx$	d	$f'(x)cosx$
73	Derivative of $sin(tanx)$ is						
a	$cos(tanx)$	b	$sec^2xcos(tanx)$	c	$-sec^2xcos(tanx)$	d	None of these
74	$\frac{d}{dx}(sin^2x + cos^2x) =$ _____						
a	1	b	0	c	-1	d	2
75	$(1+x^2)\frac{d}{dx}(\tan^{-1}x - \cot^{-1}x) =$ _____						
a	2	b	$\frac{2}{1+x^2}$	c	0	d	$\frac{-2}{1+x^2}$
76	$\frac{1}{2}\frac{d}{dx}(\tan^{-1}x - \cot^{-1}x) =$ _____						
a	$\frac{-1}{1+x^2}$	b	$\frac{1}{1+x^2}$	c	$\frac{1}{1-x^2}$	d	$\frac{-1}{1-x^2}$
77	$\frac{d}{dx}(\sec^{-1}x + \operatorname{cosec}^{-1}x) =$ _____						
a	1	b	-1	c	0	d	2
78	$\sqrt{1-x^2}\frac{d}{dx}(\cos^{-1}x + \sin^{-1}x) =$ ___						
a	$\frac{1}{\sqrt{1-x^2}}$	b	$\frac{2}{\sqrt{1-x^2}}$	c	0	d	1
79	$\frac{d}{dx}(\sin 2x + \cos 2x) =$ ____						
a	$\cos 2x + \sin 2x$	b	$\cos 2x - \sin 2x$	c	$2\cos 2x + 2\sin 2x$	d	$2(\cos 2x - \sin 2x)$
80	$\frac{d}{dx}(\sinh 2x) =$ _____						
a	$2\cosh 2x$	b	$2\sinh 2x$	c	$-2\cosh 2x$	d	$-\sinh 2x$

81	$\frac{d}{dx}(\ln \sinh x) =$ ______						
a	$\coth x$	b	$\tanh x$	c	$-\coth x$	d	$-\tanh x$
82	$\frac{d}{dx}(\cos^{-1} 3x) =$ ______						
a	$\frac{3}{\sqrt{1-9x^2}}$	b	$\frac{-3}{\sqrt{1-9x^2}}$	c	$\frac{1}{\sqrt{1-9x^2}}$	d	$\frac{-1}{\sqrt{1-9x^2}}$
83	$\frac{d}{dx}\left(\cot^{-1}\frac{x}{a}\right) =$ ______						
a	$\frac{-a}{a^2+x^2}$	b	$\frac{a^2}{a^2+x^2}$	c	$\frac{-a^2}{a^2+x^2}$	d	$\frac{-1}{a^2+x^2}$
84	$\frac{d}{dx}(\ln (sin^2 x))$						
a	$2\cot x$	b	$-2\cot x$	c	$2\tan x$	d	$-2\tan x$
85	$\frac{d}{dx}\left(a^{\sqrt{x}}\right) =$ ______						
a	$\frac{a^{\sqrt{x}} lna}{2\sqrt{x}}$	b	$a^{\sqrt{x}}$	c	$\frac{1}{2}a^{\sqrt{x}}$	d	$a^{\sqrt{x}}\sqrt{x}$
86	$\frac{d}{dx}\left(e^{x^2+1}\right) =$ ______						
a	e^{x^2+1}	b	$2xe^{x^2+1}$	c	xe^{x^2+1}	d	None of these
87	$\frac{d}{dx}(xe^x) =$ ______						
a	$xe^x + 1$	b	xe^x	c	$xe^x - 1$	d	$xe^x + e^x$
88	$\frac{d}{dx}\left(e^{tan\,2x}\right) =$ ______						
a	$2sec^2 2xe^{tan\,2x}$	b	$e^{tan\,2x}$	c	$sec^2 2xe^{tan\,2x}$	d	None of these
89	$\frac{d}{dx}3^x =?$						
a	3^x	b	$3^x ln3$	c	$3(3^x)$	d	3
90	$\frac{d}{dx}\left(e^{5x-2}\right) =$ ______						
a	$5e^{5x-2}$	b	$2e^{5x-2}$	c	e^{5x-3}	d	$5e^{5x-3}$
91	If $f(x) = 2^{2x}$ then $f'(x) =?$						
a	2^{2x-1}	b	$2^{2x}\ln 2$	c	$2^{2x+1}\ln 2$	d	$\frac{2^{2x}}{\ln 2}$
92	The differential co-efficient of $e^{\sin x}$ equals						

a	$e^{\sin x}\cos x$	b	$e^{\sin x}\sin x$	c	$\sin x\, e^{\sin x-1}$	d	$\sin x\, e^{\sin x+1}$
93	$\frac{d}{dx}(2e^{3x}) =$ _____						
a	$6e^{3x}$	b	$2e^{3x}$	c	$-6e^{3x}$	d	e^{3x}
94	$\frac{d}{dx}(e^{x}-e^{-x}) =$ ____						
a	$\sinh x$	b	$\cosh x$	c	$2\sinh x$	d	$2\cosh x$
95	If $y = 5e^{3x-4}$ then $\frac{dy}{dx} =$ _____						
a	$15e^{3x-4}$	b	e^{3x-4}	c	$5e^{3x}$	d	$-15e^{3x-4}$
96	$\frac{dy}{dx} =$ _____						
a	$\frac{dy}{dx}\times\frac{du}{dx}$	b	$\frac{dy}{dy}\times\frac{du}{dx}$	c	$\frac{dy}{du}\times\frac{du}{dx}$	d	None of these
97	If $x = at^2$ and $y = 2at$, then $\frac{dy}{dx} =$ _____						
a	$\frac{2}{y}$	b	$\frac{2a}{y}$	c	$2ay$	d	$2a$
98	If $3x + 4y - 5 = 0$, then $\frac{dy}{dx} =$ __						
a	$\frac{4}{3}$	b	$\frac{-4}{3}$	c	$\frac{3}{4}$	d	$\frac{-3}{4}$
99	If $x^2 + y^2 = a^2$, then $\frac{dy}{dx} =$ __						
a	$\frac{x}{y}$	b	$\frac{-x}{y}$	c	$\frac{y}{x}$	d	$\frac{-y}{x}$
100	Derivative of $\cos x$ w.r.t. $\cos x$ is:						
a	$-\cos x$	b	$\sin x$	c	0	d	1
101	The higher derivative of the polynomial $f(x) = \frac{1}{12}x^4 - \frac{1}{6}x^3 + \frac{1}{4}x^2 + 2x + 7$ is						
a	3	b	4	c	5	d	7
102	If $y = 3x^4 - 4x^2 + x - 2$ then $y_2 =$ ___						
a	$12x^3 - 8x + 1$	b	$36x^2 - 8$	c	$72x$	d	0
103	If $y = e^{ax}$ then y_4 is equal to:						
a	a^4e^{ax}	b	$3e^{ax}$	c	$4e^{ax}$	d	xe^{ax}
104	If $y = e^{2x}$ then y_4 is equal to:						
a	$16e^{2x}$	b	$8e^{2x}$	c	$4e^{2x}$	d	$-16e^{2x}$
105	If $y = \sin(ax + b)$ then $y_2 =$ ___						
a	a $\cos(ax + b)$	b	$-a^3\cos(ax + b)$	c	$a^4\sin(ax + b)$	d	$-a^2\sin(ax + b)$

106	If $y = \sin 3x$ then $y_2 =$ ___						
a	$3\cos x$	b	$3\cos 3x$	c	$9\sin 3x$	d	$-9\sin 3x$
107	If $y = -a\sin x$ then $y_2 =$ ___						
a	$a\sin x$	b	$a^2 cosx$	c	$-a\sin x$	d	None of these
108	If $y = a(1 + cos\theta)$, then $\frac{d^2y}{d\theta^2} =$ __						
a	$-a\sin\theta$	b	$a\cos\theta$	c	$a\sin\theta$	d	$-a\cos\theta$
109	$\frac{d^2}{dx^2}(\cosh 3x) =$ _____						
a	$3\cosh 3x$	b	$3\sinh 3x$	c	$-9\cosh 3x$	d	$9\cosh 3x$
110	$\frac{d^2}{dx^2}(2^x) =$ _____						
a	$x2^{x-1}$	b	$\ln 2^x$	c	$2^x(\ln 2)^2$	d	$x\ln 2$
111	If $f(x) = \cos x$ then $f'(\pi) =?$						
a	1	b	0	c	-1	d	2
112	If $f(x) = \sin x$ then $f''\left(\frac{\pi}{2}\right) =?$						
a	1	b	0	c	-1	d	2
113	If $f(x) = \sin x$ then $f'(\cos^{-1} x) =?$						
a	$\cos x$	b	$\sin x$	c	$-x$	d	x
114	If $f(x) = x^{2/3}$ then $f'(8) =?$						
a	$\frac{1}{2}$	b	$\frac{2}{3}$	c	$\frac{1}{3}$	d	3
115	If $f(x) = sinx$, then $f'''(\pi)$						
a	-1	b	0	c	1	d	5
116	If $f(x) = 3x^2 - 2x + 1$, then $f'(0) =$ _						
a	5	b	-2	c	1	d	2
117	If $f(x) = \tan^{-1} x$, then $f'(\cot x)$ is equal to:						
a	$\frac{1}{1+x^2}$	b	sin^2x	c	cos^2x	d	sec^2x
118	If $f(x) = x^{10}$, then $f''(1) =$ ___						
a	1	b	10	c	90	d	100
119	If both $u(x)$ and $v(x)$ are function of x then $\frac{uv'-vu'}{u^2}$ shows						
a	$\frac{d}{dx}\left(\frac{u}{v}\right)$	b	$\frac{d}{dx}\left(\frac{v}{u}\right)$	c	$\frac{d}{dx}(u.v)$	d	$\frac{d}{dx}(u+v)$

120	A series of the form $a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \cdots + a_nx^n + \cdots$ is called						
a	Maclaurin Series	b	Maclaurin's Theorem	c	Taylor Series	d	Power Series
121	The expansion $f\ (x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots + \frac{x^n}{n!}f^n(0) + \cdots$ is called						
a	Maclaurin Series	b	Maclaurin's Theorem	c	Taylor's Theorem	d	Both a & b
122	The expansion $f\ (x+h) = f(x) + f'(x)h + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \cdots + \frac{f^n(x)}{n!}h^n + \cdots$ is called						
a	Maclaurin Series	b	Maclaurin's Theorem	c	Taylor's Theorem	d	Both a & b
123	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ is expansion of						
a	$sinx$	b	$cosx$	c	e	d	e^x
124	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots$ is expansion of						
a	$sinx$	b	$cosx$	c	e	d	e^x
125	The series $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ is of						
a	$\sin x$	b	$\cos x$	c	$\tan x$	d	$-\sin x$
126	If $f(x+h) = a^{x+h}$ then $f'(x) =$ ___						
a	$a^{x+h}\ln(x+h)$	b	$a^x \ln a$	c	$a^x \ln x$	d	$a^{x+h}\ln a$
127	If $f(x+h) = \cos(x+h)$ then $f'(x) =$ ___						
a	$\cos x$	b	$-\cos x$	c	$-\sin x$	d	$\sin x$
128	Geometrically derivative represent						
a	Slope of normal line	b	Slope of tangent line	c	Slope of secant line	d	None of these
129	If $f(x) = \|x\|$ then $f'(x)$ at $x = 0$ is equal to						
a	0	b	1	c	Does not exist	d	-1
130	If f be defined on an interval (a,b) and $x_1, x_2 \in (a,b)$ such that $f(x_1) < f(x_2)$, for all $x_1 < x_2$ then f is called						
a	Increasing	b	Decreasing	c	Constant	d	None of these
131	If f be defined on an interval (a,b) and $x_1, x_2 \in (a,b)$ such that $f(x_1) > f(x_2)$, for all $x_1 < x_2$ then f is called						
a	Increasing	b	Decreasing	c	Constant	d	None of these
132	If f is increasing in an interval (a,b), $f'(c)$ is _____ for every $c \in (a,b)$						
a	Positive	b	Negative	c	Zero	d	None of these
133	If f is decreasing in an interval (a,b), $f'(c)$ is _____ for every $c \in (a,b)$						

a	Positive	b	Negative	c	Zero	d	None of these
134	If f be differentiable on (a, b), f is increasing at $x \in (a, b)$ if						
a	$f'(x) > 0$	b	$f'(x) < 0$	c	$f'(x) = 0$	d	None of these
135	If f be differentiable on (a, b), f is decreasing at $x \in (a, b)$ if						
a	$f'(x) > 0$	b	$f'(x) < 0$	c	$f'(x) = 0$	d	None of these
136	If f be differentiable on (a, b), f is neither increasing nor decreasing at $x \in (a, b)$ if						
a	$f'(x) > 0$	b	$f'(x) < 0$	c	$f'(x) = 0$	d	None of these
137	Any point where f is neither increasing nor decreasing is called _____ point.						
a	Decreasing	b	Increasing	c	Stationary	d	Maximum
138	At stationary point						
a	$f'(x) > 0$	b	$f'(x) < 0$	c	$f'(x) = 0$	d	None of these
139	Maximum and minimum values of the function is called						
a	extremum	b	Extreme value	c	Stationary	d	Both a & b
140	Let f be differentiable function in a neighborhood of c where $f'(c) = 0$ Then f has relative minima at c if						
a	$f'(x) > 0$	b	$f'(x) < 0$	c	$f''(c) > 0$	d	$f''(c) < 0$
141	Let f be differentiable function in a neighborhood of c where $f'(c) = 0$ Then f has relative maxima at c if						
a	$f'(x) > 0$	b	$f'(x) < 0$	c	$f''(c) > 0$	d	$f''(c) < 0$
142	$for\ x_1, x_2 \in (\mathrm{a}, \mathrm{b})$ f is increasing on the interval (a, b) for all $x_1 < x_2$ if						
a	$f(x_1) > f(x_2)$	b	$f(x_1) < f(x_2)$	c	$f(x_1) \leq f(x_2)$	d	None of these
143	If $c \in D_f$ and $f'(c) = 0$ or $f'(c)$ does not exist, then the number c is called ___.						
a	Increasing value	b	Decreasing value	c	Stationary value	d	Critical value
144	Which one is decreasing function?_						
a	$2 - 4x$	b	$4x - 2$	c	$4x$	d	$4x + 5$
145	$f(x) = x^2$ is increasing if						
a	$f'(x) > 0$	b	$f'(x) < 0$	c	$f'(x) \leq 0$	d	$f'(x) \geq 0$
146	The point at which $f(x) = x^2 + 2x - 3$ is neither increasing nor decreasing is						
a	$(-1, -4)$	b	$(1,4)$	c	$(1, -4)$	d	$(-1,4)$
147	The function $f(x) = 3x^2$ has relative minimum at the point						
a	$(0,0)$	b	$(0,1)$	c	$(1,1)$	d	$(-1,0)$

148	Minimum value of the function $f(x) = x^2 + 2x - 3$ is at $x =$ __						
a	-3	b	-2	c	0	d	-1
149	The maximum value of the $f(x) = \sin x + \cos x$ in the interval $[0,2\pi]$ is						
a	2	b	$\frac{1}{\sqrt{2}}$	c	$\sqrt{3}$	d	$\sqrt{2}$
150	Two positive integer whose sum is 30 and their product will be maximum are:						
a	14,16	b	15,15	c	10,20	d	12,18

MCQ's Answers Key

1	2	3	4	5	6	7	8	9	10
a	b	d	b	a	c	b	d	a	b
11	12	13	14	15	16	17	18	19	20
c	d	d	c	c	a	d	b	b	c
21	22	23	24	25	26	27	28	29	30
c	d	a	a	b	b	b	d	d	b
31	32	33	34	35	36	37	38	39	40
c	b	a	b	a	a	b	c	d	a
41	42	43	44	45	46	47	48	49	50
d	a	b	c	d	c	b	a	b	c
51	52	53	54	55	56	57	58	59	60
b	d	a	c	a	b	b	c	d	d
61	62	63	64	65	66	67	68	69	70
c	c	d	a	b	d	b	c	b	c
71	72	73	74	75	76	77	78	79	80
b	a	b	b	a	b	c	c	d	a
81	82	83	84	85	86	87	88	89	90
a	b	a	a	a	b	d	a	b	a
91	92	93	94	95	96	97	98	99	100
c	a	a	d	a	c	b	d	b	d
101	102	103	104	105	106	107	108	109	110
b	b	a	a	d	d	a	d	d	c
111	112	113	114	115	116	117	118	119	120
b	c	d	c	c	b	b	c	b	d
121	122	123	124	125	126	127	128	129	130
d	c	d	b	a	b	c	b	c	a
131	132	133	134	135	136	137	138	139	140
b	a	b	a	b	c	c	c	d	c
141	142	143	144	145	146	147	148	149	150
d	b	d	a	a	a	a	d	d	b

IMPORTANT SHORT QUESTIONS

1. Define Derivative of a function.
2. Define Differentiation.
3. Find the derivative of $f(x) = c$ by definition.
4. Find the derivative of $f(x) = \sqrt{x}\ at\ x = a$ by definition.
5. Find the derivative of $x^{2/3}$ by definition.
6. Find the derivative of $2 - \sqrt{x}$ w.r.t. 'x' by definition.
7. Find the derivative of $\frac{1}{\sqrt{x}}$ w.r.t. 'x' by definition.
8. Find the derivative of $\frac{1}{x^3}$ w.r.t. 'x' by definition.
9. Find the derivative of $\frac{1}{x-a}$ w.r.t. 'x' by definition.
10. Find the derivative of $(x+4)^{1/3}$ w.r.t. 'x' by definition.
11. Find $\frac{dy}{dx}$, if $y = \sqrt{x+2}$ by first principle.
12. Calculate $\frac{d}{dx}\left(3x^{\frac{4}{3}}\right)$
13. Find the derivative of $y = (x^2+5)(x^3+7)$ w.r.t. x.
14. Find $\frac{dy}{dx}$, if $y = \frac{(\sqrt{x}+1)(x^{3/2}-1)}{x^{1/2}-1}$
15. Differentiate $x^{-3} + 2x^{-\frac{3}{2}} + 3$ w.r.t. 'x'.
16. Differentiate $\frac{a+x}{a-x}$ w.r.t. 'x'.
17. Differentiate $\frac{2x-3}{2x+1}$ w.r.t. 'x'.
18. Differentiate $(x-5)(3-x)$ w.r.t. 'x'.
19. Differentiate $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$ w.r.t. 'x'.
20. Differentiate $\frac{(1+\sqrt{x})(x-x^{3/2})}{\sqrt{x}}$ w.r.t. 'x'.
21. Differentiate $\frac{(x^2+1)^2}{x^2-1}$ w.r.t. 'x'.
22. Differentiate $\frac{x^2+1}{x^2-3}$ w.r.t. 'x'.
23. Differentiate $\frac{2x-1}{\sqrt{x^2+1}}$ w.r.t. 'x'.
24. If $y = x^4 + 2x^2 + 2$, prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$
25. Find the derivative of $(x^3+1)^9$ w.r.t. 'x'.
26. Find $\frac{dy}{dx}$, if $x = at^2$ and $y = 2at$
27. Find $\frac{dy}{dx}$, if $x = 1 - t^2$ and $y = 3t^2 - 2t^3$
28. Find $\frac{dy}{dx}$, if $x^2 + y^2 = 4$
29. Find $\frac{dy}{dx}$, if $y^2 + x^2 - 4x = 5$
30. Find $\frac{dy}{dx}$, if $y^2 - xy - x^2 + 4 = 0$
31. Differentiate $x^2 + \frac{1}{x^2}$ w.r.t. $x - \frac{1}{x}$

32. Find $\frac{dy}{dx}$, if $y = \sqrt{x + \sqrt{x}}$
33. Find $\frac{dy}{dx}$, if $y = (3x^2 - 2x + 7)^6$
34. Find $\frac{dy}{dx}$, if $y = \sqrt{\frac{a^2+x^2}{a^2-x^2}}$
35. Find $\frac{dy}{dx}$, if $3x + 4y + 7 = 0$
36. Find $\frac{dy}{dx}$, if $xy + y^2 = 2$
37. Find $\frac{dy}{dx}$, if $x^2 - 4xy - 5y = 0$
38. Find $\frac{dy}{dx}$, if $4x^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
39. Find $\frac{dy}{dx}$, if $x = \theta + \frac{1}{\theta}$ and $y = \theta + 1$
40. Differentiate $x^2 - \frac{1}{x^2}$ w.r.t. x^4
41. Differentiate $(1 + x^2)^n$ w.r.t. x^2
42. Differentiate $\frac{x^2+1}{x^2-1}$ w.r.t. x^3
43. Differentiate sin^3x w.r.t. cos^2x
44. Prove that $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
45. Prove that $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
46. Prove that $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
47. Prove that $\frac{d}{dx}(\text{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$
48. Prove that $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
49. Prove that $\frac{d}{dx}(cot^{-1} x) = -\frac{1}{1+x^2}$
50. Differentiate $x^2 \sec 4x$ w.r.t. 'x'.
51. Differentiate $\tan^3 \theta\, sec^2\theta$ w.r.t. 'θ'.
52. Differentiate $(\sin 2\theta - \cos 3\theta)$ w.r.t. 'θ'.
53. Differentiate $\cos\sqrt{x} + \sqrt{sinx}$ w.r.t. 'x'.
54. Find $\frac{dy}{dx}$, if $y = x \cos y$
55. Find $\frac{dy}{dx}$, if $x = y \sin y$
56. Differentiate $\sin x$ w.r.t. $\cot x$
57. Differentiate sin^2x w.r.t. cos^4x
58. Differentiate $\cos^{-1}\frac{x}{a}$ w.r.t. 'x'.
59. Differentiate $\cot^{-1}\frac{x}{a}$ w.r.t. 'x'.
60. Differentiate $\frac{1}{a}\sin^{-1}\frac{x}{a}$ w.r.t. 'x'.
61. Differentiate $\sin^{-1}\sqrt{1 - x^2}$ w.r.t. 'x'.
62. Prove that $\frac{d}{dx}(a^x) = a^x lna$ by ab-initio method.
63. Find $\frac{dy}{dx}$, if $y = e^{x^2+1}$
64. Find $\frac{dy}{dx}$, if $y = a^{\sqrt{x}}$

65. Differentiate $y = a^x$ w.r.t. 'x'.
66. Find $\frac{dy}{dx}$, if $y = \log_{10}(ax^2 + bx + c)$
67. Differentiate $\ln(x^2 + 2x)$ w.r.t. 'x'.
68. Differentiate $y = e^{f(x)}$ w.r.t. 'x'.
69. Differentiate $(\ln x)^x$ w.r.t. 'x'.
70. Prove that $\frac{d}{dx}(sinhx) = coshx$
71. Prove that $\frac{d}{dx}(coshx) = sinhx$
72. Prove that $\frac{d}{dx}(tanhx) = sech^2x$
73. Find $\frac{dy}{dx}$, if $y = \tanh(x^2)$
74. Find $\frac{dy}{dx}$, if $y = \cosh^{-1}(\sec x)$
75. Find $f'(x)$ if $f(x) = e^{\sqrt{x}-1}$
76. Find $f'(x)$ if $f(x) = x^3e^{\frac{1}{x}}$
77. Find $f'(x)$ if $f(x) = e^x(1 + \ln x)$
78. Find $f'(x)$ if $f(x) = \frac{e^x}{e^{-x}+1}$
79. Find $f'(x)$ if $f(x) = \ln(e^x + e^{-x})$
80. Find $f'(x)$ if $f(x) = \frac{e^{ax}-e^{-ax}}{e^{ax}+e^{-ax}}$
81. Find $f'(x)$ if $f(x) = \ln\left(\sqrt{e^{2x} + e^{-2x}}\right)$
82. Find $\frac{dy}{dx}$, if $y = x^2 \ln \sqrt{x}$
83. Find $\frac{dy}{dx}$, if $y = x\sqrt{\ln x}$
84. Find $\frac{dy}{dx}$, if $y = \frac{x}{\ln x}$
85. Find $\frac{dy}{dx}$, if $y = x^2 \ln \frac{1}{x}$
86. Find $\frac{dy}{dx}$, if $y = \ln \sqrt{\frac{x^2-1}{x^2+1}}$
87. Find $\frac{dy}{dx}$, if $y = \ln\left(x + \sqrt{x^2 + 1}\right)$
88. Find $\frac{dy}{dx}$, if $y = \ln(9 - x^2)$
89. Find $\frac{dy}{dx}$, if $y = e^{-2x} \sin 2x$
90. Find $\frac{dy}{dx}$, if $y = e^{-x}(x^3 + 2x^2 + 1)$
91. Find $\frac{dy}{dx}$, if $y = xe^{\sin x}$
92. Find $\frac{dy}{dx}$, if $y = (x + 1)^x$
93. Find $\frac{dy}{dx}$, if $y = \sinh 3x$
94. Find $\frac{dy}{dx}$, if $y = \tanh^{-1}(\sin x)$
95. Find $\frac{dy}{dx}$, if $y = \sinh^{-1}(x^3)$
96. Find $\frac{dy}{dx}$, if $y = (\ln \tanh x)$
97. Find $\frac{dy}{dx}$, if $y = \sinh^{-1}\left(\frac{x}{2}\right)$

98. Find $f^{iv}(x)$ of $f(x) = \frac{1}{12}x^4 - \frac{1}{6}x^3 + \frac{1}{4}x^2 + 2x + 7$

99. Find $\frac{d^2y}{dx^2}$, if $y^3 + 3ax^2 + x^3 = 0$

100. Find y_2, if $y = \cos(ax + b)$

101. If $y = \sin^{-1}\frac{x}{a}$, then show that $y_2 = x(a^2 - x^2)^{-\frac{3}{2}}$

102. Find y_2, if $y = 2x^5 - 3x^4 + 4x^3 + x - 2$

103. Find y_2, if $y = (2x + 5)^{3/2}$

104. Find y_2, if $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

105. Find y_2, if $x^2 + y^2 = a^2$

106. Find y_2, if $x^3 - y^3 = a^3$

107. Find y_2, if $x = at^2, y = bt^4$

108. Find y_4, if $y = \sin 3x$

109. Find y_4, if $y = cos^3\, x$

110. Find y_4, if $y = \ln(x^2 - 9)$

111. Define Power Series.

112. Define Maclaurin Series.

113. Find the Maclaurin series for $\sin x$.

114. Expand a^x in the Maclaurin series.

115. Expand $(1 + x)^n$ in the Maclaurin series.

116. State Taylor's Theorem.

117. Apply Maclaurin series Expansion to prove that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots$

118. Apply Maclaurin series Expansion to prove that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \ldots$

119. Apply Maclaurin series Expansion to prove that $e^{2x} = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} \ldots$

120. Define Increasing Function.

121. Define Decreasing Function.

122. Define Critical Point.

123. Define Critical Value.

124. Define Relative Maxima.

125. Define Stationary Point.

126. Define Point of Inflexion.

127. Determine the intervals in which f is decreasing for $f(x) = \sin x$; $x \in (-\pi, \pi)$

128. Determine intervals in which f is increasing/decreasing for $f(x) = \cos x$; $x \in (\frac{-\pi}{2}, \frac{\pi}{2})$

129. Determine the intervals in which f is increasing for $f(x) = 4 - x^2$; $x \in (-2,2)$

130. Determine the intervals in which f is increasing for $f(x) = x^2 + 3x + 2$;$x \in (-4,1)$

131. Find the extreme values for $f(x) = x^2 - x - 2$.

132. Find the extreme values for $f(x) = 5x^2 - 6x + 2$.

133. Find two positive integers whose sum is 30 and their product will be maximum.

134. Divide 20 into two parts so that the sum of their squares will be minimum.

135. The perimeter of a triangle is 16 cm. If one side is of length 6 cm, what are length of the other sides for maximum area of the triangle?

IMPORTANT LONG QUESTIONS

1. Differentiate $\frac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$ w.r.t. 'x'.
2. Differentiate $\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$ w.r.t. 'x'.
3. If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$, show that $2x\frac{dy}{dx} + y = 2\sqrt{x}$
4. Find $\frac{dy}{dx}$, if $y = \frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}$
5. Find $\frac{dy}{dx}$, if $y = (1 + 2\sqrt{x})^3 . x^{\frac{3}{2}}$
6. Find $\frac{dy}{dx}$, if $x = \frac{a(1-t^2)}{1+t^2}, y = \frac{2bt}{1+t^2}$
7. Find $\frac{dy}{dx}$, if $x\sqrt{1+y} + y\sqrt{1+x} = 0$
8. Prove that $y\frac{dy}{dx} + x = 0$ if $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$
9. Differentiate $\sin\sqrt{x}\ w.r.t.\ x$ by ab-initio method.
10. If $y = \tan\left(2\tan^{-1}\frac{x}{2}\right)$, show that $\frac{dy}{dx} = \frac{4(1+y^2)}{(4+x^2)}$
11. Differentiate $\cos\sqrt{x}\ w.r.t.\ x$ by first principle.
12. Differentiate $\cos x^2\ w.r.t.\ x$ by first principle.
13. If $\tan y(1 + \tan x) = 1 - \tan x$, show that $\frac{dy}{dx} = -1$
14. If $x = a\cos^3\theta, y = b\sin^3\theta$, show that $a\frac{dy}{dx} + b\tan\theta = 0$
15. Find $\frac{dy}{dx}$, if $x = a(\cos t + \sin t), y = a(\sin t - t\cos t)$
16. Show that $\frac{dy}{dx} = \frac{y}{x}$ if $\frac{y}{x} = \tan^{-1}\frac{x}{y}$
17. If $y = \tan(p\tan^{-1}x)$, show that $(1 + x^2)y_1 - p(1 + y^2) = 0$
18. If $x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$. Then show that $y^2\frac{d^2y}{dx^2} + a = 0$
19. If $x = sin\theta, y = sinm\theta$, then show that $(1 - x^2)y_2 - xy_1 + m^2y = 0$
20. If $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$
21. If $y = e^{ax} \sin x$, show that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = 0$
22. If $y = (\cos^{-1}x)^2$, prove that $(1 - x^2)y_2 - xy_1 - 2 = 0$
23. If $y = a\cos(lnx) + b\,sin(lnx)$ prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$
24. Show that $cos(x + h) = cosx - hsinx - \frac{h^2}{2!}cosx + \frac{h^3}{3!}sinx + \cdots$ and evaluate $\cos 61°$.
25. Show that $2^{x+h} = 2^x\left\{1 + (\ln 2)h + \frac{(\ln 2)^2h^2}{2!} + \frac{(\ln 2)^3h^3}{3!} + \cdots\right\}$
26. Discuss the function defined as $f(x) = \sin x + \frac{1}{2\sqrt{2}}\cos 2x$ for extreme values in the interval $(0,2\pi)$
27. Find the maximum and minimum values of the function defined by the following equation occurring in the interval $[0,2\pi]$, $f(x) = \sin x + \cos x$.
28. Show that $y = \frac{lnx}{x}$ has a maximum value at $x = e$

29. Show that $y = x^x$ has a minimum value at $x = \frac{1}{e}$

30. A box with a square base and open top is to have a volume of 4 cubic dm. Find the dimensions of the box which will require the least material.

UNIT 3

Integration

DEFINITIONS + SUMMARY

DIFFERENTIAL OF A FUNCTION

Let "f " be a differentiable function define by the equation $y = f(x)$ and let δx be the arbitrary increment in x. Then the number $f'(x)\delta x$ is called the differential of the dependent variable "y" and is denoted by dy.

Thus $dy = f'(x)\delta x$

Note

(i) The increment in the dependent variable "x" is equal to its differential dx i.e., $dx = \delta x$
(ii) Instead of dy, we can write df, i.e., $df = f'(x)\,dx$ where $f'(x)$ being coefficient of differential is called **differential coefficient**.

INTEGRATION

The process of finding a such a function whose derivative is given is called ***anti-differentiation*** or ***integration***.

"c" is an arbitrary constant and it is not definite, so $\varphi(x) + c$ is called the indefinite integral of $f(x)$, that is

$$\int f(x)\,dx = \varphi(x) + c$$

- The function $f(x)$ is called the integrand.
- The symbol $\int$ is called integral sign.
- "c" is called the constant of integration.
- $\int \dots dx$ indicates that integrand is to be integrated w.r.t. x.

THEOREMS ON ANTI-DERIVATIVES

1. The integral of the product of a constant and a function is equal to the product of the constant and the integral of the function.
In symbols,

$$\int af(x)\,dx = a\int f(x)\,dx$$

where a is a constant.

2. The integral of the sum (or difference) of two functions is equal to the sum (or difference) of their integrals.
In symbols,

$$\int [f_1(x) \pm f_2(x)]\,dx = \int f_1(x)\,dx \pm \int f_2(x)\,dx$$

ANTI-DERIVATIVES OF $[f(x)]^n f'(x)$ AND $[f(x)]^{-1} f'(x)$

1. $$\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c \quad (n \neq -1)$$

2. $\int [f(x)]^{-1} f'(x)\,dx = \ln f(x) + c, \quad (f(x) > 0)$

General Form	Simple Form
In formulae 1 - 7 and 10 - 14, $a \neq 0$	
$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, (n \neq -1)$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c, (n \neq -1)$
$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + c,$	$\int \sin x\,dx = -\cos x + c$
$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + c$	$\int \cos x\,dx = \sin x + c$
$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + c$	$\int \sec^2 x\,dx = \tan x + c$
$\int \csc^2(ax+b)\,dx = -\frac{1}{a}\cot(ax+b) + c$	$\int \csc^2 x\,dx = -\cot x + c$
$\int \sec(ax+b)\tan(ax+b)\,dx = \frac{1}{a}\sec(ax+b) + c$	$\int \sec x \tan x\,dx = \sec x + c$
$\int \csc(ax+b)\cot(ax+b)\,dx = -\frac{1}{a}\csc(ax+b) + c$	$\int \csc x \cot x\,dx = \csc x + c$
$\int e^{\lambda x+\mu} dx = \frac{1}{\lambda} \times e^{\lambda x+\mu} + c, \quad (\lambda \neq 0)$	$\int e^x dx = e^x + c$
$\int a^{\lambda x+\mu} dx = \frac{1}{\lambda \ln a} . a^{\lambda x+\mu} + c, \ (a > 0, a \neq 1, \lambda \neq 0)$	$\int a^x dx = \frac{1}{\ln a} . a^x + c, \ (a > 0, a \neq 1)$
$\int \frac{1}{ax+b} dx = (ax+b)^{-1} dx = \frac{1}{a}\ln\|ax+b\| + c, (ax+b \neq 0)$	$\int \frac{1}{x} dx = \ln\|x\| + c, \quad x \neq 0$
$\int \tan(ax+b)\,dx = \frac{1}{a}\ln\|\sec(ax+b)\| + c = -\frac{1}{a}\ln\|\cos(ax+b)\| + c$	$\int \tan x\,dx = \ln\|\sec x\| + c = -\ln\|\cos x\| + c$
$\int \cot(ax+b)\,dx = \frac{1}{a}\ln\|\sin(ax+b)\| + c$	$\int \cot x\,dx = \ln\|\sin x\| + c$
$\int \sec(ax+b)\,dx = \frac{1}{a}\ln\|\sec(ax+b) + \tan(ax+b)\| + c$	$\int \sec x\,dx = \ln\|\sec x + \tan x\| + c$
$\int \csc(ax+b)\,dx = \frac{1}{a}\ln\|\csc(ax+b) - \cot(ax+b)\| + c$	$\int \csc x\,dx = \ln\|\csc x - \cot x\| + c$

INTEGRATION BY METHOD OF SUBSTITUTION

Sometimes it is possible to convert an integral into a standard form or to an easy integral by a suitable change of a variable. Now we evaluate $\int f(x)\,dx$ by the method of substitution. Let x be a function of a variable t, that is,

If $\quad x = \varphi(t)$, then $dx = \varphi'(t)dt$

Putting $x = \varphi(t)$, then $dx = \varphi'(t)dt$ we have

$$\int f(x)\,dx = \int f(\varphi(t))\,\varphi'(t)\,dt$$

INTEGRATION BY SOME USEFUL SUBSTITUTION

We list below suitable substitutions for certain expressions to be integrated.

	Expression Involving	**Suitable Substitution**
(i)	$\sqrt{a^2 - x^2}$	$x = a\sin\theta$
(ii)	$\sqrt{x^2 - a^2}$	$x = a\sec\theta$
(iii)	$\sqrt{a^2 + x^2}$	$x = a\tan\theta$
(iv)	$\sqrt{x+a}$ (or $\sqrt{x-a}$)	$\sqrt{x+a} = t$ (or $\sqrt{x-a} = t$)
(v)	$\sqrt{2ax - x^2}$	$x - a = a\sin\theta$
(vi)	$\sqrt{2ax + x^2}$	$x + a = a\sec\theta$

INTEGRATION BY PARTS

$$\int f(x)\,g'(x)dx = f(x)\int g'(x)dx - \int g(x)\,f'(x)dx = f(x)g(x) - \int g(x)\,f'(x)dx$$

This is known as the formula for integration by parts.

If we put $\quad u = f(x) \quad$ and $\quad dv = g'(x)dx$

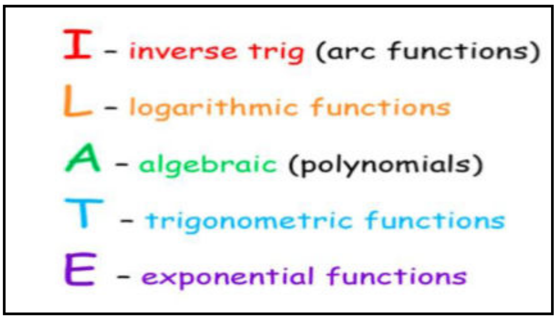
Then $\quad du = f'(x)dx \quad$ and $\quad v = g(x)$

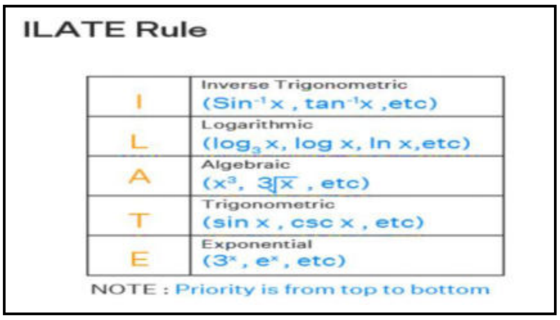
Then above equation can be written as

$$\int u\,dv = uv - \int v\,du$$

ILATE RULE

The **ILATE Rule** is a method for selecting the first and second functions when using the integration by parts method to solve integral.




ILATE Rule

I	Inverse Trigonometric (Sin⁻¹x , tan⁻¹x ,etc)
L	Logarithmic ($\log_a x$, log x, ln x,etc)
A	Algebraic (x^3, $\sqrt[3]{x}$, etc)
T	Trigonometric (sin x , csc x , etc)
E	Exponential (3^x , e^x , etc)

NOTE : Priority is from top to bottom

Note
ILATE is an acronym for Inverse, Logarithmic, Algebraic, Trigonometric, and Exponential.

INTEGRATION INVOLVING PARTIAL FRACTION

If $P(x), Q(x)$ are polynomial functions and the denominator $(Q(x) \neq 0)$, in the rational function $\frac{P(x)}{Q(x)}$, can be factorized into linear and quadratic (irreducible) factors, then the rational function is written as a sum of simpler rational functions, each of which can be integrated by methods of partial fraction which already known to us.
Here we will give examples of the following three cases when the denominator $Q(x)$ contains

Case I. Non-repeated linear factors.

$$\frac{x+7}{(x-1)(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)}$$

Case II. Repeated and non-repeated linear factors.

$$\frac{5x+7}{(x-1)(x+3)^2} = \frac{A}{(x-1)} + \frac{B}{(x+3)} + \frac{C}{(x+3)^2}$$

Case III. Linear and non-repeated irreducible quadratic factors or non-repeated irreducible quadratic factors.

$$\frac{11x+3}{(x-3)(x^2+9)} = \frac{A}{(x-3)} + \frac{Bx+C}{(x^2+9)}$$

THE DEFINITE INTEGRAL

If $f(x)$ is continuous on the interval $[a, b]$ and if $F(x)$ is any indefinite integral, then

$$\int_a^b f(x)\,dx = |F(x)|_a^b = F(b) - F(a)$$

is called ***definite integral*** of $f(x)$ between the limits "a" and "b"

- The interval $[\boldsymbol{a}, \boldsymbol{b}]$ is called ***range of integration.***
- The function $\boldsymbol{f(x)}$ is known as the ***integrand***.
- While $\boldsymbol{a}$ and $\boldsymbol{b}$ are known as ***lower*** and ***upper limits*** of integration respectively.

FUNDAMENTAL THEOREM OF CALCULUS

If $f(x)$ is continuous on the interval $[a, b]$ and $\varphi'(x) = f(x)$, that is, $\varphi(x)$ is any anti-derivative of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = \varphi(b) - \varphi(a)$$

PROPERTIES OF DEFINITE INTEGRAL

If $f(x)$ and $g(x)$ are two continuous functions on the interval $[a, b]$, then

(i) $\int_a^a f(x)\,dx = 0$

(ii) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(iii) $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$ (Where c is any constant)

(iv) $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

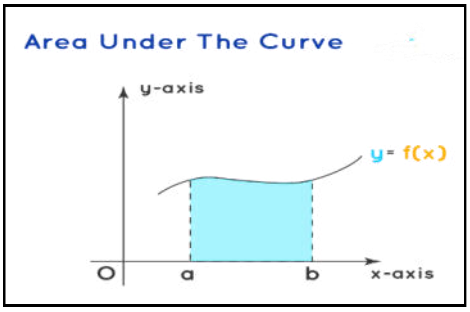
(v) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

APPLICATION OF DEFINITE INTEGRAL

Area Under the Curve

$\int_a^b f(x)\,dx$ gives the area under the cure $y = f(x)$ from $x = a$ to $x = b$ and the x-axis.

$$Area = \int_a^b f(x)\,dx$$



DIFFERENTIAL EQUATION

An equation containing at least one derivative of a dependent, variable with respect to an independent variable is called ***differential equation***.

Example: -

(i) $y\frac{dy}{dx} + 2x = 0$

(ii) $x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2x = 0$

ORDER OF DIFFERENTIAL EQUATION

The ***order*** of a differential equation is the order of the highest derivative it contains.

(i) $y\frac{dy}{dx} + 2x = 0$ (Order is 1)

(ii) $x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2x = 0$ (Order is 2)

DEGREE OF DIFFERENTIAL EQUATION

The degree of the differential equation is the highest power of the differential coefficient present in the equation.

(i) $x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2x = 0$ (Degree 1)

(ii) $y\left(\frac{dy}{dx}\right)^2 + 2x = 0$ (Degree 2)

MCQ's

Choose the correct Option.

1	If $y = x^2$ then dy is:						
a	$2x$	b	$2x\,dx$	c	$x^2\,dx$	d	$2x^2$
2	Differential of y is denoted by:						
a	dy'	b	$\frac{dy}{dx}$	c	dy	d	dx
3	If $y = x^2 - 1$ and x changes from 3 to 3.02 then $dy =$ ___						
a	0.1	b	0.12	c	0.012	d	0.21
4	If $V = x^3$, then differential of V is:						
a	$3x^2\,dx$	b	$3x^2$	c	$x^3\,dx$	d	$3x^2\,dy$
5	$f(x + \delta x) \approx$						
a	$f(x)dx$	b	$f(x) - f'(x)dx$	c	$f(x) + f'(x)dx$	d	$-f'(x)dx$
6	$\int(3x^2 + 2x)\,dx$ is equal to:						
a	$6x + 2$	b	$x^3 + x^2$	c	$3x + 2$	d	$x^3 - x^2$
7	Find dy for $y = \sqrt{x}$,when x changes from 4 to 4.41						
a	0.1	b	0.1002	c	0.1025	d	1.2
8	Find dy for $y = x^2 + 2x$, when x changes from 2 to 1.8						
a	-1.02	b	-0.012	c	-0.2	d	-1.2
9	Solve $\frac{1}{y}dy = \frac{1}{x}dx$						
a	$y = xc$	b	$y = -xc$	c	$y = x^2 + c$	d	$xy = c$
10	$d(xy) =$ ____						
a	$x\,dx + y\,dy$	b	$(x + y)dx$	c	$x\,dy + y\,dx$	d	
11	$\int \frac{\sin 2x}{4 \sin x}dx =$ ___						
a	$\sin 2x + c$	b	$2\sin 2x + c$	c	$\frac{1}{2}\sin x + c$	d	$2\sin x + c$
12	$\int \frac{1}{x\sqrt{x^2 - 1}} =$ ___						
a	$\sin^{-1} x$	b	$\tan^{-1} x$	c	$\sec^{-1} x$	d	$\operatorname{cosec}^{-1} x$
13	$\int(2x + 3)^{\frac{1}{2}}\,dx =$ ___						
a	$\frac{(2x+3)^{\frac{3}{2}}}{2} + c$	b	$\frac{(2x+3)^{\frac{3}{2}}}{3} + c$	c	$\frac{(2x+3)^{\frac{1}{3}}}{2} + c$	d	$\frac{(2x+3)^{-\frac{1}{2}}}{3} + c$
14	$\int \sec x\,dx =$ ____						
a	$\sec x + \tan x$	b	$\sec^2 x$	c	$\ln\lvert\sec x - \tan x\rvert$	d	$\ln\lvert\sec x + \tan x\rvert$
15	$\int(a - 2x)^{\frac{3}{2}}\,dx =$ _____						
a	$\frac{(a-2x)^{\frac{3}{2}}}{5} + c$	b	$\frac{(a-2x)^{\frac{5}{2}}}{5} + c$	c	$-\frac{(a-2x)^{\frac{5}{2}}}{5} + c$	d	$\frac{3(a-2x)^{\frac{5}{2}}}{5} + c$
16	$\int \cos x\,dx =$ ____						

a	$1-\sin^2 x+c$	b	$\sqrt{1-\sin^2 x}$	c	$\sin x+c$	d	$-\sin x+c$
17	$\int \tan\frac{\pi}{4}\,dx=$ ___						
a	$\ln\sin\frac{\pi}{4}$	b	x	c	$\sec^2\frac{\pi}{4}$	d	$\frac{x}{4}$
18	$\int e^{ax}\,dx=$ ___						
a	$e^{ax}+c$	b	$ae^{ax}+c$	c	$\frac{e^{ax}}{a}+c$	d	e^x+c
19	$\int \sin 5x\,dx=$ ____						
a	$-\frac{1}{5}\cos x+c$	b	$-\frac{1}{5}\cos 5x+c$	c	$\frac{1}{5}\sin x+c$	d	$\frac{1}{5}\cos 5x+c$
20	$\int \frac{a}{x}\,dx=$ _____						
a	$ax+c$	b	$a\ln\|x\|+c$	c	$-\frac{a}{x^2}+c$	d	$\frac{1}{a}\ln\|x\|+c$
21	$\int \frac{2}{x+2}\,dx=$ _____						
a	$2\ln\|x+2\|+c$	b	$\ln\|x+2\|+c$	c	$\ln\|x+2\|^2+c$	d	2
22	$\int \frac{x+2}{x+2}\,dx=$ _____						
a	$1+c$	b	$x+c$	c	$-x+c$	d	$2x$
23	Inverse of $\int \ldots dx$ is:						
a	$\frac{dx}{dy}$	b	$\frac{d}{dx}$	c	$\frac{dy}{dx}$	d	$\frac{dx}{dy}$
24	$\int \frac{1}{1+\cos x}\,dx=$ _____						
a	$\frac{1}{2}\tan\frac{x}{2}$	b	$\tan\frac{x}{2}$	c	$\cot\frac{x}{2}$	d	$\frac{1}{2}\cot\frac{x}{2}$
25	$\int \tan^2 x\,dx=$ _____						
a	$2\tan x+c$	b	$\tan x+c$	c	$\tan x+x+c$	d	$\tan x-x+c$
26	$\int \operatorname{cosec} x\,dx=$ _____						
a	$-\operatorname{cosec} x\cot x+c$	b	$\ln\|\operatorname{cosec} x-\cot x\|+c$	c	$\ln\|\operatorname{cosec} x+\cot x\|+c$	d	$\ln\|\sec x+\tan x\|$
27	$\int \tan x\,dx=$ _____						
a	$\ln\|\sec x\|+c$	b	$\ln\|\operatorname{cosec} x\|+c$	c	$\ln\|\sin x\|+c$	d	$\ln\|\cot x\|+c$
28	Anti-derivative of $\cot x$ is equal to:						
a	$\ln\|\cos x\|+c$	b	$\ln\|\sin x\|+c$	c	$-\ln\|\sin x\|+c$	d	$-\ln\|\cos x\|+c$
29	$\int \sin x\,dx=$ ____						
a	$\cos x+c$	b	$\sin x+c$	c	$-\sin x+c$	d	$-\cos x+c$
30	$\int 3^{\lambda x}\,dx=$ ___						

a	$\frac{3^{\lambda x}}{\lambda \ln 3}+c$	b	$\frac{3^{\lambda x}}{\lambda}\ln 3+c$	c	$\frac{3^{\lambda x}}{\ln 3}+c$	d	$\frac{\lambda 3^{\lambda x}}{\ln 3}+c$
31	$\int \frac{1}{ax+b}dx=$ ____						
a	$\ln\|ax+b\|+c$	b	$\frac{1}{a}\ln\|ax+b\|+c$	c	$a\ln\|ax+b\|+c$	d	$ax\ln\|ax+b\|+c$
32	$\int a^x\,dx=$ ___						
a	$\frac{a^x}{\ln a}+c$	b	a^x+c	c	$\frac{\ln a}{a^x}+c$	d	$a^x\ln a+c$
33	$\int \sin ax\,dx=$ ____						
a	$\frac{-\cos ax}{a}+c$	b	$\cos ax+c$	c	$a\,\text{cosec}\,x+c$	d	$a\sec ax+c$
34	$\int \cos 2x\,dx=$ ____						
a	$-2\sin 2x+c$	b	$2\sin 2x+c$	c	$\frac{-\sin 2x}{2}+c$	d	$\frac{\sin 2x}{2}+c$
35	$\int \sec^2 2x\,dx=$ ____						
a	$\frac{\tan 2x}{2}+c$	b	$\tan 2x+c$	c	$\frac{\tan x}{2}+c$	d	$2\tan 2x+c$
36	$\int(-\sin x)\,dx=$ ____						
a	$\cos x$	b	$\sin x$	c	$-\sin x$	d	$-\cos x$
37	$\int(\sin 3x)\,dx=$ ____						
a	$\frac{\cos 3x}{3}+c$	b	$\frac{-\cos 3x}{3}+c$	c	$3\cos 3x+c$	d	$-3\cos 3x+c$
38	$\int 3^x\,dx=$ ___						
a	3^x+c	b	$3^x\ln 3+c$	c	$\frac{3^x}{\ln 3}+c$	d	$\frac{3^x}{3\ln 3}+c$
39	$\int(e^x+1)\,dx=$ ____						
a	e^x+c	b	e^x+x+c	c	e^x+1+c	d	e^x+x^2+c
40	$\int \frac{1}{x^2}dx=$ ____						
a	$\ln x+c$	b	$\ln x^2+c$	c	$\frac{-1}{x}+c$	d	$\frac{1}{x}+c$
41	$\int \sec x\tan x\,dx=$ _____						
a	$\tan x$	b	$\sec^2 x$	c	$tan^2 x$	d	$\sec x$
42	$\int x(\sqrt{x}+1)\,dx$						
a	$\frac{2}{3}x^{3/2}+c$	b	$\frac{2}{5}x^{5/2}+\frac{x^2}{2}+c$	c	$\frac{2}{5}x^{5/2}+c$	d	$x^{3/2}+x+c$
43	$\int x^n\,dx$ for $(n\neq -1)$						

a	$\frac{x^{n+1}}{n}+c$	b	$\frac{x^{n-1}}{n-1}+c$	c	$\frac{x^{n+1}}{n+1}+c$	d	$nx^{n-1}+c$
44	$\int 5^{2x}\,dx=$ ___						
a	5^{2x}	b	$2(5^{2x})$	c	$5^{2x}\ln 5$	d	$\frac{5^{2x}}{2\ln 5}+c$
45	$\int(\sec^2\theta-\tan^2\theta)\,d\theta=$ ____						
a	$\theta+c$	b	$\sin\theta+\cos\theta+c$	c	$\tan\theta-\cot\theta+c$	d	$\cot\theta+\tan\theta+c$
46	$\int\frac{1}{\cos^2 x}dx=$ ____						
a	$\frac{1}{\sin^2 x}$	b	$\tan x+c$	c	$\sec^2 x+c$	d	$cosec^2\,x+c$
47	$\int x^{-1}\,dx=$ ___						
a	0	b	$\ln x+c$	c	$-x^{-2}+c$	d	$-\ln x+c$
48	$\int e^{\sin x}\cos x\,dx=$ ____						
a	$e^{\cos x}+c$	b	$e^{\sin x}+c$	c	$-e^{\cos x}+c$	d	$-e^{\sin x}+c$
49	$\int a^{x^2}x\,dx=$ ___						
a	$\frac{a^x}{\log_e a}+c$	b	$\frac{a^{x^2}}{2\log_e a}+c$	c	$\frac{a^{x^2}}{2\log_a e}+c$	d	$\frac{a^x}{2\log_e a}+c$
50	$\int e^{\tan x}\sec^2 x\,dx=$ ____						
a	$e^{\tan x}$	b	$e^{-\tan x}$	c	$e^{\cot x}$	d	$e^{-\cot x}$
51	$\int\frac{\ln x}{x}dx=$ ____						
a	$\ln(\ln x)$	b	$\frac{(\ln x)^2}{2}$	c	$\ln x$	d	$\frac{\ln x}{2}$
52	$\int\frac{e^{\tan^{-1}x}}{1+x^2}dx=$ ____						
a	$e^{\sec x}+c$	b	$e^{\tan x}+c$	c	$e^{cot^{-1}x}+c$	d	$e^{\tan^{-1}x}+c$
53	$\int\frac{a}{x\sqrt{x^2-1}}dx=$ _____						
a	$a\tan^{-1}x+c$	b	$-acosec^{-1}x+c$	c	$-asec^{-1}x+c$	d	$asec^{-1}x+c$
54	$\int\frac{f'(x)}{f(x)}dx=$ ____						
a	$\ln\lvert x\rvert+c$	b	$\ln\lvert f(x)\rvert+c$	c	$\ln\lvert f'(x)\rvert+c$	d	$f(x)+c$
55	$\int\sqrt{2x+3}\,(2dx)=$ ___						
a	$\frac{2}{3}(2x+3)^{3/2}$	b	$\frac{3}{2}(2x+3)^{3/2}$	c	$-(2x+3)^{3/2}$	d	$-\frac{3}{2}(2x+3)^{3/2}$
56	$\int\sin x\cos x\,dx=$ ___						
a	$\frac{\cos 2x}{2}+c$	b	$\frac{-\cos 2x}{2}+c$	c	$\frac{\sin^2 x}{2}+c$	d	$\frac{\cos^2 x}{2}+c$

57	$\int \frac{1}{x\ln x}dx$ equals:						
a	$\ln x$	b	x	c	$\ln(\ln x)$	d	$\frac{(\ln x)^2}{2}$
58	$\int \sec^2 x \tan x\, dx =$ _____						
a	$\sec x \tan^2 x$	b	$\frac{\sec^2 x}{3}+c$	c	$\frac{\tan^2 x}{2}+c$	d	$\frac{\sec^2 x \tan x\, dx}{3}$
59	$\int \sec 5x \tan 5x\, dx =$ _____						
a	$5\sec 5x + c$	b	$\frac{\sec x}{5}+c$	c	$\frac{\sec 5x}{5}+c$	d	$\frac{\tan 5x}{5}+c$
60	$\int \frac{\sec^2 x}{\sqrt{\tan x}}dx =$ ____						
a	$2\sqrt{\tan x}+c$	b	$-2\sqrt{\tan x}+c$	c	$\sqrt{\tan x}+c$	d	$\tan x + c$
61	$\int \frac{\cot x}{\ln(\sin x)}dx =$ ____						
a	$\ln\tan x + c$	b	$\ln\cot x + c$	c	$\ln\sin x + c$	d	$\ln(\ln\sin x)+c$
62	$\int \frac{e^x}{e^x+3}dx =$ ___						
a	$\ln\|e^x+3\|$	b	$e^{2x}+c$	c	e^0+c	d	$e^{2x}+3+c$
63	$\int \frac{1}{(1+x^2)\tan^{-1}x}dx =$ _____						
a	$\ln(\tan^{-1}x)+c$	b	$\ln\sin x + c$	c	$\ln(\sin x)^2+c$	d	None
64	$\int \frac{x}{\sqrt{4+x^2}}dx =$ ____						
a	$\sqrt{4+x^2}+c$	b	$\frac{1}{2}\sqrt{4+x^2}+c$	c	$\ln\sqrt{4+x^2}+c$	d	None
65	$\int \frac{1}{\sqrt{a^2-x^2}}dx =$ ____						
a	$\cos^{-1}\left(\frac{x}{a}\right)+c$	b	$\sin^{-1}\left(\frac{x}{a}\right)+c$	c	$\frac{1}{a}\cos^{-1}\left(\frac{x}{a}\right)+c$	d	$\frac{1}{a}\sin^{-1}\left(\frac{x}{a}\right)+c$
66	$\int \frac{1}{x^2+4}dx =$ ____						
a	$\frac{1}{2}\tan^{-1}\frac{x}{2}+c$	b	$\tan^{-1}\frac{x}{2}+c$	c	$2\tan^{-1}\frac{x}{2}+c$	d	$\tan^{-1}2x+c$
67	$\int \frac{1}{1+x^2}dx =$ ____						
a	$\cos^{-1}x$	b	$\cot^{-1}x$	c	$\tan^{-1}x$	d	$\sec^{-1}x$
68	$\int \frac{1}{\sqrt{5-x^2}}dx =$ ____						
a	$\sin^{-1}\left(\frac{5}{x}\right)+c$	b	$\sin^{-1}\left(\frac{x}{5}\right)+c$	c	$\sin^{-1}\left(\frac{\sqrt{5}}{x}\right)+c$	d	$\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)+c$
69	The suitable substitution to integrate $\sqrt{x^2-a^2}$:						
a	$x=a\sin\theta$	b	$x=a\cos\theta$	c	$x=a\sec\theta$	d	$x=a\tan\theta$
70	The suitable substitution to integrate $\sqrt{a^2-x^2}$:						
a	$x=a\sin\theta$	b	$x=a\cos\theta$	c	$x=a\sec\theta$	d	$x=a\tan\theta$

71	The suitable substitution to integrate $\frac{1}{x\sqrt{x^2-a^2}}$:						
a	$x = a\sin\theta$	b	$x = a\cos\theta$	c	$x = a\sec\theta$	d	$x = a\tan\theta$
72	$\int \ln x\, dx =$ __						
a	$x\ln x + x + c$	b	$x\ln x - x + c$	c	$x - x\ln x + c$	d	$-x\ln x - x + c$
73	$\int e^{-x}(\cos x - \sin x)\, dx =$ ___ −						
a	$e^{-x}\sin x + c$	b	$e^{-x}\cos x + c$	c	$e^{x}\sin x + c$	d	$e^{x}\cos x + c$
74	$\int e^{x}\left(\frac{1}{x} + \ln x\right) dx =$ ____						
a	$e^{x} + c$	b	$e^{x}\ln x + c$	c	$e^{x} + \ln x + c$	d	$\ln x + c$
75	$\int e^{x}(\cos x + \sin x)\, dx =$ ___						
a	$e^{-x}\sin x + c$	b	$e^{-x}\cos x + c$	c	$e^{x}\sin x + c$	d	$e^{x}\cos x + c$
76	$\int e^{2x}(-\sin x + 2\cos x)\, dx =$ ___						
a	$e^{2x}\cos x + c$	b	$e^{2x}\sin x + c$	c	$-e^{2x}\cos x + c$	d	$-e^{2x}\sin x + c$
77	$\int e^{ax}\left(af(x) + f'(x)\right) dx =$ ___						
a	$e^{ax}a.f(x)$	b	$e^{ax}f'(x)$	c	$e^{ax}f(x)$	d	$e^{ax}a.f'(x)$
78	$\int e^{x}(x+1)\, dx =$ ___						
a	$e^{x} + c$	b	$xe^{x} + c$	c	$-e^{x} + c$	d	$-xe^{x} - c$
79	$\int_1^2 (x^2+1)\, dx =$ ____						
a	$\frac{10}{3}$	b	$\frac{3}{10}$	c	π	d	$\frac{\pi}{2}$
80	$\int_a^x 3x^2\, dx =$ ____						
a	$x^3 + a^3$	b	$x^3 - a^3$	c	$3x^3$	d	x^3
81	$\int_0^\pi \cos x\, dx =$ ___						
a	-1	b	1	c	0	d	2
82	$\int_0^\pi \sin x\, dx =$ ___						
a	-1	b	1	c	0	d	2
83	$\int_{-\pi}^{\pi} \sin x\, dx =$ ___						
a	-1	b	1	c	0	d	2

84	$\int_0^3 \frac{dx}{x^2+9} =$ ____						
a	$\frac{\pi}{12}$	b	$\frac{12}{\pi}$	c	$-\frac{12}{\pi}$	d	$-\frac{\pi}{12}$
85	$\int_0^\pi \sec x \tan x \ dx =$ ____						
a	0	b	1	c	-2	d	2
86	$\int_0^{\frac{\pi}{4}} \cos x \ dx =$ __						
a	1	b	2	c	$\sqrt{2}$	d	$\frac{1}{\sqrt{2}}$
87	$\int_0^1 \frac{1}{1+x^2} dx =$ ____						
a	$\frac{\pi}{4}$	b	$\frac{4}{\pi}$	c	$\frac{-\pi}{4}$	d	$-\frac{4}{\pi}$
88	$\int_a^b f(x)\, dx =$						
a	$-\int_a^b f(x)\, dx$	b	$-\int_b^a f(x)\, dx$	c	$\int_b^a f(x)\, dx$	d	None
89	$\int_0^{\frac{\pi}{4}} \sec^2 x \ dx =$ __						
a	5	b	4	c	2	d	1
90	$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sec^2 x \ dx =$ __						
a	1	b	2	c	0	d	3
91	$3\int_{-\pi}^{\pi} \sin x\, dx =$ ___						
a	1	b	2	c	0	d	3
92	$\int_0^1 \lvert x \rvert\, dx =$ ___						
a	1	b	2	c	0	d	$1/2$
93	$\int_a^c f(x)\, dx + \int_c^b f(x)\, dx =$ ___						

a	$\int_a^b f(x)\,dx$	b	$\int_b^a f(x)\,dx$	c	$\int_{-a}^b f(x)\,dx$	d	$\int_c^a f(x)\,dx$
94	The area under the cure $y = f(x)$ from $x = a$ to $x = b$ and the x-axis is denoted by:						
a	$\int_a^b f(x)\,dx$	b	$\int_a^b ydx$	c	$\int_a^b f(x)\,dy$	d	Both a and b
95	The order of differential equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2 = 0$ is:						
a	1	b	2	c	0	d	3
96	The order of differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2x = 0$ is:						
a	1	b	2	c	0	d	3
97	The order of $\frac{dy}{dx} = \frac{4}{3}x^3 + x - 3$ is:						
a	1	b	2	c	0	d	3
98	The order of differential equation $y\frac{dy}{dx} + 2x = 0$ is:						
a	1	b	2	c	0	d	3
99	The solution of differential equation $\frac{dy}{dx} = -y$ is:						
a	$y = xe^{-x}$	b	$y = ce^{-x}$	c	$y = e^x$	d	$y = ce^x$
100	The solution of differential equation $y\,dx + x\,dy = 0$ is:						
a	$x + y = c$	b	$\ln xy = 0$	c	$xy = c$	d	None
101	The solution of differential equation $\frac{dy}{dx} = \sec^2 x$ is:						
a	$y = \cos x + c$	b	$y = \sec x + c$	c	$y = \sin^2 x + c$	d	$y = \tan x + c$
102	Applying initial value conditions in solution of differential equations, we get:						
a	General solution	b	Particular solution	c	No solution	d	Infinite solution
103	The solution of differential equation $\frac{dy}{dx} = 1$ is:						
a	$y = e^x + c$	b	$y = x + c$	c	$y = \ln x$	d	$y = ce^{-x}$
104	The solution of differential equation $\frac{dy}{dx} = \frac{y^2+1}{e^{-x}}$ is:						
a	$y = \tan(e^{-x} + c)$	b	$y = \tan^{-1}(e^{-x} + c)$	c	$y = \tan(e^x + c)$	d	$y = \tan^{-1}(e^x + c)$
105	The solution of differential equation $x\frac{dy}{dx} = 1 + y$ is:						
a	$y = x - 1$	b	$y = cx + 1$	c	$y = cx - 1$	d	$y = x + c$

MCQ's Answers Key

1	2	3	4	5	6	7	8	9	10
b	c	b	a	c	b	c	d	a	c
11	12	13	14	15	16	17	18	19	20
c	c	b	d	c	c	b	c	b	b
21	22	23	24	25	26	27	28	29	30
a	b	b	b	d	b	a	b	d	a
31	32	33	34	35	36	37	38	39	40
b	a	a	d	a	a	b	c	b	c
41	42	43	44	45	46	47	48	49	50
d	b	c	d	a	b	b	b	b	a
51	52	53	54	55	56	57	58	59	60
b	d	c	b	a	c	c	c	c	a
61	62	63	64	65	66	67	68	69	70
d	a	a	a	b	a	c	d	c	a
71	72	73	74	75	76	77	78	79	80
c	b	a	b	c	a	c	b	a	b
81	82	83	84	85	86	87	88	89	90
c	d	c	a	c	d	a	b	d	b
91	92	93	94	95	96	97	98	99	100
c	d	a	d	b	b	a	a	b	c
101	102	103	104	105					
d	a	b	c	c					

IMPORTANT SHORT QUESTIONS

1. Define differential equation.
2. Find δy and dy of the function $f(x) = x^2$, when $x = 2$ and $dx = 0.01$.
3. Using differentials find $\frac{dy}{dx}$ when $\frac{y}{x} - \ln x = \ln c$.
4. Find δy and dy in the case, $y = x^2 - 1$ when x changes from 3 to 3.02
5. Find δy and dy in the case, $y = x^2 + 2x$ when x changes from 2 to 1.8
6. Find δy and dy in the case, $y = \sqrt{x}$ when x changes from 4 to 4.41
7. Using differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the equation $xy + x = 4$
8. Using differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the equation $x^2 + 2y^2 = 16$
9. Using differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the equation $x^4 + y^2 = xy^2$
10. Using differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the equation $xy - \ln x = c$
11. Use differentials to approximate the value of $\sqrt[4]{17}$
12. Use differentials to approximate the value of $(31)^{1/5}$
13. Use differentials to approximate the value of $\cos 29°$
14. Find the approximate increase in the volume of a cube if the length of its each edge changes from 5 to 5.02.
15. Evaluate $\int (x+1)(x-3)\,dx$
16. Evaluate $\int x\sqrt{x^2-1}\,dx$
17. Evaluate $\int \frac{x}{x+2}\,dx$
18. Evaluate $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)}\,dx$
19. Evaluate $\int \frac{dx}{\sqrt{x+1}-\sqrt{x}}$
20. Evaluate $\int \frac{\sin x + \cos^3 x}{\cos^2 x \sin x}\,dx$
21. Evaluate $\int \frac{3-\cos 2x}{1+\cos 2x}\,dx$
22. Evaluate $\int (3x^2 - 2x + 1)\,dx$
23. Evaluate $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx$
24. Evaluate $\int x(\sqrt{x}+1)\,dx$
25. Evaluate $\int (2x+3)^{\frac{1}{2}}\,dx$
26. Evaluate $\int (\sqrt{x}+1)^2\,dx$
27. Evaluate $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$
28. Evaluate $\int \frac{3x+2}{\sqrt{x}}\,dx$
29. Evaluate $\int \frac{\sqrt{y}(y+1)}{y}\,dy$
30. Evaluate $\int \frac{(\sqrt{\theta}-1)^2}{\sqrt{\theta}}\,d\theta$

31. Evaluate $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}}\,dx$

32. Evaluate $\int \frac{e^{2x}+e^x}{e^x}\,dx$

33. Evaluate $\int \frac{1-x^2}{1+x^2}\,dx$

34. Evaluate $\int \frac{dx}{\sqrt{x+a}+\sqrt{x}}$

35. Evaluate $\int (a-2x)^{\frac{3}{2}}\,dx$

36. Evaluate $\int \frac{(1+e^x)^3}{e^x}\,dx$

37. Evaluate $\int \sin(a+b)x\,dx$

38. Evaluate $\int \sqrt{1-\cos 2x}\,dx$

39. Evaluate $\int \sin^2 x\,dx$

40. Evaluate $\int \frac{1}{1+\cos x}\,dx$

41. Evaluate $\int \frac{ax+b}{ax^2+2bx+c}\,dx$

42. Evaluate $\int \cos 3x \sin 2x\,dx$

43. Evaluate $\int \frac{\cos 2x-1}{1+\cos 2x}\,dx$

44. Evaluate $\int \tan^2 x\,dx$

45. Evaluate $\int \frac{a\,dt}{2\sqrt{at+b}}$

46. Evaluate $\int \frac{x}{\sqrt{4+x^2}}\,dx$

47. Evaluate $\int x\sqrt{x-a}\,dx$

48. Evaluate $\int \frac{\cot\sqrt{x}}{\sqrt{x}}\,dx$

49. Evaluate $\int \operatorname{cosec} x\,dx$

50. Evaluate $\int \sec x\,dx$

51. Evaluate $\int \frac{dx}{x(\ln 2x)^3}$

52. Evaluate $\int a^{x^2} x\,dx$

53. Evaluate $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$

54. Evaluate $\int \frac{-2x}{\sqrt{4-x^2}}\,dx$

55. Evaluate $\int \frac{dx}{x^2+4x+13}$

56. Evaluate $\int \frac{x^2}{4+x^2}\,dx$

57. Evaluate $\int \frac{1}{x\ln x}\,dx$

58. Evaluate $\int \frac{e^x}{e^x+3}\,dx$

59. Evaluate $\int \frac{x+b}{(x^2+2bx+c)^{1/2}}\,dx$

60. Evaluate $\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx$

61. Evaluate $\int \frac{dx}{(1+x^2)^{3/2}}$

62. Evaluate $\int \frac{1}{(1+x^2)\tan^{-1} x}\,dx$

63. Evaluate $\int \frac{\sin\theta}{1+\cos^2\theta}\,d\theta$

64. Evaluate $\int \frac{ax}{\sqrt{a^2-x^4}}\,dx$

65. Evaluate $\int \frac{\cos x}{\sin x \ln \sin x}\,dx$

66. Evaluate $\int \frac{x+2}{\sqrt{x+3}}\,dx$

67. Evaluate $\int \frac{dx}{\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x}$

68. Evaluate $\int x\cos x\,dx$

69. Evaluate $\int xe^x\,dx$

70. Evaluate $\int x\tan^2 x\,dx$

71. Evaluate $\int x^5 \ln x\,dx$

72. Evaluate the integral by parts: $\int x\sin x\,dx$

73. Evaluate the integral by parts: $\int \ln x\,dx$

74. Evaluate the integral by parts: $\int x\ln x\,dx$

75. Evaluate the integral by parts: $\int x^2\ln x\,dx$

76. Evaluate the integral by parts: $\int x^3\ln x\,dx$

77. Evaluate the integral by parts: $\int x^4\ln x\,dx$

78. Evaluate the integral by parts: $\int \tan^{-1} x\,dx$

79. Evaluate the integral by parts: $\int x^2\sin x\,dx$

80. Evaluate the integral by parts: $\int x^2\tan^{-1} x\,dx$

81. Evaluate the integral by parts: $\int x\tan^{-1} x\,dx$

82. Evaluate the integral by parts: $\int x\sin^{-1} x\,dx$

83. Evaluate the integral by parts: $\int x\sin x\cos x\,dx$

84. Evaluate the integral by parts: $\int (\ln x)^2\,dx$

85. Evaluate $\int \sec^4 x\,dx$

86. Evaluate $\int e^x\left(\frac{1}{x}+\ln x\right)dx$

87. Evaluate $\int e^x(\cos x+\sin x)\,dx$

88. Evaluate $\int e^{ax}\left(a\sec^{-1}x+\frac{1}{x\sqrt{x^2-1}}\right)dx$

89. Evaluate $\int e^{3x}\left(\frac{3\sin x-\cos x}{\sin^2 x}\right)dx$

90. Evaluate $\int e^{2x}(-\sin x+2\cos x)\,dx$

91. Evaluate $\int \frac{xe^x}{(1+x)^2}\,dx$

92. Evaluate $\int e^{-x}(\cos x-\sin x)\,dx$

93. Evaluate $\int \frac{e^{m\tan^{-1}x}}{1+x^2}\,dx$

94. Evaluate $\int \frac{2x}{1-\sin x}\,dx$

95. Evaluate $\int \frac{e^x(1+x)}{(2+x)^2}\,dx$

96. Evaluate $\int \frac{2a}{x^2-a^2}\,dx,\ (x>0)$

97. Evaluate $\int \frac{3x+1}{x^2-x+6}\,dx$

98. Evaluate $\int \frac{5x+8}{(x+3)(2x-1)}\,dx$

99. Evaluate $\int \frac{(a-b)x}{(x-a)(x-b)}dx$

100. Evaluate $\int \frac{2x}{x^2-a^2}dx$

101. Define Definite Integral.

102. Give two properties of definite integral.

103. State Fundamental Theorem of Calculus.

104. Prove that $\int_a^b f(x)dx = -\int_b^a f(x)dx$

105. Evaluate $\int_{-1}^{3}(x^3+3x^2)\,dx$

106. Evaluate $\int_1^2 \frac{x^2+1}{x+1}\,dx$

107. Evaluate $\int_0^{\frac{\pi}{4}} \sec x\,(\sec x + \tan x)\,dx$

108. Evaluate $\int_0^{\frac{\pi}{6}} x\cos x\,\,dx$

109. Evaluate $\int_1^e x\ln x\,dx$

110. If $\int_{-2}^{1} f(x)dx = 5$, $\int_{-2}^{1} g(x)dx = 4$ then evaluate the integral

(i) $\int_{-2}^{1}[2f(x)+3g(x)]dx$ (ii) $\int_{-2}^{1} 3f(x)dx - 2\int_{-2}^{1} g(x)dx$

111. Evaluate $\int_1^2 (x^2+1)\,dx$

112. Evaluate $\int_{-1}^{1}\left(x^{\frac{1}{3}}+1\right)dx$

113. Evaluate $\int_{-2}^{0}\frac{1}{(2x-1)^2}dx$

114. Evaluate $\int_{-6}^{2}\sqrt{3-x}\,dx$

115. Evaluate $\int_1^{\sqrt{5}}\sqrt{(2t-1)^3}\,dt$

116. Evaluate $\int_2^{\sqrt{5}} x\sqrt{x^2-1}\,dx$

117. Evaluate $\int_1^2 \frac{x}{x^2+2}dx$

118. Evaluate $\int_2^3\left(x-\frac{1}{x}\right)^2 dx$

119. Evaluate $\int_0^3 \frac{dx}{x^2+9}$

120. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos t\,dt$

121. Evaluate $\int_1^2 \ln x\,dx$

122. Evaluate $\int_0^2\left(e^{\frac{x}{2}}-e^{-\frac{x}{2}}\right)dx$

123. Evaluate $\int_0^{\frac{\pi}{3}}\cos^2\theta\sin\theta\,dx$

124. Evaluate $\int_{-1}^{5}|x-3|\,dx$

125. Find the area bounded by the curve $y=4-x^2$ and the x-axis.

126. Find the area bounded by the curve $y=x^3+3x^2$ and the x-axis.

127. Find the area between the x-axis and the curve $y=x^2+1$ from $x = 1$ to $x = 2$.

128. Find the area, above the x-axis and under the curve $y=5-x^2$ from $x=-1$ to $x=2$.

129. Find the area below the curve $y=3\sqrt{x}$ and above the x-axis between $x=1$ and $x=4$.

130. Find the area bounded by cos function from $x=-\frac{\pi}{2}$ to $x=\frac{\pi}{2}$

131. Find the area between the x-axis and the curve $y = 4x - x^2$
132. Find the area above x-axis, bounded by the curve $y^2 = 3 - x$ from $x = -1$ to $x = 2$.
133. Find the area between the x-axis and the curve $y = \cos\frac{1}{2}x$ from $x = -\pi$ to π.
134. Find the area between the x-axis and the curve $y = \sin 2x$ from $x = 0$ to $x = \frac{\pi}{3}$.
135. Define Differential Equation.
136. Define Order of Differential Equation.
137. Solve the differential equation $\frac{dy}{dx} = \frac{y^2+1}{e^{-x}}$
138. Check $y = \tan(e^x + c)$ is a solution of the differential equation $\frac{dy}{dx} = \frac{y^2+1}{e^{-x}}$
139. Solve the differential equation $\frac{dy}{dx} = -y$
140. Solve the differential equation $y\,dx + x\,dy = 0$
141. Solve the differential equation $\frac{dy}{dx} = \frac{1-x}{y}$
142. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x^2}$
143. Solve the differential equation $\sin y \operatorname{cosec} x \frac{dy}{dx} = 1$
144. Solve the differential equation $x\,dy + y(x-1)\,dx = 0$
145. Solve the differential equation $\frac{x^2+1}{y+1} = \frac{x}{y}.\frac{dy}{dx}$
146. Solve the differential equation $\frac{1}{x}\frac{dy}{dx} = \frac{1}{2}(1 + y^2)$
147. Solve the differential equation $1 + \cos x \tan y \frac{dy}{dx} = 0$
148. Solve the differential equation $\sec x + \tan y \frac{dy}{dx} = 0$
149. Solve the differential equation $(e^x + e^{-x})\frac{dy}{dx} = e^x - e^{-x}$
150. Find the general solution of the equation $\frac{dy}{dx} - x = xy^2$

IMPORTANT LONG QUESTIONS

1. Evaluate $\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}}, \begin{pmatrix} x+a>0 \\ x+b>0 \end{pmatrix}$
2. Evaluate $\int \sqrt{1+\sin x}\, dx$
3. Evaluate $\int \frac{1}{\sqrt{a^2+x^2}}\, dx$
4. Show that $\int \frac{1}{\sqrt{x^2-a^2}}\, dx = \ln\left(x+\sqrt{x^2-a^2}\right)+c$
5. Show that $\int \sqrt{a^2-x^2}\, dx = \frac{a^2}{2}\sin^{-1}\frac{x}{a}+\frac{x}{2}\sqrt{a^2-x^2}+c$
6. Evaluate $\int \frac{dx}{\sqrt{7-6x-x^2}}$
7. Evaluate $\int \frac{x}{x^4+2x^2+5}\, dx$
8. Evaluate $\int \frac{\sqrt{2}}{\sin x+\cos x}\, dx$
9. Evaluate $\int \ln\left(x+\sqrt{x^2+1}\right) dx$
10. Evaluate $\int \sqrt{a^2+x^2}\, dx$
11. Evaluate $\int \sin^4 x\, dx$
12. Evaluate $\int \frac{e^x(1+\sin x)}{1+\cos x}\, dx$
13. Evaluate the integral by parts: $\int x^3 \cos x\, dx$
14. Evaluate the integral by parts: $\int x \sin^{-1} x\, dx$
15. Evaluate the integral by parts: $\int e^x \sin x \cos x\, dx$
16. Evaluate the integral by parts: $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\, dx$
17. Evaluate $\int x^3 \cos x\, dx$
18. Evaluate $\int \tan^3 x \sec x\, dx$
19. Evaluate $\int x^3 e^{5x}\, dx$
20. Evaluate $\int e^{-x} \sin 2x\, dx$
21. Evaluate $\int e^{2x} \cos 3x\, dx$
22. Evaluate $\int \operatorname{cosec}^3 x\, dx$
23. Evaluate the indefinite integral $\int \sqrt{x^2-a^2}\, dx$
24. Evaluate the indefinite integral $\int \sqrt{4-5x^2}\, dx$
25. Evaluate the indefinite integral $\int \sqrt{3-4x^2}\, dx$
26. Evaluate the indefinite integral $\int \sqrt{x^2+4}\, dx$
27. Evaluate $\int \frac{7x-1}{(x-1)^2(x+1)}\, dx$
28. Evaluate $\int \frac{2x}{x^6-1}\, dx$
29. Evaluate $\int \frac{x^2+3x-34}{x^2+2x-15}\, dx$
30. Evaluate $\int \frac{2x^3-3x^2-x-7}{2x^2-3x-2}\, dx$
31. Evaluate $\int \frac{4+7x}{(1+x)^2(2+3x)}\, dx$
32. Evaluate $\int \frac{2x^2}{(x-1)^2(x+1)}\, dx$
33. Evaluate $\int \frac{x-2}{(x+1)(x^2+1)}\, dx$

34. Evaluate $\int \frac{1+4x}{(x-3)(x^2+4)}\,dx$

35. Evaluate $\int \frac{12}{x^3+8}\,dx$

36. Evaluate $\int_0^{\sqrt{3}} \frac{x^3+9x+1}{x^2+9}\,dx$

37. Evaluate $\int_{-1}^{2} (x+|x|)\,dx$

38. Evaluate $\int_0^{\sqrt{7}} \frac{3x}{\sqrt{x^2+9}}\,dx$

39. Evaluate $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

40. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos^2\theta\cot^2\theta\,dt$

41. Evaluate $\int_0^{\frac{\pi}{4}} \cos^4 t\,dt$

42. Evaluate $\int_0^{\frac{\pi}{4}} (1+\cos^2\theta)\tan^2\theta\,d\theta$

43. Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sec\theta}{\sin\theta+\cos\theta}\,d\theta$

44. Evaluate $\int_{\frac{1}{8}}^{1} \frac{\left(x^{1/3}+2\right)^2}{x^{2/3}}\,dx$

45. Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin x-1}{\cos^2 x}\,dx$

46. Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{1+\sin x}\,dx$

47. Find the area bounded by the curve $f(x) = x^3 - 2x^2 + 1$ and the x-axis is in the 1st quadrant.

48. Determine the area bounded by the parabola $y = x^2 + 2x - 3$ and the x-axis.

49. Find the area bounded by the curve $y = x^3 - 4x$ and the x-axis.

50. Find the area between the x-axis and the curve $y = \sqrt{2ax - x^2}$ when $a > 0$.

51. Solve the differential equation $\frac{1}{x}\frac{dy}{dx} - 2y = 0$

52. Solve $2e^x \tan y + (1-e^x)\sec^2 y\,dy = 0$

53. Solve the differential equation $\frac{dy}{dx} + \frac{2xy}{2y+1} = x$

54. Solve the differential equation $(x^2 - yx^2)\frac{dy}{dx} + y^2 + xy^2 = 0$

55. Solve the differential equation $\sec^2 x\tan y\,dx + \sec^2 y\tan x\,dy = 0$

56. Solve the differential equation $\left(y - x\frac{dy}{dx}\right) = 2\left(y^2 + \frac{dy}{dx}\right)$

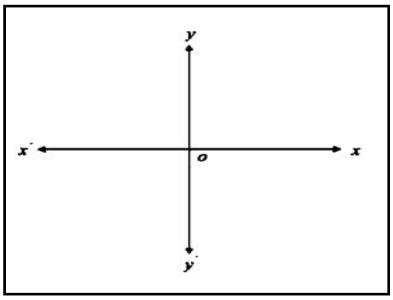
57. Solve the differential equation $y - x\frac{dy}{dx} = 3\left(1 + x\frac{dy}{dx}\right)$

UNIT 4

Introduction to Analytic Geometry

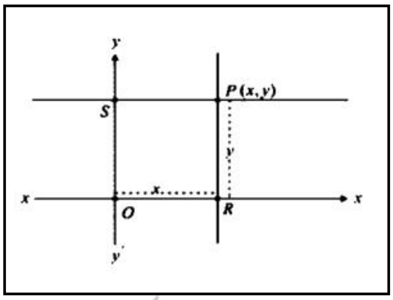
DEFINITIONS + SUMMARY

COORDINATE SYSTEM



Draw in a plane two mutually perpendicular number lines $x'\,x$ and $y'\,y$, one horizontal and the other vertical. Let their point of intersection be O, to which we call the **origin** and the real number 0 of both the lines is represented by O. The two lines are called the **coordinate axes**. The horizontal line $x'Ox$ is called the ***x*-axis** and the vertical line $y'Oy$ is called the ***y*-axis**.

Suppose P is any point in the plane. Then ***P*** can be **located** by using an ordered pair of real numbers. Through P draw lines parallel to the coordinates axes meeting $x-axis$ at R and $y-axis$ at S. Let the directed distance $\overline{OR}=x$ and the directed distance $\overline{OS}=y$.

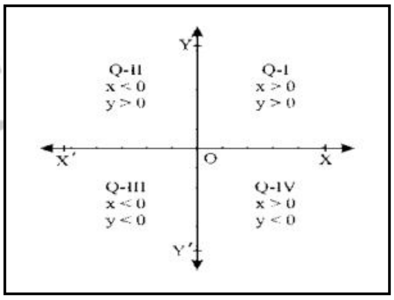


The ordered pair (x,y) gives us enough information to locate the point P. Thus, with every point P in the plane, we can associate an ordered pair of real numbers (x,y) and we say that P has **coordinates** (x,y). It may be noted that x and y are the directed distances of P from the $y-axis$ and the $x-axis$ respectively. The reverse of this technique also provides method for associating exactly one point in the plane with any ordered pair (x,y) of real numbers. This method of pairing of in a one-to-one fashion the points in a plane with ordered pairs of real numbers is called the **two-dimensional rectangular** (or **Cartesian**) **coordinate system**.

Note

If (x,y) are the coordinates of a point P, then the first member (component) of the ordered pair is called the ***x* - coordinate** or **abscissa** of P and the second member of the ordered pair is called the ***y* - coordinate** or **ordinate** of P. Note that abscissa is always first element and the ordinate is second element in an ordered pair.

QUADRANTS



The coordinate axes divide the plane into four equal parts called **quadrants**. They are defined as follows:

Quadrant I: All points (x,y) with $x>0, y>0$
Quadrant II: All points (x,y) with $x<0, y>0$
Quadrant III: All points (x,y) with $x<0, y<0$
Quadrant IV: All points (x,y) with $x>0, y<0$

DISTANCE FORMULA

Let $A(x_1,y_1)$ and $B(x_2,y_2)$ be the points in the plane, then distance ***d*** is given by

$$d=|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

THE RATIO FORMULA (POINT DIVIDING THE LINE-SEGMENT IN A GIVEN RATIO)

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be the two given points in a plane. The coordinates of the point dividing the line segment AB in the ratio $k_1: k_2$ are $\left(\frac{k_1x_2+k_2x_1}{k_1+k_2}, \frac{k_1y_2+k_2y_1}{k_1+k_2}\right)$.Where k_1, k_2 are positive integer.

Let $P(x, y)$ be the points that divides AB in the ratio $k_1: k_2$

(i) If the directed distances AP and PB have the same sign, then their ratio is positive and P is said to divide AB **internally.**

$$\left(\frac{k_1x_2 + k_2x_1}{k_1 + k_2}, \frac{k_1y_2 + k_2y_1}{k_1 + k_2}\right)$$

(ii) If the directed distances AP and PB have opposite signs i.e. P is beyond AB. then their ratio is negative and P is said to divide AB **externally**.

$$\left(\frac{k_1x_2 - k_2x_1}{k_1 - k_2}, \frac{k_1y_2 - k_2y_1}{k_1 - k_2}\right)$$

MID-POINT FORMULA

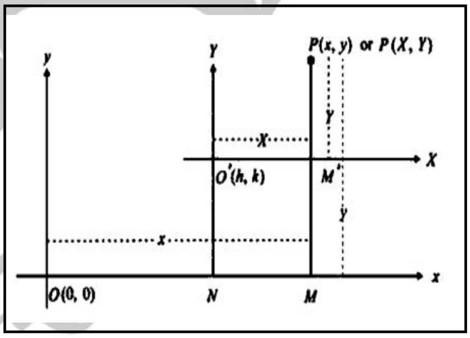
Let $P(x, y)$ be the points that divides AB in the ratio $k_1: k_2$. If $k_1: k_2 = 1: 1$, then P becomes the mid-point of $\overline{AB}$ and coordinates of P are given as

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Note

- If $k_1: k_2 = 1: 1$, then the Ratio Formula becomes the Mid-Point Formula.
- The centroid of a triangle $\Delta\, ABC$ is a point that divides each median in the ratio 2: 1
- Median of a triangle are concurrent.
- Bisectors of angles of a triangle are concurrent.

TRANSLATION OF AXES



Let xy −coordinate system be given and O' (h, k) be any point in the plane. Through O' draw two mutually perpendicular lines $O'X, O'Y$ such that $O'X$ is parallel to Ox . The new axes $O'X$ and $O'Y$ are called **translation** of the Ox- and Oy-axes through the point O'. In translation of axes, origin is shifted to another point in the plane but the axes remain parallel to the old axes.

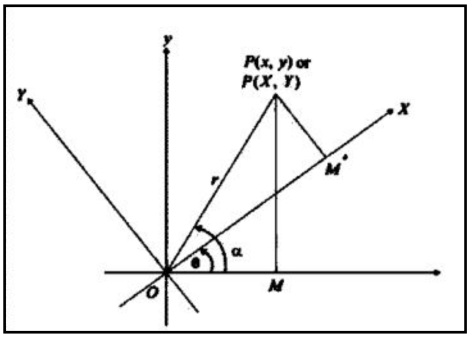
If P be a point with coordinates (x, y) referred to xy −coordinate system and the axes be translated through the point $O'(h, k)$ and $O'X, O'Y$ be the new axes. If P has coordinates (X, Y) referred to the new axes, then

$$X = x - h\,, Y = y - k \quad \text{or} \quad P(X, Y) = (x - h, y - k)$$

If P has coordinates (x, y) referred to the old axes, then

$$x = X + h\,, y = Y + k \quad \text{or} \quad P(x, y) = (X + h, Y + k)$$

ROTATION OF AXES



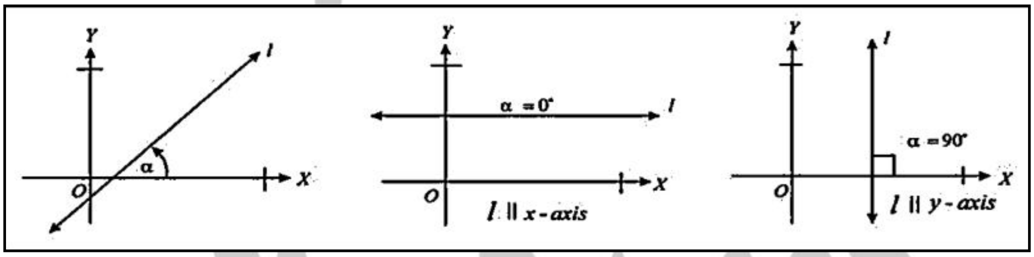
Let xy −coordinate system be given. We rotate Ox and Oy about the origin through an angle $\theta(0 < \theta < 90°)$ so that the new axes are OX and OY. This process is called rotation of the axes. Let a point P have coordinates (x, y) referred to the xy-system and axes be rotated about origin through an angle θ and $O'X, O'Y$ be the new axes.
If P has coordinates (X, Y) referred to the new axes, then

$$X = x\cos\theta + y\sin\theta, \quad Y = y\cos\theta - x\sin\theta$$

$$\text{Or } P(X,Y) = (x\cos\theta + y\sin\theta, y\cos\theta - x\sin\theta)$$

INCLINATION OF A LINE

The angle α measured anti-clock wise from positive x-axis to a non-horizontal straight line l is called ***inclination of the line***.



Note

- If α is the inclination of the line then $0° < \alpha < 180°$
- If a line l is parallel to x-axis, then $\alpha = 0°$
- If a line l is parallel to y-axis, then $\alpha = 90°$

SLOPE OR GRADIENT OF THE LINE

If α is the inclination of the line, $\tan\alpha$ is called its ***slope*** or ***gradient*** of a line. It is generally denoted by m.

Thus $m = \tan\alpha$

Note

- If a line l is horizontal (parallel to x-axis), then its slope $m = 0$
- If a line l is vertical (parallel to y-axis), then its slope $m = undefined = \infty = \frac{1}{0}$
- If α is the inclination of the line l and $0° < \alpha < 90°$, then slope "m" is positive.
- If α is the inclination of the line l and $90° < \alpha < 180°$, then slope "m" is negative.

SLOPE OF A STRAIGHT LINE JOINING TWO POINTS

If a non-vertical line l with inclination α passes through two points $A(x_1, y_1)$ and $B(x_2, y_2)$, then its slope "m" is

$$m = \tan\alpha = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

Note

If l_1 and l_2 be two lines with slope m_1 and m_2 respectively. Then

(i) Parallel iff $m_1 = m_2 \quad \Leftrightarrow \quad l_1 \parallel l_2$

(ii) Perpendicular iff $m_1.m_2 = -1$ or $m_1 = -\frac{1}{m_2} \Leftrightarrow \quad l_1 \perp l_2$

COLLINEAR

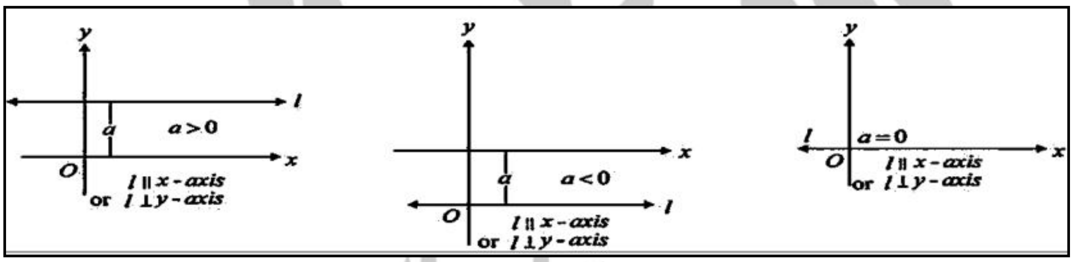
Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be three points.

If Slope of AB $=$ Slope of BC then A, B, C are Collinear Points.

EQUATION OF THE LINE PARALLEL TO X-AXIS

If l is parallel to x-axis remain at a constant distance (say a) from x-axis. Let $P(x, y)$ be any point on the line l. So, all the points on this line satisfy the equation.

$$y = a$$



Note

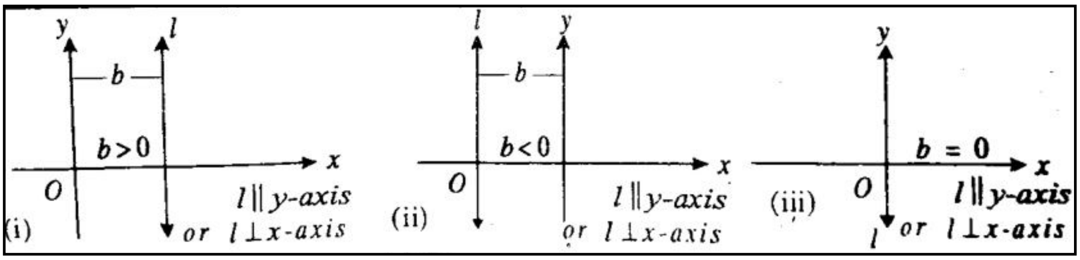
- If $a > 0$, then the line l is above x-axis
- If $a < 0$, then the line l is below x-axis
- If $a = 0$, then the line l becomes the x-axis

Equation of x-axis is $y = 0$

EQUATION OF THE LINE PARALLEL TO Y-AXIS

If l is parallel to y-axis remain at a constant distance (say b) from y-axis. Let $P(x, y)$ be any point on the line l. So, all the points on this line satisfy the equation.

$$x = b$$



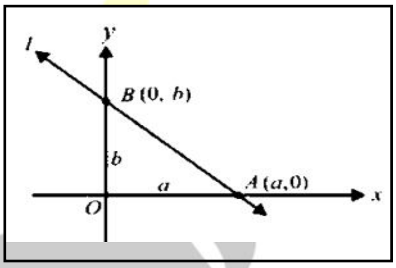
Note

- If $b > 0$, then the line l is above y-axis
- If $b < 0$, then the line l is below y-axis
- If $b = 0$, then the line l becomes the y-axis

Equation of y-axis is $\quad x = 0$

INTERCEPTS

- If a line intersects x-axis at $(a, 0)$, then a is called x-intercept of the line.
- If a line intersects y-axis at $(0, b)$, then b is called y-intercept of the line.



SLOPE-INTERCEPT FORM OF EQUATION OF A STRAIGHT LINE

Equation of a non-vertical straight line l with slope "m" and y-intercept "c" is

$$y = mx + c$$

POINT-SLOPE FORM OF EQUATION OF A STRAIGHT LINE

Equation of a non-vertical straight line l with slope "m" passes through the point $P(x_1, y_1)$ is

$$y - y_1 = m(x - x_1)$$

The equation of the line through the origin $O(0,0)$ having slope "m" is

$$y = mx$$

TWO-POINT FORM OF EQUATION OF A STRAIGHT LINE

Equation of a non-vertical straight line l with slope "m" passes through the point $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \Rightarrow \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\boldsymbol{OR}\ y - y_2 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2) \Rightarrow \frac{y - y_2}{y_2 - y_1} = \frac{x - x_2}{x_2 - x_1}$$

$$\boldsymbol{OR}\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

SYMMETRIC FORM OF EQUATION OF A STRAIGHT LINE

Equation of a non-vertical straight line l passes through the point $P(x_1, y_1)$ with inclination α is

$$\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha} = r\ (say)$$

This is called ***symmetric*** form of equation of the line.

TWO-INTERCEPT FORM OF EQUATION OF A STRAIGHT LINE

Equation of a non-vertical straight line l whose non-zero x-intercept and y-intercepts are "a" and "b" respectively is

$$\frac{x}{a} + \frac{y}{b} = 1$$

NORMAL FORM OF EQUATION OF A STRAIGHT LINE

Equation of a non-vertical straight line l, such that length of the perpendicular from the origin to l is "p" and "α" is the inclination of this perpendicular is

$$x\cos\alpha + y\sin\alpha = p$$

Note

The linear equation $ax + by + c = 0$ in two variables x and y represents a straight line.

A linear equation in two variables x and y is

$$\boxed{ax + by + c = 0}$$

Where a, b and c are constants and a and b are not simultaneously zero.

POSITION OF A POINT WITH RESPECT TO A LINE

Let $P(x_1, y_1)$ be appoint in the plane not lying on the line $l: ax + by + c = 0$ ---- (1)

(a) Above the line (1) if $\quad ax_1 + by_1 + c > 0$

(b) Below the line (1) if $\quad ax_1 + by_1 + c < 0$

CONDITION OF CONCURRENCY OF THREE STRAIGHT LINES

Three non-parallel lines

$l_1: a_1x + b_1y + c_1 = 0$, $\quad l_2: a_2x + b_2y + c_2 = 0$, $\quad l_3: a_3x + b_3y + c_3 = 0$

Are concurrent iff $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & a_3 & c_3 \end{vmatrix} = 0$

Note

- Two non-parallel lines intersect each other at one and only one point.
- An infinite number of lines can pass through a point.
- Altitudes of a triangle are concurrent.
- Right bisectors of a triangle are concurrent.

DISTANCE OF A POINT FROM A LINE

The distance "d" from the points $P(x_1, y_1)$ to the line $l: ax + by + c = 0$ is given by

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

If the point $P(x_1, y_1)$ lies on the line $l: ax + by + c = 0$, then distance "d" is zero.

DISTANCE BETWEEN TWO PARALLEL LINES

The distance between two parallel lines is the distance from any point on one of the lines to the other line.

AREA OF TRIANGLE

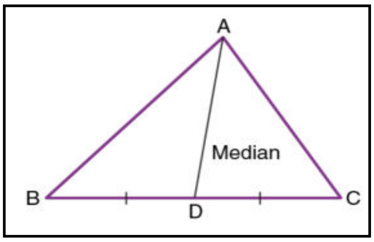
Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be three points. Then area of a triangle ΔABC is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

If the points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear, then $\Delta = 0$.

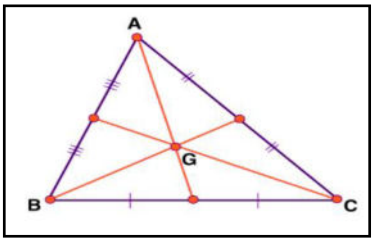
MEDIAN OF A TRIANGLE

The median of a triangle is a line segment from the vertex to the midpoint of the opposite side. Because a triangle has three vertexes, it has also three medians.

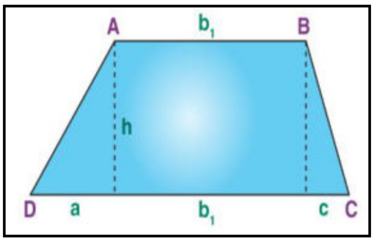


CENTROID OF A TRIANGLE

The point at which three medians of a triangle intersects is called ***centroid*** of a triangle. In figure the point **G** is Centroid.



TRAPEZIUM



A quadrilateral having two parallel and two non-parallel sides is called ***trapezium***.

Area of trapezoidal region= $\frac{1}{2}(sum\ of\ \parallel sides)(distance\ between\ \parallel sides)$

From figure Area of Trapezium $= \frac{1}{2}(AB + DC)(h)$

ANGLE BETWEEN TWO LINES

Let l_1 and l_2 be two non-vertical lines such that they are not perpendicular to each axes. If m_1 and m_2 are the lopes of l_1 and l_2 respectively, then the angle θ from l_1 to l_2 is given by:

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

Corollary 1. $l_1 \parallel l_2$ if and only if $m_1 = m_2$

$$\Rightarrow \theta = 0°$$

Corollary 2. $l_1 \perp l_2$ if and only if $1 + m_1 m_2 = 0$

$$\Rightarrow \theta = \infty$$

HOMOGENEOUS EQUATION

Let $f(x, y) = 0$ be any equation in the variables x any y, equation $f(x, y) = 0$ is called a ***homogeneous equation of degree n*** (a positive integer) if

$$f(kx, ky) = k^n f(x, y) \text{ for some real number } k.$$

Every homogeneous second-degree equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines through the origin. The lines are

- **(i)** Real and distinct, if $h^2 > ab$
- **(ii)** Real and Coincident, if $h^2 = ab$
- **(iii)** Imaginary, if $h^2 < ab$

MEASURE OF ANGLE BETWEEN THE LINES REPRESENTED BY $ax^2 + 2hxy + by^2 = 0$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

- Two lines are parallel, if $\theta = 0°$, so that $\tan\theta = 0$ which implies $\boldsymbol{h^2 - ab = 0}$ which is **condition for the lines to be coincident.**
- Two lines are orthogonal ($\perp$), if $\theta = 90°$, so that $\tan\theta = \infty$ (undefined), which implies $\boldsymbol{a + b = 0}$. Hence the condition for $ax^2 + 2hxy + by^2 = 0$ to represent a pair of orthogonal (perpendicular) lines is that **sum of the coefficients of x^2 and y^2 is 0.**

MCQ's

Choose the correct Option.

1	If (x, y) are the coordinates of a point P, then the first member of the order pair is called						
a	x-coordinate	b	Abscissa	c	Ordinate	d	Both a and b
2	If (x, y) are the coordinates of a point P, then the second member of the order pair is called						
a	x-coordinate	b	Abscissa	c	Ordinate	d	Both a and b
3	If $x > 0, y > 0$ then $P(x, y)$ lie in _____ quadrant.						
a	I	b	II	c	III	d	IV
4	If $x < 0, y > 0$ then $P(x, y)$ lie in _____ quadrant.						
a	I	b	II	c	III	d	IV
5	If $x < 0, y < 0$ then $P(x, y)$ lie in _____ quadrant.						
a	I	b	II	c	III	d	IV
6	If $x > 0, y < 0$ then $P(x, y)$ lie in _____ quadrant.						
a	I	b	II	c	III	d	IV
7	Distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is define as						
a	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	b	$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$	c	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	d	Both a and b
8	If point $C(-5, 3)$ is the centre of the circle and $P(7, -2)$ lies on the circle then radius of the circle is						
a	12	b	13	c	15	d	0
9	Mid-point of the line segment $A(x_1, y_1)$ and $B(x_2, y_2)$ is						
a	$\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$	b	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	c	$\left(\frac{x_1 + y_1}{2}, \frac{x_2 + y_2}{2}\right)$	d	None
10	If $x > 0$ then $P(x, y)$ lie in						
a	Left half plane	b	Right half plane	c	x-axis	d	y-axis
11	If y> 0 then $P(x, y)$ lie in						
a	Left half plane	b	Right half plane	c	x-axis	d	y-axis
12	Distance between the points $(0, 0)$ and $(3, 4)$ is						
a	6	b	5	c	7	d	8
13	Distance between the points $(1,2)$ and $(2,2)$ is						
a	3	b	5	c	1	d	8

14	Distance between the origin and $(4,5)$ is						
a	$\sqrt{31}$	b	$\sqrt{41}$	c	$\sqrt{51}$	d	$\sqrt{64}$
15	Coordinate of the mid-point of the joining the points $(5,7)$ and $(7,5)$ are						
a	$(6,5)$	b	$(5,5)$	c	$(5,6)$	d	$(6,6)$
16	Given the points $A(7,5)$ and $B(-6,1)$. The mid-point of the segment AB is						
a	$(1,6)$	b	$(13,4)$	c	$\left(\frac{1}{2},3\right)$	d	$(-13,-4)$
17	In translation of axis, origin is shifted to another point in the plane but axes remain						
a	same	b	Parallel to old axes	c	Perpendicular to old axes	d	None
18	The angle α measured anti-clock wise from positive x-axis to a non-horizontal straight line l is called						
a	Slope of line	b	Gradient of line	c	Inclination of line	d	None
19	The inclination of the line is always measures						
a	Clock-wise	b	Anti-clockwise	c	Both a & b	d	None
20	If α is the inclination of the line then						
a	$0° < \alpha < 90°$	b	$0° < \alpha < 180°$	c	$0° < \alpha < 270°$	d	None
21	If a line l is parallel to x-axis, then its inclination $\alpha =$____?						
a	$1°$	b	$0°$	c	$90°$	d	$180°$
22	If a line l is parallel to y-axis, then its inclination $\alpha =$____?						
a	$1°$	b	$0°$	c	$90°$	d	$180°$
23	If α is the inclination of the line then slope of line is						
a	$\cot\alpha$	b	$\sin\alpha$	c	$\cos\alpha$	d	$\tan\alpha$
24	Slope of a line is represented by						
a	s	b	m	c	α	d	None
25	If a line l is horizontal (parallel to x-axis), then its slope is						
a	1	b	0	c	90	d	∞
26	If a line l is vertical (parallel to y-axis), then its slope is						
a	1	b	0	c	90	d	∞
27	If α is the inclination of the line l and $0° < \alpha < 90°$, then slope "m" is						
a	positive	b	negative	c	Zero	d	None

28	If α is the inclination of the line l and $90° < \alpha < 180°$, then slope "m" is						
a	positive	b	negative	c	zero	d	None
29	If a non-vertical line l with inclination α passes through two points $A(x_1, y_1)$ and $B(x_2, y_2)$, then its slope "m" is						
a	$\frac{y_2 - y_1}{x_2 - x_1}$	b	$\frac{y_1 - y_2}{x_1 - x_2}$	c	$\frac{y_1 - y_2}{x_2 - x_1}$	d	Both a & b
30	If Slope of AB = Slope of BC then A, B, C are						
a	Linear	b	Collinear	c	Perpendicular	d	None
31	If l_1 and l_2 be two lines with slope m_1 and m_2 respectively. Then $l_1 \parallel l_2$ if						
a	$m_1 m_2 = 0$	b	$m_1 m_2 = 1$	c	$m_1 m_2 = -1$	d	$m_1 = m_2$
32	If l_1 and l_2 be two lines with slope m_1 and m_2 respectively. Then $l_1 \perp l_2$ if						
a	$m_1 m_2 = 0$	b	$m_1 m_2 = 1$	c	$m_1 m_2 = -1$	d	$m_1 = m_2$
33	If a point $P(x, y)$ lie on line l is parallel to x-axis remain at a constant distance "a" from x-axis then its equation is						
a	$y = a$	b	$x = a$	c	$y = b$	d	$x = b$
34	If a point $P(x, y)$ lie on line l is parallel to y-axis remain at a constant distance "b" from y-axis then its equation is						
a	$y = a$	b	$x = a$	c	$y = b$	d	$x = b$
35	Equation of x-axis is						
a	$x = 0$	b	$y = 0$	c	$x = a$	d	$y = b$
36	Equation of y-axis is						
a	$x = 0$	b	$y = 0$	c	$x = a$	d	$y = b$
37	If a line intersects x-axis at $(a, 0)$, then a is called ____ of the line.						
a	x-intercept	b	y-intercept	c	Origin	d	None
38	If a line intersects y-axis at $(0, b)$, then b is called ____ of the line.						
a	x-intercept	b	y-intercept	c	Origin	d	None
39	$y = mx + c$ is equation of the line in _____ form.						
a	Normal	b	Symmetric	c	Slope-point	d	Slope-intercept
40	$y = mx$ is equation of the line passes through						
a	x-axis	b	y-axis	c	Origin	d	None
41	y-intercept of the line $y = 2x + 5$						
a	5	b	2	c	0	d	-5

42	$y-y_1=m(x-x_1)$ is called equation of line in ____ form.						
a	Normal	b	Symmetric	c	Point-slope	d	Slope-intercept
43	The point of concurrency of the medians of a triangle is called ___.						
a	In-centre	b	Centroid	c	Circumcircle	d	Orthocenter
44	Medians of a triangle are ____.						
a	Collinear	b	Concurrent	c	Perpendicular	d	Parallel
45	Bisectors of angles of a triangle are ____						
a	Collinear	b	Concurrent	c	Perpendicular	d	Parallel
46	Equation of a non-vertical straight line l passes through the point $P(x_1,y_1)$ with inclination α is ___ **OR** Symmetric form of equation of straight line is ____						
a	$\frac{x+x_1}{\cos\alpha}=\frac{y+y_1}{\sin\alpha}$	b	$\frac{x-x_1}{\cos\alpha}=\frac{y-y_1}{\sin\alpha}$	c	$y=mx+c$	d	None
47	Equation of straight line l if its x-intercept is "a"and y-intercepts is "b" is						
a	$\frac{x-x_1}{\cos\alpha}=\frac{y-y_1}{\sin\alpha}$	b	$y=mx+c$	c	$\frac{x}{a}+\frac{y}{b}=1$	d	None
48	Normal form of equation of straight line is						
a	$\frac{x-x_1}{\cos\alpha}=\frac{y-y_1}{\sin\alpha}$	b	$y-y_1=m(x-x_1)$	c	$x\cos\alpha+y\sin\alpha=p$	d	None
49	The linear equation $ax+by+c=0$ in two variables x and y represents						
a	Circle	b	Parabola	c	Straight line	d	Ellipse
50	Slope of the line $5x-12y+39=0$ is equal to						
a	$\frac{5}{12}$	b	$-\frac{5}{12}$	c	$\frac{5}{39}$	d	$-\frac{5}{39}$
51	The point $(-2,4)$ lie on the line $4x+5y-3=0$						
a	Above	b	Below	c	On the lie	d	None
52	Two non-parallel lines intersect each other at						
a	One point	b	Two points	c	Three points	d	Four points
53	If two lines are parallel then their intersecting point						
a	Exists	b	Not exist	c	Is origin	d	None
54	Altitudes of a triangle are						
a	Collinear	b	Concurrent	c	Perpendicular	d	Parallel
55	Right bisectors of a triangle are						
a	Collinear	b	Concurrent	c	Perpendicular	d	Parallel
56	The distance "d" from the points $P(x_1,y_1)$ to the line $l: ax+by+c=0$ is						

a	$\frac{\lvert ax+by-c\rvert}{\sqrt{a^2+b^2}}$	b	$\frac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$	c	$\frac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2-b^2}}$	d	$\frac{\lvert ax_1-by_1-c\rvert}{\sqrt{a^2+b^2}}$
57	If the point $P(x_1,y_1)$ lies on the line $l: ax+by+c=0$, then distance "d" is						
a	$\frac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$	b	$\frac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2-b^2}}$	c	Zero	d	1
58	If the points $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are collinear, then its area is						
a	1	b	0	c	2	d	3
59	If area of triangle ABC is zero then points are						
a	Linear	b	Concurrent	c	Collinear	d	None
60	Equation of horizontal line through $(7,-9)$						
a	$x=7$	b	$y=-9$	c	$y=7$	d	$x=-9$
61	Equation of vertical line through $(-5,3)$						
a	$x=-5$	b	$y=3$	c	$y=-5$	d	$x=3$
62	Equation of line bisecting the first and third quadrant is						
a	$x=y$	b	$x=-y$	c	$x-y=1$	d	None
63	The point of the intersection of the altitudes of a triangle is called						
a	Centroid	b	In-centre	c	Ortho-centre	d	Circum-centre
64	Every homogeneous equation $ax^2+2hxy+by^2=0$ represents two real lines passes through the origin (0,0) if						
a	$h^2-ab>0$	b	$h^2-ab<0$	c	$h^2-ab=0$	d	None
65	Every homogeneous equation $ax^2+2hxy+by^2=0$ represents two imaginary lines passes through the origin (0,0) if						
a	$h^2-ab>0$	b	$h^2-ab<0$	c	$h^2-ab=0$	d	None
66	Every homogeneous equation $ax^2+2hxy+by^2=0$ represents two real and coincident lines passes through the origin (0,0) if						
a	$h^2-ab>0$	b	$h^2-ab<0$	c	$h^2-ab=0$	d	None
67	Slope of line perpendicular to $2x-3y+1=0$ is						
a	$\frac{2}{3}$	b	$-\frac{2}{3}$	c	$\frac{3}{2}$	d	$-\frac{3}{2}$
68	Slope of the line $3x+2y-8=0$ is						
a	$\frac{2}{3}$	b	$-\frac{2}{3}$	c	$\frac{3}{2}$	d	$-\frac{3}{2}$
69	The angle between two lines represented by $ax^2+2hxy+by^2=0$ is $\theta=$ _						

a	$\tan^{-1}\left(\frac{2\sqrt{h^2-ab}}{a+b}\right)$	b	$\tan^{-1}\left(\frac{2\sqrt{h^2+ab}}{a+b}\right)$	c	$\tan^{-1}\left(\frac{2\sqrt{h^2-ab}}{a-b}\right)$	d	$\tan^{-1}\left(\frac{\sqrt{h^2-ab}}{a+b}\right)$
70	Slope of the line passes through $(2,6)$ and $(8,-6)$ is						
a	5	b	2	c	4	d	3
71	Slope of line with inclination $60°$ is						
a	0	b	1	c	$1/\sqrt{3}$	d	$\sqrt{3}$
72	P is mid-point of AB then P divides AB in ratio						
a	$1:1$	b	$2:2$	c	$1:2$	d	Both a & b
73	Every homogeneous equation $ax^2+2hxy+by^2=0$ represents two straight lines						
a	Through the origin	b	Not through the origin	c	Parallel to each other	d	Perpendicular to each other
74	Slope of line with inclination $30°$ is						
a	0	b	1	c	$1/\sqrt{3}$	d	$\sqrt{3}$
75	The point of concurrency of median of a triangle is called						
a	Centroid	b	In-centre	c	Ortho-centre	d	Circum-centre
76	Equation of straight line passes through the point $(-8,5)$ having slope undefined is						
a	$x-8=0$	b	$x+8=0$	c	$y+5=0$	d	None
77	Equation of the line having x-intercept: -3 and y-intercept: 4 is						
a	$x-y+2=0$	b	$4x-y+36=0$	c	$2x-4y=0$	d	$x-y=0$
78	The length of the perpendicular from $(0,0)$ to $15y-8x+3=0$						
a	$\frac{3}{17}$	b	$\frac{17}{3}$	c	3	d	0
79	The point of intersection of the line $5x+7y=35$ and $3x-7y=21$ is						
a	$(0,7)$	b	$(7,0)$	c	$(2,3)$	d	$(1,2)$
80	Two lines $5x+7y=35$ and $3x-7y=21$ are						
a	Parallel	b	Perpendicular	c	Neither parallel nor perpendicular	d	None
81	The perpendicular distance from the point $P(-4,7)$ to the line $6x-4y+9=0$ is						
a	$49/\sqrt{52}$	b	$\sqrt{52}$	c	$1/\sqrt{52}$	d	0
82	If two lines l_1 and l_2 having slopes $-7/3$ and $5/2$ respectively. Then the angle l_1 to l_2 is						

a	-1	b	1	c	3	d	0
83	If a line passes through the points $(4,6)$ and $(4,8)$ then its inclination is						
a	$60°$	b	$30°$	c	$45°$	d	$90°$
84	$P\left(\frac{3}{7},-\frac{5}{7}\right)$ lie in _____ quadrant.						
a	I	b	II	c	III	d	IV
85	The distance of point $(-2,3)$ from y-axis is:						
a	2	b	-2	c	-3	d	3
86	The points $A(-5,-2)$. $B(5,-4)$ are ends of a diameter of a circle. The centre will be:						
a	$(0,3)$	b	$(0,-3)$	c	$(5,2)$	d	$(-5,4)$
87	If distance between two points $(3,1)$ and $(k,2)$ is "1", then value of $k=$ __						
a	-3	b	3	c	1	d	2
88	The centroid of a triangle divides each median in the ratio:						
a	$2:1$	b	$1:2$	c	$3:1$	d	$1:3$
89	The distance between the points $(1,2)$ and $(2,1)$ is:						
a	$\sqrt{3}$	b	$\sqrt{5}$	c	$\sqrt{2}$	d	$\sqrt{7}$
90	The distance between the points $(0,0)$ and $(1,2)$ is:						
a	0	b	2	c	$\sqrt{3}$	d	$\sqrt{5}$
91	The perpendicular distance of the line $3x+4y+10=0$ from $(0,0)$						
a	0	b	1	c	2	d	10
92	The slope of the line through the points $(-2,4)$ and $(5,11)$ is:						
a	-1	b	0	c	1	d	2
93	y-intercept of line $2x-y-4=0$ is:						
a	2	b	-2	c	4	d	-4
94	Equation of the line passing through $(5,-7)$ having slope undefined is:						
a	$y=-7$	b	$x=5$	c	$x=-5$	d	$y=7$
95	Equation of line bisecting II and IV quadrant.						
a	$y=x$	b	$y=-x$	c	$y=\frac{1}{x}$	d	$x+y=1$
96	The distance between the points $(3,1)$ and $(-2,-4)$ is:						
a	$3\sqrt{2}$	b	$5\sqrt{2}$	c	$4\sqrt{2}$	d	5
97	If $(3,5)$ is the mid-point of $(5,y)$ and $(x,7)$ then $x=?$, $y=?$						

a	$x=1, y=1$	b	$x=-3, y=1$	c	$x=1, y=3$	d	$x=-5, y=2$
98	Equation of line with slope -2, y-intercept 3 is:						
a	$x-2y=3$	b	$3x+2y=2$	c	$2x+y=3$	d	$x+3y=2$
99	Slope of the line $5x+7y=35$ is:						
a	$\frac{5}{7}$	b	$\frac{7}{5}$	c	35	d	$-\frac{5}{7}$
100	The distance of point $(3,7)$ from x-axis is:						
a	7	b	3	c	-3	d	-7
101	Distance between the points $(2,3)$ and $(3,2)$ is:						
a	$\sqrt{2}$	b	2	c	1	d	$2\sqrt{2}$
102	Equation of line passing through $(-2,5)$ having slope 0 is:						
a	$c=5$	b	$y=5$	c	$x=-2$	d	$x=2$
103	If $(4,-2),(-2,4),(4,10)$ are vertices of triangle than its centroid is:						
a	$(-2,4)$	b	$(2,4)$	c	$(2,-4)$	d	$(-2,-4)$
104	Centroid of triangle with vertices $A(2,1), B(-1,3)$ and $C(-1,-4)$ is:						
a	$(3,1)$	b	$(0,0)$	c	$(2,2)$	d	$(-2,5)$
105	Equation of line having slope -5, y-intercept -7 is:						
a	$5x+y+7=0$	b	$5x-y+7=0$	c	$5x+y-7=0$	d	$7x+y+5=0$

MCQ's Answers Key

1	2	3	4	5	6	7	8	9	10
d	c	a	b	c	d	d	b	b	b
11	12	13	14	15	16	17	18	19	20
d	b	c	b	d	c	b	c	b	b
21	22	23	24	25	26	27	28	29	30
b	c	d	b	b	d	a	d	d	b
31	32	33	34	35	36	37	38	39	40
d	c	a	d	b	a	a	b	d	c
41	42	43	44	45	46	47	48	49	50
a	c	b	b	b	b	c	c	c	a
51	52	53	54	55	56	57	58	59	60
a	a	b	b	b	b	c	b	c	b
61	62	63	64	65	66	67	68	69	70
a	a	c	a	b	c	d	d	a	b
71	72	73	74	75	76	77	78	79	80
d	d	a	c	a	b	b	a	b	c
81	82	83	84	85	86	87	88	89	90
a	a	d	d	b	b	b	a	c	d
91	92	93	94	95	96	97	98	99	100
c	c	d	b	b	b	c	c	d	a
101	102	103	104	105					
a	b	b	b	a					

IMPORTANT SHORT QUESTIONS

1. Show that the points $A\ (-1, 2), B\ (7, 5)$ and $C\ (2, -6)$ are vertices of a right triangle.
2. The point $C\ (-5, 3)$ is the centre of a circle and $P\ (7, -2)$ lies on the circle. What is the radius of the circle?
3. Find the coordinates of the point that divides the join of $A\ (-6, 3)$ and $B\ (5, -2)$ in the ratio 2 : 3. (i) internally (ii) externally
4. Find the distance and midpoint of $A\ (3, 1), B\ (-2, -4)$.
5. Find the distance and midpoint of $A\ (-8, 3), B\ (2, -1)$.
6. Find the distance and midpoint of $A\left(-\sqrt{5}, -\frac{1}{3}\right), B\left(-3\sqrt{5}, 5\right)$.
7. Is $(\sqrt{176}, 7)$ is at a distance of 15 units from origin?
8. Show that the points $A\ (0, 2), B(\sqrt{3}, -1)$ and $C\ (0, -2)$ are vertices of a right triangle.
9. Show that the points $A\ (3, 1), B\ (-2, -3)$ and $C\ (2, 2)$ are vertices of an isosceles triangle.
10. Find h such that $A\ (-1, h), B\ (3, 2)$ and $C\ (7, 3)$ are collinear.
11. The points $A\ (-5, -2)$ and $B\ (5, -4)$ are ends of a diameter of a circle. Find the Centre and radius of the circle.
12. Find the points trisecting the join of $A\ (-1, 4)$ and $B\ (6, 2)$.
13. Find the point three-fifth of the way along the line segment from $A\ (-5, 8)$ to $B\ (5, 3)$.
14. Find the point P on the join of A (1, 4) and B (5, 6) that is twice as far from A as B is from A and lies on the same side of A as B does.
15. The coordinates of a point P are $(-6, 9)$. The axes are translated through the point $O'\ (-3, 2)$. Find the coordinates of P referred to the new axes.
16. The two points $P(-2,6), O'(-3, 2)$ are given in xy -coordinate system. Find the XY −coordinates of P.
17. The two points $P(-6, -8), O'\ (-4, -6)$ are given in xy-coordinate system. Find the XY −coordinates of P.
18. The coordinates of two points $P\ (-5, -3), O'\ (-2, -6)$ are given in the XY-coordinate system. Find the coordinates of P in xy −coordinate system.
19. Find the XY −coordinates of the point P with given xy −coordinates. $P\ (5, 3), \theta = 45°$.
20. Show that the points $A(-3, 6), B(3, 2)$ and $C(6, 0)$ are collinear.
21. Show that the triangle with vertices $A\ (1, 1), B\ (4, 5)$ and $C\ (12, -1)$ is a right triangle.
22. Find an equation of the straight line if its slope is 2 and y-intercept is 5.
23. Find an equation of the straight line if it is perpendicular to a line with slope -6 and its y-intercept is $\frac{4}{3}$.
24. Find an equation of line through the points $(-2, 1)$ and $(6, -4)$.
25. Write Intercept Form of Equation of a Straight Line.
26. Write down an equation of the line which cuts the x-axis at $(2, 0)$ and y-axis at $(0, -4)$.
27. Find equation of line passing through (2,3), having slope -1.
28. The length of perpendicular from the origin to a line is 5 units and the inclination of this perpendicular is 120°. Find the slope and y-intercept of the line.
29. Convert the equation $5x - 12y + 39 = 0$ into
    (i) Slope intercept form (ii) Two-intercept form (iii) Normal form
30. Check whether the following lines are concurrent or not.
    $$3x - 4y - 3 = 0, 5x + 12y + 1 = 0, 32x + 4y - 17 = 0$$
31. Find the distance between the parallel lines $2x - 5y + 13 = 0$ & $2x - 5y + 6 = 0$
32. Find the area of the triangular region with vertices $(a, b + c), (a, b - c)$ and $(-a, c)$.

33. Find the area of the triangle with vertices $A\ (1,4), B\ (2,-3)$ and $C\ (3,-10)$.
34. Define Trapezium.
35. Define Centroid of a triangle.
36. Define Medians of a triangle.
37. Find the slope and inclination of the line joining the points $(-2,4),(5,11)$.
38. Find the slope and inclination of the line joining the points $(3,-2)$, $(2,7)$.
39. Find the slope and inclination of the line joining the points $(4,6),(4,8)$.
40. By means of slopes, show that the points $(-1,-3),(1,5),(2,9)$ lie on the same line.
41. By means of slopes, show that the points $(4\ ,-5),(7,5),(10,15)$ lie on the same line.
42. By means of slopes, show that the points $(-4,6),(3,8),(10,10)$ lie on the same line.
43. Find k so that the line joining $A\ (7,3), B\ (k,-6)$ and the line joining $C\ (-4,5), D\ (-6,4)$ are (i) parallel (ii) perpendicular.
44. Find an equation of the horizontal line through $(7,-9)$.
45. Find an equation of the vertical line through $(-5,3)$.
46. Find an equation of the line bisecting the first and third quadrants.
47. Find an equation of the line bisecting the second and fourth quadrants.
48. Find an equation of the line through $A\ (-6,5)$ having slope 7.
49. Find an equation of the line through $(8,-3)$ having slope 0.
50. Find an equation of the line through $(-8\ ,5)$ having slope undefined.
51. Find an equation of the line through $(-5,-3)$ and $(9,-1)$.
52. Find an equation of the line having y-intercept: -7 and slope: -5.
53. Find an equation of the line having x-intercept: -3 and y-intercept: 4.
54. Find an equation of the line having x-intercept: -9 and slope: -4.
55. Find an equation of the line through $(-4,-6)$ and perpendicular to line having slope $\frac{-3}{2}$.
56. Find an equation of the line through $(11,-5)$ and parallel to a line with slope -24.
57. Convert $2x-4y+11=0$ into (i) Slope intercept form (ii) two intercept form
58. Convert $4x+7y-2=0$ into (i) Slope intercept form (ii) two intercept form
59. Convert $15y-8x+13=0$ into (i) two intercept form (ii) normal form
60. Show that the lines $2x+y-3=0\ ,4x+2y+5=0$ are parallel.
61. Check whether the two lines $12x+35y-7=0\ ,105x-36y+11=0$ are parallel or perpendicular.
62. Find the distance between two parallel lines $3x-4y+3=0\ ,3x-4y+7=0$
63. Find an equation of the line through $(-4,7)$ and parallel to the line $2x-7y+4=0$.
64. Check whether the point $(5,8)$ lies above or below the line $2x-3y+6=0$.
65. Check whether the point $(-7,6)$ lies above or below the line $4x+3y-9=0$.
66. Find the distance from the point $P(6,-1)$ to the line $6x-4y+9=0$.
67. Find the area of the triangular region whose vertices are $A\ (5,3), B\ (-2,2), C\ (4,2)$.
68. Find the area of the triangle with vertices $A(2,3), B(-1,1)$ and $C(4,-5)$.
69. Find the angle from the line with slope $\frac{-7}{3}$ to the line with slope $\frac{5}{2}$.
70. Check whether the lines are concurrent or not?
$3x+4y-7=0\ ,2x-5y+8=0\ ,x+y-3=0$
71. Find the point of intersection of the lines; $x-2y+1=0\ ,2x-y+2=0$
72. Find the point of intersection of the lines; $3x+y+12=0\ ,x+2y-1=0$
73. Find the point of intersection of the lines; $x+4y-12=0\ ,x-3y+3=0$
74. Determine the value of p such that the lines $2x-3y-1=0,\ 3x-y-5=0$ and $3x+py+8=0$ meet at a point (concurrent).
75. Show that the lines $4x-3y-8=0\ ,3x-4y-6=0\ ,x-y-2=0$ are concurrent.
76. Define Homogeneous Equation.

77. Find an equation of each of the lines represented by $20x^2 + 17xy - 24y^2 = 0$.
78. Find measure of the angle between the lines represented by $x^2 - xy - 6y^2 = 0$
79. Find equation of two lines represented by $10x^2 - 23xy - 5y^2 = 0$.
80. Find equation of two lines represented by $3x^2 + 7xy + 2y^2 = 0$.
81. Find equation of two lines represented by $9x^2 + 24xy + 16y^2 = 0$.
82. Find equation of two lines represented by $2x^2 + 3xy - 5y^2 = 0$.
83. Find equation of two lines represented by $6x^2 - 19xy + 15y^2 = 0$.
84. Find measure of angle between the two lines represented by $10x^2 - 23xy - 5y^2 = 0$.
85. Find measure of angle between the two lines represented by $2x^2 + 3xy - 5y^2 = 0$.
86. Find measure of angle between the two lines represented by $3x^2 + 7xy + 2y^2 = 0$.

IMPORTANT LONG QUESTIONS

1. Find h such that the points $A(\sqrt{3},-1), B\ (0,2)$ and $C\ (h,-2)$ are vertices of a right triangle with right angle at the vertex A.
2. Find h such that the points $A\ (h\ ,1), B\ (2,7)$ and $C\ (-6,-7)$ are vertices of a right triangle with right angle at the vertex A.
3. Find the point which is equidistant from the points $A\ (5,3), B\ (-2,2)$ and $C\ (4,2)$. What is the radius of the circumcircle of the ΔABC?
4. The points $(4,-2), (-2,4)$ and $(5,5)$ are the vertices of a triangle. Find in-Centre of the triangle.
5. Prove that the linear equation $ax + by + c = 0$ in two variables x and y represents a straight line.
6. The three points $A\ (7,-1), B\ (-2,2)$ and $C(1,4)$ are consecutive vertices of a parallelogram. Find the fourth vertex.
7. Find an equation of the perpendicular bisector of the segment joining the points A (3,5) and $B\ (9,8)$.
8. Find equations of the sides, altitudes and medians of the triangle whose vertices are $A(-3,2), B(5,4)$ and $C(3,-8)$.
9. The points $A(-1,2), B(6,3)$ and $C(2,-4)$ are vertices of a triangle. Show that the line joining the midpoint D of AB and the midpoint E of AC is parallel to BC and $DE = \frac{1}{2}BC$.
10. Find an equation of the line through $(5,-8)$ and perpendicular to the join of $A\ (-15,-8), B\ (10,7)$.
11. Find equations of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x- and y-intercepts of each is 3.
12. One vertex of a parallelogram is $(1,4)$, the diagonals intersect at $(2,1)$ and the sides have slopes 1 and $\frac{-1}{7}$. Find the other three vertices.
13. Find the angles of the triangle whose vertices are $A\ (-5,4), B\ (-2,-1), C\ (7,-5)$.
14. Find an equation of the line through the point $(2,-9)$ and the intersection of the lines
$$2x + 5y - 8 = 0 \text{ and } 3x - 4y - 6 = 0$$
15. Find an equation of the line through the intersection of the lines $x - y - 4 = 0$ and $7x + y + 20 = 0$ and parallel to the line $6x + y - 14 = 0$
16. Find an equation of the line through the intersection of the lines $x + 2y + 3 = 0$, $3x + 4y + 7 = 0$ and making equal intercepts on the axes.
17. Find an equation of the line through the intersection of $16x - 10y - 33 = 0$; $12x + 14y + 29 = 0$ and the intersection of $x - y + 4 = 0\ ; x - 7y + 2 = 0$
18. Find the condition that the lines $y = m_1x + c_1; y = m_2x + c_2;\ y = m_3x + c_3$ are concurrent.
19. Show that the lines $4x - 3y - 8 = 0\ ,\ 3x - 4y - 6 = 0\ ,\ x - y - 2 = 0$ are concurrent and the third line bisects the angle formed by the first two lines.
20. The vertices of a triangle are $A\ (-2,3), B\ (-4,1)$ and $C\ (3,5)$. Find coordinates of the (i) centroid (ii) orthocenter (iii) circumcenter of the triangle
Are these three points collinear?
21. Find the coordinates of the vertices of the triangle formed by the lines
$$x - 2y - 6 = 0, \quad 3x - y + 3 = 0, \quad 2x + y - 4 = 0$$
Also find measures of the angles of the triangle.
22. Find the interior angles of the triangle whose vertices are $A(-2,11), B(-6,-3), C(4,-9)$.

23. Find the interior angles of the triangle whose vertices are $A(6,1), B(2,7), C(-6,-7)$.
24. Find the interior angles of the triangle whose vertices are $A(2,-5), B(-4,-3), C(-1,5)$.
25. Find the area of the region bounded by the triangle whose sides are

$$7x - y - 10 = 0, 10x + y - 14 = 0, 3x + 2y + 3 = 0.$$

26. Find a joint equation of the straight lines through the origin and perpendicular to the lines represented by $x^2 + xy - 6y^2 = 0$
27. Find equation of two lines represented by $x^2 + 2xy\,sec\alpha + y^2 = 0$ and also find measure of angle between them.
28. Find a joint equation of the straight lines through the origin and perpendicular to the lines represented by $x^2 - 2xytan\alpha - y^2 = 0$
29. Find a joint equation of the straight lines through the origin and perpendicular to the lines represented by $ax^2 + 2hxy + by^2 = 0$
30. Find the area of the region bounded by:

$$10x^2 - xy - 21y^2 = 0 \text{ and } x + y + 1 = 0$$

UNIT 5

Linear Inequalities and Linear Programming

DEFINITIONS + SUMMARY

INEQUALITY

Inequalities are expressed by the following four symbols;

(i) $>$ (greater than)
(ii) $<$ (less than)
(iii) $\geq$ (greater than or equal to)
(iv) $\leq$ (less than or equal to)

Example: - $(i)\ ax < b\ (ii)\ 2x - y > 0\ (iii)\ 5x - y \geq 0\ (iv)\ x + 2y \leq 3$

The following operations will not affect the order (or sense) of inequality while changing it to simpler equivalent form:

(i) Adding or subtracting a constant to each side of it.
(ii) Multiplying or dividing each side of it by a positive constant.

> ***Note***
> The order (or sense) of an inequality is changed by multiplying or dividing its each side by a negative constant.

LINEAR INEQUALITY

A Linear Inequality in two variables x any y can be one of the following forms:

$$ax + by < c\ ;\ ax + by > c\ ;\ \ ax + by \geq c\ ;\ \ ax + by \leq c$$

Where a, b and c are constants and a, b are not both zero.

SOLUTION OF LINEAR INEQUALITY

A solution of a linear inequality in x and y is an ordered pair of numbers which satisfies the inequality.

Example: - The ordered pair (1, 1) is a solution of the inequality $x + 2y < 6$

Because $1 + 2(1) = 3 < 6$ which is true.

> ***Note***
> (i) There are infinitely many ordered pairs that satisfy the inequality
> (ii) Graph of linear inequality is the half plane.
> (iii) The linear equation $ax + by = c$ is called ***"associated or corresponding equation"*** of linear inequalities $ax + by < c\ ;\ ax + by > c\ ;\ \ ax + by \geq c\ ;\ \ ax + by \leq c$

CORNER POINT OR VERTEX

A point of a solution region where two of its boundary lines intersect, is called a ***corner point*** or ***vertex*** of the solution region.

PROBLEM CONSTRAINT

Tackling a certain problem from everyday life each linear inequality concerning the problem is named as ***problem constraint***.

PROBLEM CONSTRAINTS

The system of linear inequalities involved in the problem concerned are called ***problem constraints.***

NON-NEGATIVE CONSTRAINTS

The variables used in the system of linear inequalities relating to the problems of everyday life are non-negative and are called ***non-negative constraints.***

DECISION VARIABLES

The non-negative constraints which are used to taking a decision are called decision variables.

FEASIBLE REGION

A region which is restricted to the first quadrant is referred to as a ***feasible region*** for the set of given constraints.

FEASIBLE SOLUTION

Each point of the feasible region is called a ***feasible solution*** of the system of linear inequalities (or for the set of a given constraints).

FEASIBLE SOLUTION SET

A set consisting of all the feasible solutions of the system of linear inequalities are called a ***feasible solution set***.

CONVEX REGION

If the line segment obtained by joining any two points of a region lies entirely within the region, then the region is called ***Convex.***

OBJECTIVE FUNCTION

A function which is to be maximized or minimized is called an ***objective function***.

OPTIMAL SOLUTION

The feasible solution which maximizes or minimizes the objective function is called the ***optimal solution***.

THEOREM OF LINEAR PROGRAMMING

The theorem of linear programming states that the maximum and minimum values of the objective function occur at corner points of the feasible region.

PROCEDURE FOR DETERMINING OPTIMAL SOLUTION

(i) Graph the solution set of linear inequality constraints to determine feasible region.
(ii) Find the corner points of the feasible region.
(iii) Evaluate the objective function at each corner point to find the optimal solution.

MCQ's

Choose the correct Option.

1	An expression involving any one of the symbols, $<, >, \leq$ and $\geq$ is called						
a	Equation	b	Inequality	c	Identity	d	Linear equation
2	$ax + b < c$ is linear inequality in ______ variables.						
a	one	b	two	c	three	d	four
3	The solution set of the inequality $ax + by < c$ is the						
a	Half plane	b	Whole plane	c	Quadrant of a plane	d	Circle
4	Solution of the inequality is						
a	Finite	b	Infinite	c	Three	d	Four
5	The graph of inequality $ax + by < c$ is						
a	Circle	b	Half plane	c	Straight line	d	Both b & c
6	Graph of the inequality $x + 2y < 6$ lies ____						
a	Opposite to origin	b	Towards origin	c	In 1st quadrant	d	In 2nd quadrant
7	$2x - 8 < 0$ is						
a	Equation	b	Identity	c	Inequality	d	Curve
8	The graph of $2x \geq 3$ lies in						
a	Upper half plane	b	Lower half plane	c	Left half plane	d	Right half plane
9	The graph of the equation $2y = -3$ is						
a	Horizontal line	b	Vertical line	c	Inclined line	d	Line through origin
10	For the inequalities $2x + y \leq 10$ and $x + 4y \leq 12$, the corner point is						
a	(5,10)	b	(12,3)	c	(4,2)	d	(10,12)
11	$x = 5$ is the solution of						
a	$2x - 3 > 0$	b	$2x + 3 < 0$	c	$x + 4 < 0$	d	$x < 0$
12	$x = 4$ is the solution of						
a	$-2x + 3 > 0$	b	$x + 3 > 0$	c	$x - 3 < 0$	d	$x + 3 < 0$
13	$x = 2$ is the solution of						
a	$2x - 1 \leq 0$	b	$2x - 1 \geq 0$	c	$x - 1 \leq 0$	d	$x + 1 \leq 0$

14	$x = -1$ is the solution of the inequality						
a	$2x+3 \le 0$	b	$2x+3>0$	c	$x-2>0$	d	$2x+1>0$
15	$x=5$ is not the solution of						
a	$x+4>0$	b	$2x+3<0$	c	$x-4>0$	d	$x+7>0$
16	$x=0$ is not in the solution of inequality						
a	$2x+3>0$	b	$2x+3<0$	c	$x+4>0$	d	$x+6>0$
17	$(0,0)$ is the solution of inequality						
a	$x+y>2$	b	$2x-y>4$	c	$x-y<1$	d	$2x+y>10$
18	$(1,2)$ is the solution of inequality						
a	$x-y<4$	b	$x-y>4$	c	$x-y=4$	d	$x-y=0$
19	$(0,0)$ is the solution of inequality						
a	$7x+2y>3$	b	$x-3y>0$	c	$x+2y<6$	d	$x-3y<0$
20	$(0,1)$ is the solution of inequality						
a	$x-3y>0$	b	$x-5y>0$	c	$x+y>0$	d	$x<0$
21	$(1,0)$ is solution of inequality						
a	$7x+2y<8$	b	$3x+y>6$	c	$x-y<0$	d	$-3x+y>0$
22	$(1,-3)$ is the solution of						
a	$x+y>0$	b	$x+y<0$	c	$x+y=0$	d	$x-y=0$
23	$(1,1)$ is the solution of inequality						
a	$x+y<1$	b	$2x+y<1$	c	$2x-y<1$	d	$x-y<1$
24	Point $(1,2)$ satisfy the inequality						
a	$2x+y>5$	b	$2x+y<3$	c	$2x+y<5$	d	$2x+y\ge 5$
25	$(0,2)$ is the solution of inequality						
a	$3x+5y>7$	b	$3x+5y<7$	c	$x>0$	d	$x<0$
26	The point $(1,3)$ lies in the solution region of the inequality:						
a	$x+y<0$	b	$x+y<2$	c	$x+y<2$	d	$x-y<0$
27	$2x+y<6$ is satisfied by which point?						
a	$(3,1)$	b	$(1,3)$	c	$(0,7)$	d	$(4,0)$
28	Solution of the inequality $2x+y<5$ is						
a	$(2,1)$	b	$(1,2)$	c	$(2,3)$	d	$(5,0)$
29	$(1,0)$ is not the solution of inequality						

a	$9x+2y<8$	b	$-x+3y<0$	c	$3x+5y<6$	d	$3x+5y<4$
30	Which one satisfies the inequality $x+2y<6$?						
a	(4,1)	b	(1,3)	c	(1,4)	d	(3,1)
31	$2x+3y<5$ is satisfied by						
a	(1,1)	b	(1,2)	c	(2,3)	d	$(-1,1)$
32	(1,0) is solution of inequality						
a	$9x+2y<8$	b	$-x+3y<0$	c	$3x+5y<0$	d	$3x+y<0$
33	Point satisfies $x-y<2$						
a	(3,1)	b	$(1,-1)$	c	$(0,-2)$	d	$(-1,1)$
34	Which of the following ordered pairs does not satisfy $4x-3y<2$						
a	(3,0)	b	(1,1)	c	$(-2,1)$	d	(0,0)
35	Which one is not a solution of in-equality $2x+3y > 0$:						
a	$(-1,-2)$	b	(1,2)	c	(2,3)	d	(0,1)
36	Solution set of inequality $2x - 3 \geq 0$ equals:						
a	$\left[\frac{3}{2},\infty\right]$	b	$\left[\frac{3}{2},\infty\right[$	c	$\left[\frac{2}{3},\infty\right]$	d	$\left[\frac{2}{3},\infty\right[$
37	(3,2) is not solution of inequality						
a	$x+y>2$	b	$3x+5y>7$	c	$x+y\leq 1$	d	$3x-5y<3$
38	The point $(-1,2)$ satisfies the inequality						
a	$x-y>4$	b	$x-y\geq 4$	c	$x+y<4$	d	$x+y>4$
39	(1,0) is not solution of inequality						
a	$7x+2y<8$	b	$x-3y<0$	c	$3x+2y<8$	d	$7x+2y>6$
40	The graph of $4y\geq 5$ will be _____ half plane.						
a	Lower	b	Upper	c	Left	d	Right
41	The associated equation of inequality $x + 2y < 6$ is:						
a	$x + 2y = 6$	b	$x - 2y = 6$	c	$x + 2y = -6$	d	$x - 2y = -6$
42	The non-negative constraints are called						
a	Free variables	b	Decision variables	c	Vertex	d	Convex
43	An order pair which satisfied the inequality is called						
a	Solution	b	Point	c	Variable	d	None of these
44	The solution region restricted to 1st quadrant is called						

a	Solution region	b	Feasible region	c	Optimal region	d	None of these
45	A point of a solution region where two of its boundary lines intersect, is called						
a	Boundary points	b	Single point	c	Zero points	d	Corner point
46	The values of the variables which satisfies the inequality are called						
a	Solution set	b	Constraints	c	Constants	d	None of these
47	The variables used in inequality are called						
a	Solution	b	Constraints	c	Constants	d	None of these
48	A point which is used to determine solution area is called						
a	Corner point	b	Test point	c	Optimal point	d	Point
49	Feasible region is always lie in _____ quadrant.						
a	I	b	II	c	III	d	IV
50	A linear inequality concerning the problem from everyday life is named						
a	Problem constraint	b	Problem constraints	c	Non-negative constrains	d	Linear programming
51	The system of linear inequalities involved in the problem concerned are called						
a	Problem constraints	b	Solution	c	Non-negative constrains	d	Linear programming
52	A function which is to be maximized or minimized is called						
a	Objective function	b	Objective solution	c	Feasible region	d	None of these
53	The process used to maximize or minimize is called						
a	Optimization	b	Solution	c	Procedure	d	None of these
54	The feasible solution which maximizes or minimizes the objective function is called						
a	Optimal solution	b	Objective function	c	Solution	d	Objective solution
55	The maximum and minimum values of the objective function occur in the feasible region at						
a	Corner point	b	Boundary point	c	Origin	d	None of these

MCQ's Answers Key

1	2	3	4	5	6	7	8	9	10
b	a	a	b	d	b	c	d	a	c
11	12	13	14	15	16	17	18	19	20
a	b	b	b	b	b	c	a	c	c
21	22	23	24	25	26	27	28	29	30
a	b	d	c	a	d	b	b	a	d
31	32	33	34	35	36	37	38	39	40
d	b	d	a	a	b	c	c	b	b
41	42	43	44	45	46	47	48	49	50
a	b	a	b	d	a	b	b	a	a
51	52	53	54	55					
a	a	a	a	a					

IMPORTANT SHORT QUESTIONS

1. Define Linear Inequality?
2. What do you know about half plane?
3. If a non-vertical line divides a plane into two parts, then write the name of that two planes?
4. Define Associated Equation?
5. Graph the inequality $x + 2y < 6$.
6. Graph the linear inequalities $2x \geq -3$ in xy-plane.
7. Graph the linear inequalities $y \leq 2$ in xy-plane.
8. Define Corner Point or Vertex?
9. Show that the ordered pair (1,1) is a solution of inequality $x + 2y < 6$.
10. Graph the solution set of the following linear inequality in xy-plane. $2x + y \leq 6$.
11. Graph the solution set of the following linear inequality in xy-plane. $3x + 7y \geq 21$.
12. Graph the solution set of the following linear inequality in xy-plane. $3x - 2y \geq 6$.
13. Graph the solution set of the following linear inequality in xy-plane. $5x - 4y \leq 20$.
14. Graph the solution set of the following linear inequality in xy-plane. $2x + 1 \geq 0$.
15. Graph the solution set of the following linear inequality in xy-plane. $3y - 4 \leq 0$.
16. Graph the solution set of the following linear inequality in xy-plane.

$$3x + 7y \geq 21, \quad x - y \leq 2$$

17. Graph the solution set of the following linear inequality in xy-plane.

$$4x - 3y \leq 12, \quad x \geq \frac{-3}{2}$$

18. Indicate the solution region of the following systems of linear inequalities by shading:

$$x + y \leq 5, \ -2x + y \leq 2, \ x \geq 0.$$

19. Define Problem Constraint.
20. Define Problem Constraints.
21. Define non-negative Constraints.
22. Define Decision Variables.
23. Define Feasible Region.
24. Define Feasible Solution.
25. Define Feasible Solution Set.
26. Define Convex Region.
27. Define Objective Function.
28. Define Optimal Solution.
29. State the Theorem of Linear Programming Problem.
30. How would you obtain the optimal solution?

IMPORTANT LONG QUESTIONS

1. Indicate the solution region of the following systems of linear inequalities by shading:
$3x+7y \le 21,\ 2x-y \ge -3,\ x \ge 0.$
2. Graph the solution region of the following system of linear inequalities and find the corner points
$$2x-3y \le 6,\ 2x+3y \le 12,\ x \ge 0$$
3. Graph the solution region of the following system of linear inequalities and find the corner points
$$x+y \le 5,\quad -2x+y \le 2,\ y \ge 0$$
4. Graph the feasible region of the following system of linear inequalities and find the corner points
$$2x-3y \le 6,\ 2x+3y \le 12,\ x \ge 0, y \ge 0$$
5. Graph the feasible region of the following system of linear inequalities and find the corner points
$$x+y \le 5,\quad -2x+y \le 2,\ x \ge 0, y \ge 0$$
6. Graph the feasible region of the following system of linear inequalities and find the corner points
$$x+y \le 5,\quad -2x+y \ge 2,\ x \ge 0, y \ge 0$$
7. Graph the feasible region of the following system of linear inequalities and find the corner points
$$3x+7y \le 21,\quad x-y \le 3,\ x \ge 0, y \ge 0$$
8. Graph the feasible region of the following system of linear inequalities and find the corner points
$$2x+3y \le 18,\quad 2x+y \le 10,\quad x+4y \le 12.\ \ x \ge 0, y \ge 0$$
9. Graph the feasible region of the following system of linear inequalities and find the corner points
$$2x+3y \le 18,\quad x+4y \le 12,\quad 3x+y \le 12.\ \ x \ge 0, y \ge 0$$
10. Graph the feasible region of the following system of linear inequalities and find the corner points
$$x+3y \le 15,\quad 2x+y \le 12,\quad 4x+3y \le 24.\ \ x \ge 0, y \ge 0$$
11. Maximize $f(x,y) = 2x + 5y$; subject to the constraints:
$$2y-x \le 8;\quad x-y \le 4;\quad x \ge 0;\quad y \ge 0$$
12. Maximize $f(x,y) = x + 3y$; subject to the constraints:
$$2x+5y \le 30;\quad 5x+4y \le 20;\quad x \ge 0;\quad y \ge 0$$
13. Maximize $z = 2x + 3y$; subject to the constraints:
$$3x+4y \le 12;\quad 2x+y \le 4;\quad 2x-y \le 4;\quad x \ge 0;\quad y \ge 0$$
14. Minimize $z = 2x + y$; subject to the constraints:
$$x+y \ge 3;\quad 7x+5y \le 35;\quad x \ge 0;\quad y \ge 0$$
15. Maximize the function defined as; $f(x,y) = 2x+3y$; subject to the constraints:
$$2x+y \le 8;\quad x+2y \le 14;\quad x \ge 0;\quad y \ge 0$$
16. Minimize $z = 3x + y$; subject to the constraints:
$$3x+5y \ge 15;\quad x+3y \ge 9;\quad x \ge 0;\quad y \ge 0$$

UNIT 6

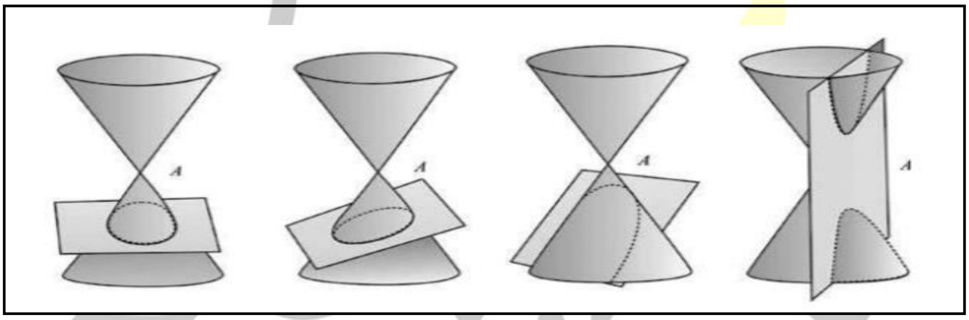
Conic Section

DEFINITIONS + SUMMARY

CONIC SECTIONS

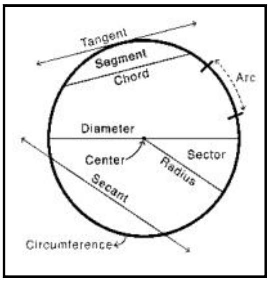
Conic sections or simply **conics**, are the curves obtained by cutting a (double) right circular cone by a plane. Let *RS* be a line through the centre *C* of a given circle and perpendicular to its plane. Let *A* be a fixed point on *RS*. All lines through *A* and points on the circle generate a ***right circular cone***. The lines are called rulings or generators of the cone. The surface generated consists of two parts, called ***nappes***, meeting at the fixed-point *A*, called the **vertex** or **apex** of the cone. The line ***RS*** is called ***axis*** of the cone.

- If the cone is cut by a plane perpendicular to the axis of the cone, then the section is a **circle**.
- If the cutting plane is slightly tilted and cuts only one nappe of the cone, the resulting section is an **ellipse**.
- If the intersecting plane is parallel to a generator of the cone, but intersects its one nappe only, the curve of intersection is a **parabola**.
- If the cutting plane is parallel to the axis of the cone and intersects both of its nappes, then the curve of intersection is a **hyperbola**.



CIRCLE

The set of all points in the plane that are equally distant from a fixed point is called a circle. The fixed point is called the ***centre*** of the circle and the distance from the **center** of the circle to any point on the circle is called the ***radius*** of the circle.
A line segment whose end points lie on a circle is called a ***chord*** of the circle. A ***diameter*** of a circle is a chord containing the centre of the circle.



EQUATION OF CIRCLE IN STANDARD FORM

If $C(h,k)$ is centre of a circle, $\boldsymbol{r}$ its radius and $P(x,y)$ any point on the circle then equation of circle is given as

$$(x-h)^2+(y-k)^2=r^2 \quad ---(i)$$

- If the centre of the circle is origin, then equation (i) reduces to $x^2+y^2=r^2$.
- If $r=0$, the circle is called a ***point circle*** which consists of the centre only.
- $x=r\cos\theta\,, y=r\sin\theta$ are called ***parametric equation*** of the circle.

GENERAL FORM OF AN EQUATION OF A CIRCLE

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ is called ***general form*** of an equation of a circle with centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

> **Note**
> Every second-degree equation in two variables "x" and "y" in which coefficient of x^2 and y^2 is same and contains no term involving the product xy, represents the circle.

EQUATION OF TANGENT LINE TO THE CIRCLE

A **tangent** to a curve is a line that touches the curve without cutting through it.

The equation of tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at the point $P(x_1, y_1)$ is given by

$$y - y_1 = -\frac{x_1+g}{y_1+f}(x - x_1) \quad \textbf{or} \quad xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$

EQUATION OF NORMAL LINE TO THE CIRCLE

The **normal** to the curve at $P(x_1, y_1)$ is the line through $P(x_1, y_1)$ perpendicular to the tangent to the curve at P.
The equation of normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at the point $P(x_1, y_1)$ is given by

$$y - y_1 = \frac{y_1+f}{x_1+g}(x - x_1) \quad \textbf{or} \quad (y - y_1)(x_1 + g) = (x - x_1)(y_1 + f)$$

> **Note**
> The line $y = mx + c$ intersects the circle $x^2 + y^2 = r^2$ at the most two points.

THE POSITION OF THE POINT WITH RESPECT TO THE CIRCLE

- The point $P(x_1, y_1)$ lies outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ if
  $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c > 0$
- The point $P(x_1, y_1)$ lies inside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ if
  $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c < 0$
- The point $P(x_1, y_1)$ lies on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ if
  $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0$

LENGTH OF THE TANGENT TO A CIRCLE

Let $P(x_1, y_1)$ be a point outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ then length of point $P(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is given by

$$\text{Length of the tangent} = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

PROPERTIES OF CIRCLE

- Length of a diameter of the circle $x^2 + y^2 = a^2$ is $2a$.
- Perpendicular dropped from the centre of a circle on a chord bisects the chord.
- The perpendicular bisector of any chord of a circle passes through the centre of the circle.
- The line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.
- Congruent chords of a circle are equidistant from the centre.
- Measure of the central angle of a minor arc is double the measure of the angle subtended in the corresponding major arc.
- An angle in a semi-circle is a right angle.
- The tangent to a circle at any point of the circle is perpendicular to the radial segment at that point.
- The perpendicular at the outer end of a radial segment is tangent to the circle.
- Normal lines of a circle pass through the centre of the circle.
- The straight line drawn from the centre of a circle perpendicular to a tangent passes through the point of tangency.
- The midpoint of the hypotenuse of a right triangle is the circumcentre of the triangle.
- The perpendicular dropped from a point of a circle on a diameter is a mean proportional between the segments into which it divides the diameter.

CONIC SECTION

Let L be a fixed line in a plane and F be a fixed point not on the line L. Suppose $|PM|$ denotes the perpendicular distance of a point $P(x, y)$ from the line L. The set of all points P in the plane such that

$$\frac{|PF|}{|PM|} = e \text{ (a positive constant)}$$

is called a **conic section**.

(i) If $e = 1$, then the conic is a **parabola**.
(ii) If $0 < e < 1$, then the conic is an **ellipse**.
(iii) If $e > 1$, then the conic is a **hyperbola**.

The fixed line L is called a **directrix** and the fixed-point F is called a **focus** of the conic.
The number e is called the **eccentricity** of the conic.

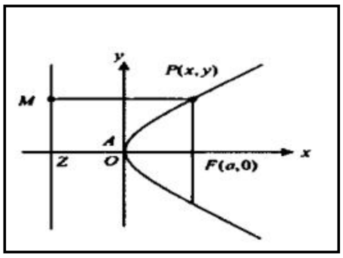
PARABOLA

Let $e = 1$ and F be a fixed point and L is fixed line not containing F. Let $P(x, y)$ be the point in the plane and $|PM|$ be the perpendicular distance of a point $P(x, y)$ from the line L. The set of all points P such that

$$\frac{|PF|}{|PM|} = 1 \text{ or } |PF| = |PM|$$

is called parabola.

OR The set of all points in a plane which is equidistant from a given fixed line in the plane is called ***Parabola***.



- The fixed point is called ***focus*** of the parabola.
- The fixed line is called ***directrix*** of the parabola.

STANDARD EQUATION OF PARABOLA

$$y^2 = (x+a)^2 - (x-a)^2 = 4ax \;\; or \;\; y^2 = 4ax$$

DEFINITIONS

(i) The line through the focus and perpendicular to the directrix is called ***axis*** of the parabola.
(ii) The point where the axis meets the parabola is called ***vertex*** of the parabola.
(iii) In parabola, the fixed point is called ***focus*** of the parabola.
(iv) In parabola, the fixed line is called ***directrix*** of the parabola
(v) A line passing through vertex and perpendicular to the axis of parabola is called ***tangent at vertex*** of parabola.
(vi) Line joining two distinct points on a parabola is called a ***chord*** of the parabola.
(vii) A chord passing through the focus of a parabola is called a ***focal chord*** of the parabola.
(viii) The focal chord perpendicular to the axis of the parabola is called ***latus rectum*** of the parabola.

PARAMETRIC EQUATION OF PARABOLA

The point $(at^2, 2at)$ lies on the parabola $y^2 = 4ax$ for any real t.
$x = at^2, y = 2at$ are called parametric equation of the parabola $y^2 = 4ax$.

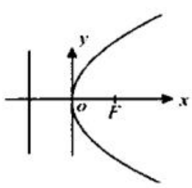
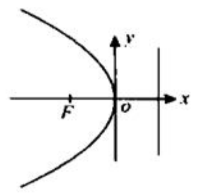
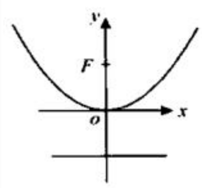
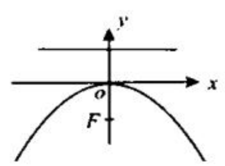
GENERAL FORM OF AN EQUATION OF PARABOLA

Let $P(x, y)$ be any point on the parabola having $F(h, k)$ as focus and M be the point on directrix $lx + my + n = 0$

By definition, equation of parabola is given by

$$|PM| = \frac{|lx+my+n|}{\sqrt{l^2+m^2}} \;\; or \;\; (x-h)^2 + (y-k)^2 = \frac{(lx+my+n)^2}{l^2+m^2}$$

Summary of Standard Parabolas

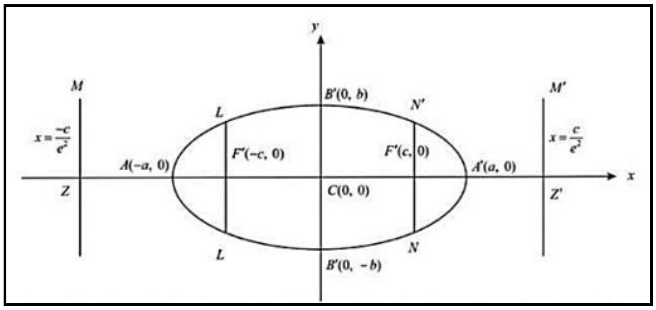
Sr.No.	1	2	3	4
Equation	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Vertex	(0,0)	(0,0)	(0,0)	(0,0)
Graph				

ELLIPSE

Let $0 < e < 1$ and F be a fixed point and L is fixed line not containing F. Let $P(x, y)$ be the point in the plane and $|PM|$ be the perpendicular distance of a point $P(x, y)$ from the line L. The set of all points P such that

$$\frac{|PF|}{|PM|} = e \quad (0 < e < 1)$$

is called an ellipse.



OR The set of all points P in a plane, such that distance of each point from a fixed point bears a constant ratio (less than one) to the distance from a fixed line is called an ellipse.

The number e is **eccentricity** of the ellipse, F a **focus** and L a **directrix**.

DEFINITIONS

Let F' and F be two foci of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - - - (1)$$

- The midpoint C of FF' is called the ***Centre*** of the ellipse. In case of (1) Centre is $C(0,0)$.
- The intersection of (1) with the line joining the foci are obtained by setting $y = 0$ into (1).These are the points $A'(-a, 0)$ and $A(a, 0)$. The points A and A' are called ***vertices*** of the ellipse.
- The line segment $AA' = 2a$ is called the ***major axis*** of the ellipse. The line through the centre of (1) and perpendicular to the major axis has its equation as $x = 0$. It meets (1) at points $B'(0, b)$ and $B(0, -b)$. The line segment $BB' = 2b$ is called the ***minor axis*** of the ellipse and B', B are some-times called the ***covertices*** of the ellipse.
- The length of the major axis is greater than the length of the minor axis.
- Foci of an ellipse always lie on the major axis.
- Each of the focal chords LFL' and $NF'N'$ perpendicular to the major axis of an ellipse is called a ***latusrectum*** of the ellipse. Thus, there are two ***laterarecta*** of an ellipse. The length of each latus-rectum is $\frac{2b^2}{a}$
- If the foci lie on the y-axis with coordinates $(0, -ae)$ and $(0, ae)$, then equation of the ellipse is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, a > b$$

Note

In each ellipse:

(i) Length of major axis= $2a$,
(ii) Length of minor axis= $2b$,
(iii) Foci lie on the major axis.
(iv) Length of Latusrectum $= \frac{2b^2}{a}$

STANDARD FORM OF THE ELLIPSE

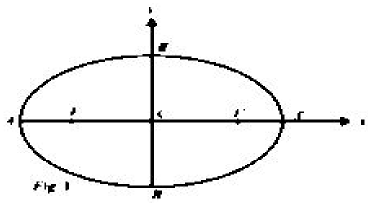
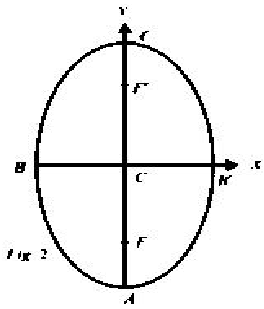
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

PARAMETRIC EQUATION OF ELLIPSE

The point $(a\cos\theta\,, b\sin\theta)$ lies on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ for all real θ.

$x=a\cos\theta\,, y=b\sin\theta$ are called parametric equation of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Summary of standard Ellipses

Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ $c^2=a^2-b^2$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1, a>b$ $c^2=a^2-b^2$
Foci	$(\pm c, 0)$	$(0, \pm c)$
Directrices	$x=\pm\frac{c}{e^2}$	$y=\pm\frac{c}{e^2}$
Major axis	$y=0$	$x=0$
Vertices	$(\pm a, 0)$	$(0, \pm a)$
Convertices	$(0, \pm b)$	$(\pm b, 0)$
Centre	$(0, 0)$	$(0, 0)$
Eccentricity	$e=\frac{c}{a}<1$	$e=\frac{c}{a}<1$
Graph		

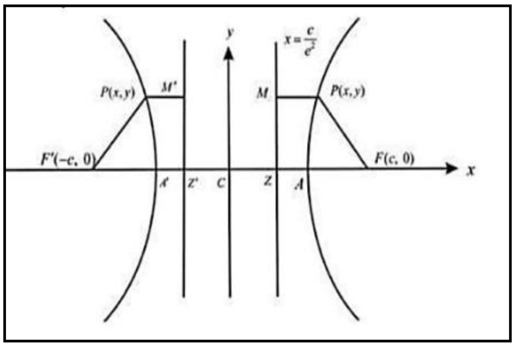
HYPERBOLA

Let $e>1$ and F be a fixed point and L is fixed line not containing F. Let $P(x,y)$ be the point in the plane and $|PM|$ be the perpendicular distance of a point $P(x,y)$ from the line L. The set of all points P such that

$$\frac{|PF|}{|PM|}=e>1$$

is called hyperbola.

OR The set of all points P in a plane, such that distance of each point from a fixed point bears a constant ratio (greater than one) to the distance from a fixed line is called hyperbola.



STANDARD FORM OF THE HYPERBOLA

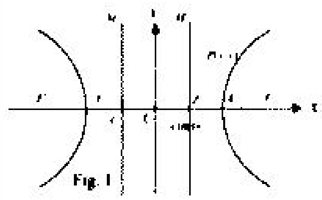
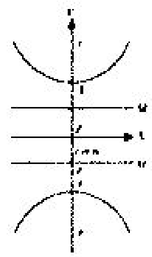
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

PARAMETRIC EQUATION OF THE HYPERBOLA

The point $(a \sec\theta, b \tan\theta)$ lies on the ellipse $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ for all real θ.

$x = a \sec\theta, y = b \tan\theta$ are called parametric equation of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Summary of Standard Hyperbolas

Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Foci	$(\pm c, 0)$	$(0, \pm c)$
Directrices	$x = \pm\frac{c}{e^2}$	$y = \pm\frac{c}{e^2}$
Transverse axis	$y = 0$	$x = 0$
Vertices	$(\pm a, 0)$	$(0, \pm a)$
Eccentricity	$e = \frac{c}{a} > 1$	$e = \frac{c}{a} > 1$
Centre	(0, 0)	(0, 0)
Graph		

MCQ's

Choose the correct Option.

1	The centre of the circle $(x-1)^2+(y+3)^2=3$ is:						
a	$(-1,-3)$	b	$(-1,3)$	c	$(1,3)$	d	$(1,-3)$
2	Equation of circle with centre at origin and $\sqrt{5}$ radius is:						
a	$x^2+y^2=\sqrt{5}$	b	$x^2+y^2=5$	c	$x^2+y^2=25$	d	$x^2-y^2=5$
3	Radius of the circle $x^2+y^2+2gx+2fy+c=0$ is:						
a	$\sqrt{g^2+f^2+c}$	b	$\sqrt{g^2-f^2-c}$	c	$\sqrt{g^2+f^2-c}$	d	$\sqrt{-g^2+f^2-c}$
4	The centre of the circle having equation $x^2+y^2+12x-10y=0$ is:						
a	$(6,5)$	b	$(-6,5)$	c	$(5,6)$	d	$(6,-6)$
5	An angle in semi-circle is of measure:						
a	$30°$	b	$40°$	c	$60°$	d	$90°$
6	The focus of parabola $y^2=4ax$ is:						
a	$(0,a)$	b	$(0,-a)$	c	$(a,0)$	d	$(-a,0)$
7	The latusrectum of a parabola $y^2=4ax$ is:						
a	$x=-a$	b	$y=-a$	c	$x=a$	d	$y=a$
8	The directrix of parabola $x^2=-8y$ is:						
a	$x+2=0$	b	$x-2=0$	c	$y+2=0$	d	$y-2=0$
9	The end points of the major axis of the ellipse are called its:						
a	Foci	b	Vertices	c	Covertices	d	Directrix
10	The mid-point of line segment joining the foci of an ellipse is called:						
a	Centre	b	Vertex	c	Directrix	d	Major axis
11	Centre of the circle $x^2+y^2+2gx+2fy+c=0$ is:						
a	$(-g,-f)$	b	$(-g,f)$	c	$(g,-f)$	d	(g,f)
12	Length of a diameter of the circle $x^2+y^2=a^2$ is:						
a	$2a$	b	$-2a$	c	$4a$	d	0
13	An angle in a semi-circle is:						
a	Right angle	b	Acute angle	c	Obtuse angle	d	None
14	The midpoint of the hypotenuse of a right triangle is the ____ of the triangle.						
a	Circum-Centre	b	In-Centre	c	Ortho-Centre	d	None
15	Length of Latusrectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is:						
a	$4a$	b	$\frac{2a^2}{b}$	c	$\frac{2b^2}{a}$	d	$\frac{b^2}{a}$
16	Length of Latusrectum of the parabola $y^2=4ax$ is:						
a	$4a$	b	$\frac{2a^2}{b}$	c	$\frac{2b^2}{a}$	d	$\frac{b^2}{a}$
17	The end points of the minor axis of the ellipse are called its:						
a	Foci	b	Vertices	c	Covertices	d	Directrix
18	Vertices of ellipse with equation $x^2+4y^2=16$ are:						
a	$(\pm 4,0)$	b	$(0,\pm 4)$	c	$(\pm 2,0)$	d	$(0,\pm 2)$
19	Asymptotes are very useful in graphing:						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola
20	$x=r\cos\theta$, $y=r\sin\theta$ are called parametric equation of the ____.						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola

21	$x = at^2, y = 2at$ are called parametric equation of the____.						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola
22	$x = a\cos\theta, y = b\sin\theta$ are called parametric equation of the ______.						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola
23	$x = a\sec\theta, y = b\tan\theta$ are called parametric equation of the____.						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola
24	$(x-h)^2 + (y-k)^2 = r^2$ is the standard form of the___?						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola
25	$y^2 = 4ax$ is the standard form of the ___?						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola
26	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is the standard form of the ___?						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola
27	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is the standard form of the ___?						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola
28	Vertex of the parabola $y^2 = 4ax$ is:						
a	$(0,0)$	b	$(0,a)$	c	$(a,0)$	d	(a,a)
29	The coordinates of the vertices of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is:						
a	$(\pm b, 0)$	b	$(0, \pm b)$	c	$(\pm a, 0)$	d	$(0, \pm a)$
30	Vertices of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ is:						
a	$(\pm b, 0)$	b	$(0, \pm b)$	c	$(\pm a, 0)$	d	$(0, \pm a)$
31	Length of latusrectum of the ellipse $\frac{x^2}{36} + \frac{y^2}{25} = 1$ is:						
a	$\frac{25}{6}$	b	$\frac{25}{3}$	c	$\frac{25}{36}$	d	$\frac{3}{25}$
32	Length of the diameter of the circle $(x+5)^2 + (y-8)^2 = 12$ is:						
a	$4\sqrt{3}$	b	$2\sqrt{3}$	c	12	d	24
33	Set of all points equidistance from a fixed-point form.						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola
34	Length of latusrectum of the parabola $x^2 = 5y$ is:						
a	5	b	20	c	$5/4$	d	10
35	If $a = b$ then equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ represent.						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola
36	Foci of $\frac{x^2}{25} + \frac{y^2}{16} = 1$						
a	$(\pm 4, 0)$	b	$(\pm 5, 0)$	c	$(\pm 3, 0)$	d	$(0, \pm 3)$
37	Axis of parabola $x^2 = 4ay$ is:						
a	$y = 0$	b	$x = 0$	c	$x = y$	d	$x = 1$
38	Centre of the circle $x^2 + y^2 - 6x + 4y + 13 = 0$						
a	$(3,-2)$	b	$(-3,-2)$	c	$(-3,2)$	d	$(3,2)$
39	Length of the diameter of the circle $(x-5)^2 + (y-3)^2 = 8$ is:						
a	64	b	16	c	$2\sqrt{2}$	d	$4\sqrt{2}$
40	The line $y = mx + c$ will be tangent to the circle $x^2 + y^2 = a^2$ if:						
a	$\frac{a}{m}$	b	$\pm\sqrt{1-m^2}$	c	$c = \pm a\sqrt{1+m^2}$	d	$c = \pm a\sqrt{m^2-1}$
41	Radius of the circle $(x-5)^2 + (y-3)^2 = 8$ is:						

a	64	b	4	c	2	d	$2\sqrt{2}$
42	Foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are:						
a	$(\pm a, 0)$	b	$(0, \pm a)$	c	$(\pm ae, 0)$	d	$(0, \pm ae)$
43	If a circle of a line intersects in two points, then line is called:						
a	Chord	b	Secant	c	Radius	d	Diameter
44	What is the length of latusrectum of the parabola $(y-2)^2=-4(x-7)$?						
a	8	b	$1/4$	c	4	d	16
45	Length of major axis of the ellipse is:						
a	$2a$	b	$2b$	c	$4a$	d	$4b$
46	Length of minor axis of the ellipse is:						
a	$2a$	b	$2b$	c	$4a$	d	$4b$
47	Foci of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is:						
a	$(a, \pm c)$	b	$(0,0)$	c	$(\pm c, 0)$	d	$(0,0)$
48	Foci of hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ is:						
a	$(\pm 4,0)$	b	$(0, \pm 4)$	c	$(\pm 2,0)$	d	$(0, \pm 2)$
49	The two separate part of hyperbola are called:						
a	Foci	b	Vertices	c	Branches	d	Directions
50	Length of major and minor axis of the ellipse $4x^2+9y^2=36$ are:						
a	6,4	b	4,6	c	3,2	d	2,3
51	Eccentricity of the parabola is:						
a	$e = 1$	b	$0 < e < 1$	c	$e > 1$	d	$e = 0$
52	Eccentricity of the ellipse is:						
a	$e = 1$	b	$0 < e < 1$	c	$e > 1$	d	$e = 0$
53	Eccentricity of the hyperbola is:						
a	$e = 1$	b	$0 < e < 1$	c	$e > 1$	d	$e = 0$
54	The conic is called circle if:						
a	$e = 1$	b	$0 < e < 1$	c	$e > 1$	d	$e = 0$
55	Two circles are said to be concentric circles if they have same:						
a	Radius	b	Diameter	c	Chord	d	Centre
56	Opening parabola of $x^2=-16y$ is:						
a	Downward	b	Upward	c	Leftward	d	Rightward
57	Vertices of $\frac{y^2}{16}-\frac{x^2}{49}=1$ is:						
a	$(\pm 4,0)$	b	$(0, \pm 4)$	c	$(\pm 7,0)$	d	$(0, \pm 7)$
58	Length of diameter of circle $x^2+y^2=9$.						
a	6	b	3	c	9	d	4
59	Focus of parabola $x^2=-16y$ is:						
a	$(0,4)$	b	$(4,0)$	c	$(0,-4)$	d	$(-4,0)$
60	Parabola having equation $x^2=4ay$ opens:						
a	Downward	b	Upward	c	Leftward	d	Rightward
61	Conic sections are the curve obtained by cutting a cone by___						
a	A plane	b	A line	c	Two lines	d	A sphere
62	The length of diameter of the circle $x^2+y^2-4x-12=0$ is:						
a	6	b	7	c	8	d	9
63	The length of tangent from $(0,1)$ to $x^2+y^2+6x-3y+3=0$ is:						

a	2	b	3	c	4	d	1
64	The coordinates of vertex of parabola $x+8-y^2+2y=0$ will be___						
a	$(-9,1)$	b	$(9,1)$	c	$(9,-1)$	d	$(-9,-1)$
65	The line $y=mx+c$ intersects the circle $x^2+y^2=r^2$ at the most						
a	One point	b	Two points	c	Three points	d	Four points
66	If the cone is cut by a plane perpendicular to the axis of the cone, then the section is a						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola
67	If the intersecting plane is parallel to a generator of the cone, but intersects its one nappe only, the curve of intersection is a						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola
68	If the cutting plane is slightly tilted and cuts only one nappe of the cone, the resulting section is						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola
69	If the cutting plane is parallel to the axis of the cone and intersects both of its nappes, then the curve of intersection is						
a	Circle	b	Parabola	c	Ellipse	d	Hyperbola
70	The line through the focus and perpendicular to the directrix is called _of the parabola.						
a	Axis	b	Focus	c	Directrix	d	Vertex
71	In parabola, the fixed point is called____ of the parabola.						
a	Axis	b	Focus	c	Directrix	d	Vertex
72	In parabola, the fixed line is called ____of the parabola						
a	Axis	b	Focus	c	Directrix	d	Vertex
73	The point where the axis meets the parabola is called _____of the parabola.						
a	Axis	b	Focus	c	Directrix	d	Vertex
74	A chord passing through the focus of a parabola is called a _____ of the parabola						
a	Focal chord	b	Latusrectum	c	Directrix	d	Vertex
75	The focal chord perpendicular to the axis of the parabola is called __ of the parabola.						
a	Focal chord	b	Latusrectum	c	Directrix	d	Vertex
76	Axis of the parabola $y^2=4ax$ is:						
a	$x=0$	b	$x=a$	c	$y=0$	d	$x=-a$
77	Foci of the ellipse always lie on:						
a	Major axis	b	Minor axis	c	x-axis	d	y-axis
78	Axis of the parabola $y^2=-4ax$ is						
a	$x=0$	b	$x=a$	c	$y=0$	d	$x=-a$
79	Axis of the parabola $x^2=-4ay$ is						
a	$x=0$	b	$x=a$	c	$y=0$	d	$x=-a$
80	Centre of hyperbola is:						
a	$(0,0)$	b	$(0,1)$	c	$(1,1)$	d	$(1,0)$

MCQ's Answers Key

1	2	3	4	5	6	7	8	9	10
d	b	c	b	d	c	c	d	b	a
11	12	13	14	15	16	17	18	19	20
a	a	a	a	c	a	c	a	d	a
21	22	23	24	25	26	27	28	29	30
b	c	d	a	b	c	d	a	c	c
31	32	33	34	35	36	37	38	39	40
b	a	a	a	a	c	b	a	d	c
41	42	43	44	45	46	47	48	49	50
d	c	a	c	a	b	c	a	c	a
51	52	53	54	55	56	57	58	59	60
a	b	c	d	d	a	b	a	c	b
61	62	63	64	65	66	67	68	69	70
a	c	d	a	b	a	b	c	d	a
71	72	73	74	75	76	77	78	79	80
b	c	d	a	b	c	a	c	a	a

IMPORTANT SHORT QUESTIONS

1. Define Conic Sections.
2. Define Circle and its Centre.
3. Write equation of circle in standard form and in general form.
4. Write an equation of the circle with Centre $(-3, 5)$ and radius 7.
5. Show that the equation $5x^2 + 5y^2 + 24x + 36y + 10 = 0$ represents a circle. Also find its centre and radius.
6. Find an equation of the circle having the join of $A\ (x_1, y_1)$ and $B\ (x_2, y_2)$ as a diameter.
7. Find an equation of the circle with centre at $(5, -2)$ and radius 4.
8. Find an equation of the circle with centre at $\left(\sqrt{2}, -3\sqrt{3}\right)$ and radius $2\sqrt{2}$.
9. Find an equation of the circle with ends of a diameter at $(-3, 2)$ and $(5, -6)$.
10. Find the centre and radius of the circle with $x^2 + y^2 + 12x - 10y = 0$.
11. Find the centre and radius of the circle with $5x^2 + 5y^2 + 14x + 12y - 10 = 0$.
12. Find the centre and radius of the circle with $x^2 + y^2 - 6x + 4y + 13 = 0$.
13. Find the centre and radius of the circle with $4x^2 + 4y^2 - 8x + 12y - 25 = 0$.
14. Determine whether the point $P(-5, 6)$ lies outside, on or inside the circle: $$x^2 + y^2 + 4x - 6y - 12 = 0$$
15. Find the condition that the line $y = mx + c$ touches circle $x^2 + y^2 = a^2$ at a single point.
16. Find the length of the tangent from the point $P(-5, 10)$ to the circle $$5x^2 + 5y^2 + 14x + 12y - 10 = 0.$$
17. Write down equations of tangent and normal to the circle $x^2 + y^2 = 25$ at $(4,3)$.
18. Write down equations of tangent and normal to the circle $$3x^2 + 3y^2 + 5x - 13y + 2 = 0 \text{ at } \left(1, \frac{10}{3}\right)$$
19. Check the position of the point $(5,6)$ with respect to the circle $x^2 + y^2 = 81$
20. Check the position of the point $(5,6)$ with respect to the circle $$2x^2 + 2y^2 + 12x - 8y + 1 = 0$$
21. Find the length of the tangent drawn from the point $(-5,4)$ to the circle $$5x^2 + 5y^2 - 10x + 15y - 131 = 0$$
22. Define eccentricity of the conic.
23. Define Parabola.
24. Define axis and vertex of the parabola.
25. Define latus rectum of the parabola.
26. Find the focus, vertex and directrix of the parabola $y^2 = 8x$
27. Find the focus, vertex and directrix of the parabola $x^2 = -16y$
28. Find the focus, vertex and directrix of the parabola $x^2 = 5y$
29. Find the focus, vertex and directrix of the parabola $y^2 = -12x$
30. Find the focus, vertex and directrix of the parabola $x^2 = 4(y - 1)$
31. Find the focus, vertex and directrix of the parabola $y^2 = -8(x - 3)$
32. Find the focus, vertex and directrix of the parabola $(x - 1)^2 = 8(y + 2)$
33. Find the focus, vertex and directrix of the parabola $y = 6x^2 - 1$
34. Write an equation of the parabola with Focus $(-3, 1)$; directrix $x = 3$

35. Write an equation of the parabola with Focus $(2, 5)$; directrix $y = 1$
36. Write an equation of the parabola with Focus $(-3, 1)$; directrix $x - 2y - 3 = 0$
37. Write an equation of the parabola with Focus $(-1, 0)$; vertex $(-1, 2)$
38. Write an equation of the parabola with Directrix $x = -2$; Focus $(2, 2)$
39. Define Latus rectum of Ellipse.
40. Derive equation of ellipse in standard form.
41. Find focus and eccentricity of ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$
42. Find an equation of the ellipse with foci $(\pm 3,0)$ and minor axis of length 10.
43. Find an equation of the ellipse with foci $(0, -1)$ and $(0, -5)$ and major axis of length 6.
44. Find an equation of the ellipse with foci $(-3\sqrt{3}, 0)$ and vertices $(\pm 6,0)$.
45. Find an equation of the ellipse with Vertices $(-1,1), (5,1)$; foci $(4,1)$ and $(0,1)$.
46. Find an equation of the ellipse with Vertices $(0, \pm 5)$, eccentricity $\frac{3}{5}$.
47. Find an equation of the ellipse with Centre $(0,0)$, focus $(0, -3)$, vertex $(0,4)$.
48. Find the centre, foci, eccentricity, vertices and directrices of the ellipse $x^2 + 4y^2 = 16$.
49. Find the centre, foci, eccentricity, vertices and directrices of the ellipse $9x^2 + y^2 = 18$.
50. Find the centre, foci, eccentricity, vertices and directrices of the ellipse $25x^2 + 9y^2 = 225$.
51. Write standard equation of hyperbola.
52. Define transverse axis of the hyperbola.
53. Find an equation of the hyperbola whose foci are $(\pm 4, 0)$ and vertices $(\pm 2, 0)$.
54. Find the foci and eccentricity of the hyperbola $\frac{y^2}{16} - \frac{x^2}{49} = 1$
55. Find an equation of the hyperbola with centre $(0,0)$, focus $(6,0)$ vertex $(4,0)$.
56. Find an equation of the hyperbola with foci $(\pm 5,0)$, vertex $(3,0)$.
57. Find an equation of the hyperbola with foci $(0, \pm 6)$, $e = 2$.
58. Find an equation of the hyperbola with foci $(0, \pm 9)$, directrices $y = \pm 4$
59. Find the centre, foci, eccentricity, vertices and equation of directrices of the hyperbola

$$x^2 - y^2 = 9$$

60. Find equation of tangent and normal $y^2 = 4ax$ at the point (x_1, y_1).
61. Write equation of tangent and normal to the parabola $x^2 = 16y$.
62. Find equation of tangent and normal of $y^2 = 4ax$ at $(at^2, 2at)$.
63. Find equation of tangent and normal of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a\cos\theta, b\sin\theta)$.
64. Find equation of tangent and normal of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(a\sec\theta, b\tan\theta)$.
65. Find the point of intersection of the conics

$$x^2 + y^2 = 8 \quad \text{and} \quad x^2 - y^2 = 1$$

66. Find the point of intersection of the conics

$$3x^2 - 4y^2 = 12 \quad \text{and} \quad 3y^2 - 2x^2 = 7$$

67. Define rotation of axes.
68. Identify the conic $4x^2 - 4xy + y^2 - 6 = 0$.

IMPORTANT LONG QUESTIONS

1. Find an equation of the circle which passes through the points $A(5,10), B(6,9)$ and $C(-2,3)$
2. Find an equation of the circle passing through the points $A(1,2)$ and $B(1,-2)$ and touching the line $x + 2y + 5 = 0$.
3. Write an equation of the circle that passes through the points $A(4,5), B(-4,-3), C(8,-3)$.
4. Write an equation of the circle that passes through the points $A(-7,7), B(5,-1), C(10,0)$.
5. Write an equation of the circle that passes through the points $A(5,6), B(-3,2), C(3,-4)$.
6. Find an equation of the circle passing through $A(3,-1), B(0,1)$ and having centre at $4x - 3y - 3 = 0$.
7. Find an equation of the circle passing through $A(-3,1)$ with radius 2 and centre at $2x - 3y + 3 = 0$.
8. Find an equation of the circle passing through $A(5,1)$ and tangent to the line $2x - y - 10 = 0$ at $B(3,-4)$.
9. Find an equation of the circle passing through $A(1,4), B(-1,8)$ and tangent to the line $x + 3y - 3 = 0$.
10. Find an equation of a circle of radius a and lying in the second quadrant such that it is tangent to both the axes.
11. Show that the lines $3x - 2y = 0$ and $2x + 3y - 13 = 0$ are tangents to the circle $x^2 + y^2 + 6x - 4y = 0$.
12. Show that the circles $x^2 + y^2 + 2x - 2y - 7 = 0$ and $x^2 + y^2 - 6x + 4y + 9 = 0$ touch externally.
13. Show that the circles $x^2 + y^2 + 2x - 8 = 0$ and $x^2 + y^2 - 6x + 6y - 46 = 0$ touch internally.
14. Write equations of two tangents from $(2,3)$ to the circle $x^2 + y^2 = 9$.
15. Find the length of the chord cut off from the line $2x + 3y = 13$ by the circle $x^2 + y^2 = 26$
16. Find the coordinates of the points of intersection of the line $x + 2y = 6$ with the circle: $x^2 + y^2 - 2x - 2y - 39 = 0$
17. Find equations of the tangents to the circle $x^2 + y^2 = 2$ parallel to the $x - 2y + 1 = 0$.
18. Find equations of the tangents to the circle $x^2 + y^2 = 2$ perpendicular to the line $3x + 2y = 6$.
19. Find equation of the tangents drawn from $(0,5)$ to $x^2 + y^2 = 16$.
20. Find equation of the tangents drawn from $(-1,2)$ to $x^2 + y^2 + 4x + 2y = 0$.
21. Find equation of the tangents drawn from $(-7,-2)$ to $(x+1)^2 + (y-2)^2 = 26$.
22. Find an equation of the parabola whose focus is $F(-3,4)$ & directrix is $3x - 4y + 5 = 0$.
23. Find the focus, vertex and directrix of the parabola $x + 8 - y^2 + 2y = 0$
24. Find the focus, vertex and directrix of the parabola $x^2 - 4x - 8y + 4 = 0$
25. Write an equation of the parabola with axis parallel to y-axis, the points $(0,3), (3,4)$ and $(4,11)$ lie on the graph.
26. Find an equation of the parabola having its focus at the origin and directrix, parallel to the (i) x-axis (ii) y-axis.
27. Show that an equation of the parabola with focus at $(a\cos\alpha, a\sin\alpha)$ and directrix $x\cos\alpha + y\sin\alpha + a = 0$ is $(x\sin\alpha - y\cos\alpha)^2 = 4a(x\cos\alpha + y\sin\alpha)$
28. Show that the ordinate at any point P of the parabola is a mean proportional between the length of the latus rectum and the abscissa of P.

29. Show that the equation $9x^2 - 18x + 4y^2 + 8y - 23 = 0$ represents an ellipse. Find its elements (Centre, Focus, Eccentricity, Covertices).

30. Find an equation of the ellipse with foci $(\pm\sqrt{5}, 0)$ and passing through the point $\left(\frac{3}{2}, \sqrt{3}\right)$.

31. Find an equation of the ellipse with Centre $(2, 2)$, major axis parallel to y-axis and of length 8 units, minor axis parallel to x-axis and of length 6 units.

32. Find an equation of the ellipse with Centre $(0, 0)$, major axis horizontal, the points $(3, 1)$, $(4, 0)$ lie on the graph.

33. Find the Centre, foci, eccentricity, vertices and directrices of the ellipse

$$\frac{(2x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$$

34. Find the Centre, foci, eccentricity, vertices and directrices of the ellipse

$$x^2 + 16x + 4y^2 - 16y + 76 = 0$$

35. Prove that the lactusrectum of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.

36. Find Centre, foci, eccentricity and vertices of hyperbola

$$4x^2 - 8x - y^2 - 2y - 1 = 0$$

37. Find the Centre, foci, eccentricity, vertices and equation of directrices of the hyperbola

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

38. Find the Centre, foci, eccentricity, vertices and equation of directrices of the hyperbola

$$\frac{y^2}{16} - \frac{x^2}{9} = 1$$

39. Find the Centre, foci, eccentricity, vertices and equation of directrices of the hyperbola

$$\frac{y^2}{4} - x^2 = 1$$

40. Find the Centre, foci, eccentricity, vertices and equation of directrices of the hyperbola

$$\frac{(y+2)^2}{9} - \frac{(x-2)^2}{16} = 1$$

41. Find the Centre, foci, eccentricity, vertices and equation of directrices of the hyperbola

$$9x^2 - 12x - y^2 - 2y + 2 = 0$$

42. Find the Centre, foci, eccentricity, vertices and equation of directrices of the hyperbola

$$4y^2 + 12y - x^2 + 4x + 1 = 0$$

43. Find the Centre, foci, eccentricity, vertices and equation of directrices of the hyperbola

$$9x^2 - y^2 - 36x - 6y + 18 = 0$$

44. Find an equation of tangent to the parabola $y^2 = -6x$ which is parallel to the line $2x + y + 1 = 0$. Also find the point of tangency.

45. Find the point of intersection of $\frac{x^2}{18} + \frac{y^2}{8} = 1$ and $\frac{x^2}{3} - \frac{y^2}{3} = 1$

UNIT 7

Vectors

DEFINITIONS + SUMMARY

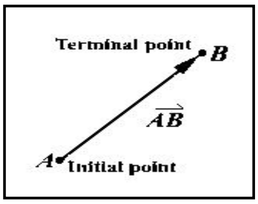
SCALAR

A **scalar quantity**, or simply a **scalar**, is one that possesses only magnitude. It can be specified by a number along with unit.
Example: - Mass, time, density, temperature, length, volume, speed and work etc.

VECTOR

A **vector quantity**, or simply a **vector**, is one that possesses both magnitude and direction. *Example:* - Displacement, velocity, acceleration, weight, force, momentum, electric and magnetic fields etc.

GEOMETRICAL INTERPRETATION OF VECTOR



Geometrically, a vector is represented by a directed line segment $\overrightarrow{AB}$ with A its initial point and B its terminal point. It is often found convenient to denote a vector by an arrow and is written either as $\overrightarrow{AB}$ or as a boldface symbol like $\boldsymbol{v}$ or in underlined form $\underline{v}$.

MAGNITUDE OR LENGTH OF A VECTOR

The magnitude of a vector $\overrightarrow{AB}$ or $\underline{v}$ is its absolute value and is written as $|\overrightarrow{AB}|$ or $|\underline{v}|$ or simply AB or v.

If $\underline{v} = x\underline{i} + y\underline{j} + z\underline{k}$ then $|\underline{v}| = \sqrt{x^2 + y^2 + z^2}$

PROPERTIES OF MAGNITUDE OF A VECTOR

Let $\underline{v}$ be a vector in plane or in space and c be a real number, then

(i) $|\underline{v}| \geq 0$ and $|\underline{v}| = 0$ if and only if $\underline{v} = 0$

(ii) $|c\underline{v}| = |c||\underline{v}|$

UNIT VECTOR

A vector whose magnitude is one is called ***Unit Vector***. Unit vector of vector $\underline{v}$ is written as $\hat{v}$ (read as v hat) and is defined as $\hat{v} = \frac{\underline{v}}{|\underline{v}|}$.

ZERO OR NULL VECTOR

If terminal point B of a vector $\overrightarrow{AB}$ coincides with its initial point A, then magnitude $AB = 0$ and $|\overrightarrow{AB}| = 0$, which is called zero or null vector.

NEGATIVE OF A VECTOR

Two vectors are said to be negative of each other if they have same magnitude but opposite direction. If $\overrightarrow{AB} = \underline{v}$ then $\overrightarrow{BA} = -\overrightarrow{AB} = -\underline{v}$ and $|\overrightarrow{BA}| = |-\overrightarrow{AB}|$

EQUAL VECTORS

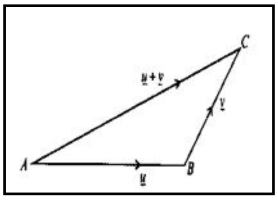
Two vectors $\underline{u}$ and $\underline{v}$ are said to be equal if they have same magnitude and direction

i.e., $\underline{u} = \underline{v}$

PARALLEL VECTORS

Two vectors $\underline{u}$ and $\underline{v}$ are parallel if and only if they are non-zero scalar multiple of each other.
i.e., $\underline{u} = k\underline{v}$.

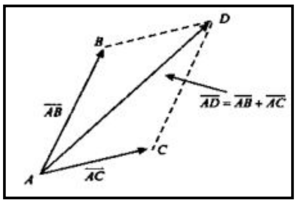
TRIANGLE LAW OF VECTOR ADDITION



If two vectors $\underline{u}$ and $\underline{v}$ are represented by the two sides AB and BC of a triangle such that the terminal point of $\underline{u}$ coincide with the initial point of $\underline{v}$, then the third side AC of the triangle gives vector sum $\underline{u} + \underline{v}$, that is

$$\overline{AB} + \overline{BC} = \overline{AC} \Rightarrow \underline{u} + \underline{v} = \overline{AC}$$

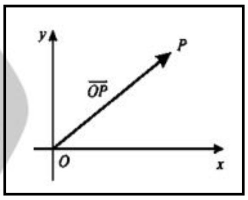
PARALLELOGRAM LAW OF VECTOR ADDITION



If two vectors $\underline{u}$ and $\underline{v}$ are represented by two adjacent sides AB and AC of a parallelogram as shown in the figure, then diagonal AD give the sum or resultant of AB and AC, that is

$$\overline{AD} = \overline{AB} + \overline{AC} = \underline{u} + \underline{v}$$

POSITION VECTOR



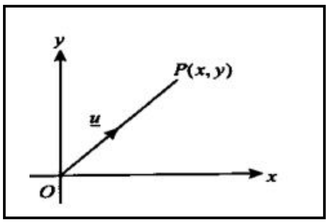
The vector, whose initial point is the origin O and whose terminal point is P, is called the position vector of the point P and is written as $\overline{OP}$.

THE RATIO FORMULA

Let A and B be two points whose position vectors (p.v.) are $\underline{a}$ and $\underline{b}$ respectively. If a point P divides AB in the ratio $p : q$, then the position vector of P is given by

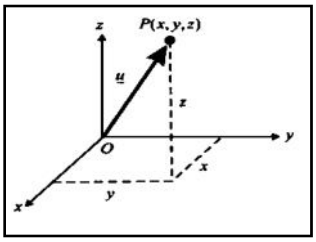
$$\underline{r} = \frac{q\underline{a} + p\underline{b}}{p + q}$$

VECTOR IN PLANE



The set of all ordered pairs $[x, y]$ of real numbers, together with the rules of addition and scalar multiplication, is called the ***set of vectors*** in $\boldsymbol{R^2}$.
For the vector $\underline{u} = [x, y] = x\underline{i} + y\underline{j}$. x and y are called components of $\underline{u}$. Where $\underline{i}$ and $\underline{j}$ are the unit vectors along x-axis and y-axis respectively.

VECTOR IN SPACE



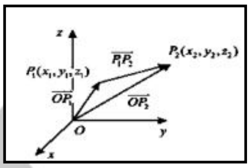
The set of all ordered triples $[x, y, z]$ of real numbers, together with the rules of addition and scalar multiplication, is called the ***set of vectors*** in $\boldsymbol{R^3}$.
For the vector $\underline{u} = [x, y, z] = x\underline{i} + y\underline{j} + z\underline{k}$. x, y and z are called components of $\underline{u}$. Where $\underline{i}, \underline{j}$ and $\underline{k}$ are the unit vectors along x-axis, y-axis and z-axis respectively.

PROPERTIES OF VECTORS

Vectors, both in the plane and in space, have the following properties:
Let $\underline{u}$, $\underline{v}$ and $\underline{w}$ be vectors in the plane or in space and let $a, b \in R$, then

- **(i)** $\underline{u} + \underline{v} = \underline{v} + \underline{u}$ (Commutative Property)
- **(ii)** $(\underline{u} + \underline{v}) + \underline{w} = \underline{u} + (\underline{v} + \underline{w})$ (Associative Property)
- **(iii)** $\underline{u} + (-1)\underline{u} = \underline{u} - \underline{u} = 0$ (Inverse for vector addition)
- **(iv)** $a(\underline{v} + \underline{w}) = a\underline{v} + a\underline{w}$ (Distributive Property)
- **(v)** $a(b\underline{u}) = (ab)\underline{u}$ (Scalar Multiplication)

DISTANCE BETWEEN TWO POINTS IN SPACE

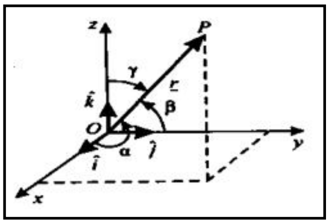


If $\overline{OP_1}$ and $\overline{OP_2}$ are the position vectors of points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$, then

Distance between P_1 and $P_2 = |\overline{P_1P_2}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

This is called ***Distance Formula*** between two points P_1 and P_2 in $\boldsymbol{R^3}$.

DIRECTION ANGLES AND DIRECTION COSINES OF A VECTOR



Let $\underline{r} = \overline{OP} = x\underline{i} + y\underline{j} + z\underline{k}$ be a non-zero vector let α, β and γ denote the angles formed between $\underline{r}$ and the unit coordinate vectors $\underline{i}, \underline{j}$ and $\underline{k}$ respectively.
Such that

$$0 \le \alpha \le \pi, \quad 0 \le \beta \le \pi, \quad \text{and} \quad 0 \le \gamma \le \pi$$

(i) The angles α, β and γ are called the ***direction angles.***
(ii) The numbers $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called ***direction cosines*** of a vector $\underline{r}$.
If $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ then direction cosine of a vector $\underline{r}$ are given as

$$\cos\alpha = \frac{x}{|\underline{r}|}, \cos\beta = \frac{y}{|\underline{r}|}, \cos\gamma = \frac{z}{|\underline{r}|}$$

Important Result

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

SCALAR PRODUCT OF TWO VECTORS

Definition 1:

Let two non-zero **vectors** $\underline{u}$ and $\underline{v}$, in the plane or in space, have same initial point. The **dot or Scalar** product of $\underline{u}$ and $\underline{v}$, written as $\underline{u}.\underline{v}$, is defined by

$$\underline{u}.\underline{v} = |\underline{u}||\underline{v}|\cos\theta \quad \text{or} \quad \underline{u}.\underline{v} = uv\cos\theta$$

Where θ is the angle between $\underline{u}$ and $\underline{v}$ and $0 \le \theta \le \pi$.

Definition 2:

(i) If $\underline{u} = a_1\underline{i} + b_1\underline{j}$ and $\underline{v} = a_2\underline{i} + b_2\underline{j}$ are two non-zero vectors in plane. The dot product $\underline{u}.\underline{v}$ is defined by

$$\underline{u}.\underline{v} = a_1a_2 + b_1b_2$$

(ii) If $\underline{u} = a_1\underline{i} + b_1\underline{j} + c_1\underline{k}$ and $\underline{v} = a_2\underline{i} + b_2\underline{j} + c_2\underline{k}$ are two non-zero vectors in space. The dot product $\underline{u}.\underline{v}$ is defined by

$$\underline{u}.\underline{v} = a_1a_2 + b_1b_2 + c_1c_2$$

Note

The dot product is also referred to the **scalar** product or the **inner** product.

$$\underline{i}.\underline{i} = \underline{j}.\underline{j} = \underline{k}.\underline{k} = 1$$

$$\underline{i}.\underline{j} = \underline{j}.\underline{k} = \underline{k}.\underline{i} = 0$$

PERPENDICULAR (ORTHOGONAL) VECTORS

Two non-zero **vectors** $\underline{u}$ and $\underline{v}$ are said to be perpendicular if and only if $\underline{u}.\underline{v} = 0$.

$$\underline{u}.\underline{v} = |\underline{u}||\underline{v}|\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

PROPERTIES OF DOT PRODUCT

Let $\underline{u}$, $\underline{v}$ and $\underline{w}$ be vectors and c be a real number, then

(i) $\underline{u}.\underline{v} = 0 \Rightarrow \underline{u} = 0 \; or \; \underline{v} = 0$

(ii) $\underline{u}.\underline{v} = \underline{v}.\underline{u}$ (Commutative Property)

(iii) $\underline{u}.(\underline{v} + \underline{w}) = \underline{u}.\underline{v} + \underline{u}.\underline{w}$ (Distributive Property)

(iv) $(c\underline{u}).\underline{v} = c(\underline{u}.\underline{v})$ (c is scalar)

ANGLE BETWEEN TWO VECTORS

The angle between two vectors $\underline{u}$ and $\underline{v}$ is determined from the definition of dot product, that is

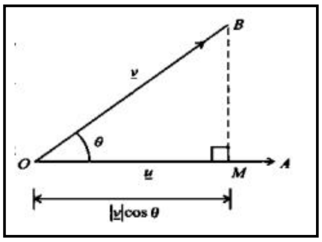
$$\underline{u}.\underline{v} = |\underline{u}||\underline{v}|\cos\theta \Rightarrow \cos\theta = \frac{\underline{u}.\underline{v}}{|\underline{u}||\underline{v}|}$$

Corollaries:

(i) If $\theta = 0 \; or \; \pi$, vectors $\underline{u}$ and $\underline{v}$ are collinear.

(ii) If $\theta = \frac{\pi}{2}$, then $\underline{u}.\underline{v} = 0$. Vectors $\underline{u}$ and $\underline{v}$ are perpendicular (orthogonal).

PROJECTION OF ONE VECTOR UPON ANOTHER VECTOR



Let $\overline{OA} = \underline{u}$ and $\overline{OB} = \underline{v}$ be two vectors such that θ is the angle between them and

$$0 \le \theta \le \pi$$

Draw a perpendicular $\overline{BM}$ on $\overline{OA}$ ($\overline{BM} \perp \overline{OA}$). Then $\overline{OM}$ is called projection of $\underline{v}$ along $\underline{u}$.

- Projection of $\underline{v}$ along $\underline{u} = \frac{\underline{u}.\underline{v}}{|\underline{u}|}$
- Projection of $\underline{u}$ along $\underline{v} = \frac{\underline{u}.\underline{v}}{|\underline{v}|}$

VECTOR PRODUCT OF TWO VECTORS

Definition 1:

Let $\underline{u}$ and $\underline{v}$ be two non-zero vectors**.** The **Cross or Vector** product of $\underline{u}$ and $\underline{v}$, written as $\underline{u} \times \underline{v}$, is defined by

$$\underline{u} \times \underline{v} = |\underline{u}||\underline{v}| \sin\theta\, \hat{n}$$

Where θ is the angle between $\underline{u}$ and $\underline{v}$ such that $0 \le \theta \le \pi$. And $\hat{n}$ is a unit vector perpendicular to the plane containing $\underline{u}$ and $\underline{v}$.

Definition 2:

If $\underline{u} = a_1\underline{i} + b_1\underline{j} + c_1\underline{k}$ and $\underline{v} = a_2\underline{i} + b_2\underline{j} + c_2\underline{k}$ are two non-zero vectors in space. The cross product $\underline{u} \times \underline{v}$ is defined by

$$\underline{u} \times \underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

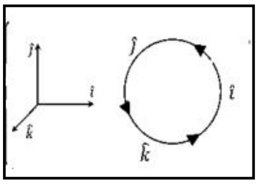
Note

The vector product is also referred to the **cross** product.

$$\underline{i} \times \underline{i} = \underline{j} \times \underline{j} = \underline{k} \times \underline{k} = 0$$

$$\underline{i} \times \underline{j} = \underline{k}, \quad \underline{j} \times \underline{k} = \underline{i}, \quad \underline{k} \times \underline{i} = \underline{j}$$

$$\underline{j} \times \underline{i} = -\underline{k}, \quad \underline{k} \times \underline{j} = -\underline{i}, \quad \underline{i} \times \underline{k} = -\underline{j}$$



PARALLEL VECTORS

Two non-zero **vectors** $\underline{u}$ and $\underline{v}$ are said to be parallel if and only if $\underline{u} \times \underline{v} = 0$.

$$\underline{u} \times \underline{v} = |\underline{u}||\underline{v}| \sin\theta\, \hat{n} = 0 \Rightarrow \theta = 0, \pi$$

Note

Zero Vector is both parallel and perpendicular to every vector.

PROPERTIES OF CROSS PRODUCT

Let $\underline{u}$, $\underline{v}$ and $\underline{w}$ be vectors and k be a real number, then

(i) $\underline{u} \times \underline{v} = 0 \Rightarrow \underline{u} = 0 \; or \; \underline{v} = 0$

(ii) $\underline{u} \times \underline{v} = -\underline{v} \times \underline{u}$ (Commutative Property)

(iii) $\underline{u} \times (\underline{v} + \underline{w}) = \underline{u} \times \underline{v} + \underline{u} \times \underline{w}$ (Distributive Property)

(iv) $\underline{u} \times (k\underline{v}) = (k\underline{u}) \times \underline{v} = k(\underline{u} \times \underline{v})$ (k is scalar)

(v) $\underline{u} \times \underline{u} = 0$

AREA OF PARALLELOGRAM

Let $\underline{u}$ and $\underline{v}$ be two non-zero vectors and θ is the angle between $\underline{u}$ and $\underline{v}$. Then $|\underline{u}|$ and $|\underline{v}|$ represents the length of adjacent sides of a parallelogram, then

$$\text{Area of Parallelogram} = \text{base} \times \text{height}$$

$$\text{Area of Parallelogram} = |\underline{u} \times \underline{v}|$$

AREA OF TRIANGLE

$$\text{Area of Triangle} = \frac{1}{2}(\text{Area of Parallelogram})$$

$$\text{Area of Triangle} = \frac{1}{2}|\underline{u} \times \underline{v}|$$

Where $\underline{u}$ *and* $\underline{v}$ are adjacent sides of a triangle.

SCALAR TRIPLE PRODUCT OF VECTORS

Let $\underline{u} = a_1\underline{i} + b_1\underline{j} + c_1\underline{k}$, $\underline{v} = a_2\underline{i} + b_2\underline{j} + c_2\underline{k}$ and $\underline{w} = a_3\underline{i} + b_3\underline{j} + c_3\underline{k}$ be three vectors.

The scalar triple product of vectors $\underline{u}$, $\underline{v}$ and $\underline{w}$ is defined by

$$\underline{u}.(\underline{v} \times \underline{w}) \text{ or } \underline{v}.(\underline{w} \times \underline{u}) \text{ or } \underline{w}.(\underline{u} \times \underline{v})$$

$$\underline{u}.(\underline{v} \times \underline{w}) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Note

The scalar triple product $\underline{u}.(\underline{v} \times \underline{w})$ is written as

$$\underline{u}.(\underline{v} \times \underline{w}) = [\underline{u} \quad \underline{v} \quad \underline{w}]$$

If we take vectors in cyclic order then

$$\underline{u}.(\underline{v} \times \underline{w}) = \underline{v}.(\underline{w} \times \underline{u}) = \underline{w}.(\underline{u} \times \underline{v})$$

The dot and cross are interchangeable.

$$\underline{u}.(\underline{v} \times \underline{w}) = \underline{u} \times (\underline{v}.\underline{w})$$

The value of product changes if the order is not cyclic.

VOLUME OF PARALLELEPIPED

The scalar triple product $\underline{u}.(\underline{v} \times \underline{w})$ represents the volume of parallelepiped having $\underline{u}$, $\underline{v}$ and $\underline{w}$ are conterminous edges.

$$\text{Volume of Parallelepiped} = \underline{u}.(\underline{v} \times \underline{w}) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

VOLUME OF TETRAHEDRON

The volume of tetrahedron ABCD having $\underline{u}$, $\underline{v}$ and $\underline{w}$ are conterminous edges is given by

$$\text{Volume of Tetrahedron} = \frac{1}{6}(\underline{u} \times \underline{v}).\underline{w} = \frac{1}{6}[\underline{u} \quad \underline{v} \quad \underline{w}]$$

PROPERTIES OF SCALAR TRIPLE PRODUCT

(i) If $\underline{u}$, $\underline{v}$ and $\underline{w}$ are coplanar then the scalar triple product of vectors is zero.

i.e., the vectors $\underline{u}$, $\underline{v}$ and $\underline{w}$ are coplanar $\Leftrightarrow$ $\underline{u}.(\underline{v} \times \underline{w}) = 0$

(ii) If any two vectors of scalar triple product are equal, then its value is zero.

i.e., $[\underline{u} \quad \underline{u} \quad \underline{w}] = [\underline{u} \quad \underline{v} \quad \underline{v}] = 0$

WORK DONE

If a constant force $\underline{F}$, applied to a body, acts at an angle θ to the direction of motion, then the work done by $\underline{F}$ is defined to be the product of the component of $\underline{F}$ in the direction of the displacement and the distance that the body moves.

$$\text{Work done} = (\text{Force})\ (\text{Dispalcement}) = \underline{F}.\underline{d}$$

$$\text{Work done} = \underline{F}.\overline{AB}$$

MOMENT OF FORCE

Let a force $\underline{F}$ act at a point P then moment of $\underline{F}$ about O is given by

$$\text{Moment of Force} = \overline{OP} \times \underline{F} = \underline{r} \times \underline{F}$$

MCQ's

Choose the correct Option.

1	A quantity which has only magnitude is called:						
a	Vector	b	Norm	c	Scalar	d	None
2	A quantity which has both magnitude and direction is called:						
a	Vector	b	Norm	c	Scalar	d	None
3	Mass, time and work are example of						
a	Vector	b	Norm	c	Scalar	d	None
4	Displacement, velocity and acceleration are examples of						
a	Vector	b	Norm	c	Scalar	d	None
5	Geometrically a vector is represented by						
a	$\|V\|$	b	$\hat{v}$	c	Directed line segment	d	None
6	The magnitude of a vector $\underline{v}$ is represented by						
a	$\|\underline{v}\|$	b	v	c	$\hat{v}$	d	Both a and b
7	The unit vector of a vector $\underline{v}$ is defined as						
a	$\|\underline{v}\|$	b	v	c	$\frac{\underline{v}}{\|\underline{v}\|}$	d	Both a and b
8	If terminal point B of a vector $\overrightarrow{AB}$ coincides with its initial point A, which is called						
a	Zero vector	b	Null vector	c	Parallel vector	d	Both a and b
9	Two vectors have same magnitude but opposite direction is called						
a	Zero vector	b	Unit vector	c	Negative of each other	d	Equal vector
10	Two vectors have same magnitude and same direction is called						
a	Zero vector	b	Unit vector	c	Negative of each other	d	Equal vector
11	Two non-zero vectors $\underline{u}$ and $\underline{v}$ are said to be parallel if						

a	$\underline{u} \times \underline{v} = 0$	b	$\underline{u} = k\underline{v}$	c	$\underline{u}.\underline{v} = 0$	d	Both a and b
12	If initial point of a vector is considered with origin is called ___ vector.						
a	Position	b	Parallel	c	Perpendicular	d	None
13	The set of all ordered pairs $[x, y]$ of real numbers, together with the rules of addition and scalar multiplication is called						
a	Vectors in R^2	b	Vectors in R^3	c	Vectors in R^n	d	None
14	If $\underline{v} = x\underline{i} + y\underline{j} + z\underline{k}$ then magnitude of $\underline{v}$ is define as						
a	$\sqrt{x^2 + y^2}$	b	$\sqrt{x^2 - y^2 - z^2}$	c	$\sqrt{x^2 + y^2 + z^2}$	d	None
15	If $\underline{u}$ and $\underline{v}$ be vectors then $\underline{u} + \underline{v} = \underline{v} + \underline{u}$ is known as _____ property.						
a	Associative	b	Commutative	c	Distributive	d	None
16	If $\underline{u}$ $\underline{v}$ and $\underline{w}$ be vectors then $(\underline{u} + \underline{v}) + \underline{w} = \underline{u} + (\underline{v} + \underline{w})$ is known as___ property.						
a	Associative	b	Commutative	c	Distributive	d	None
17	$\underline{i}$ is called unit vector along						
a	x-axis	b	y-axis	c	z-axis	d	xy-plane
18	$\underline{j}$ is called unit vector along						
a	x-axis	b	y-axis	c	z-axis	d	xy-plane
19	$\underline{k}$ is called unit vector along						
a	x-axis	b	y-axis	c	z-axis	d	xy-plane
20	If α be direction angle then						
a	$0° \leq \alpha \leq 90°$	b	$0° \leq \alpha \leq 180°$	c	$0° \leq \alpha < 90°$	d	None
21	If $\underline{v}$ be a vector and c be a real number then $\lvert c\underline{v} \rvert =$ __						
a	$c\underline{v}$	b	$\underline{v}$	c	$c\lvert\underline{v}\rvert$	d	$\lvert c \rvert \lvert \underline{v} \rvert$
22	If α, β and γ are the direction angles then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$ __						
a	0	b	-1	c	1	d	2
23	Direction cosines of x-axis is						
a	(1,0,0)	b	(0,1,0)	c	(0,0,1)	d	(1,1,1)
24	Direction cosines of y-axis is						
a	(1,0,0)	b	(0,1,0)	c	(0,0,1)	d	(1,1,1)
25	Direction cosines of z-axis is						
a	(1,0,0)	b	(0,1,0)	c	(0,0,1)	d	(1,1,1)

26	If $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ makes angles α, β and γ with x-axis, y-axis and z-axis then these angles are called:						
a	Direction ratio	b	Direction cosine	c	Angles of vector	d	Direction angles
27	If $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ makes angles α, β and γ with x-axis, y-axis and z-axis. Then the numbers $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called:						
a	Direction ratio	b	Direction cosine	c	Angles of vector	d	Direction angles
28	If $\underline{u}$ and $\underline{v}$ are two vectors and θ be angle between them, then their dot product is defined as $\underline{u}.\underline{v} =$ __						
a	$\underline{u}.\underline{v}\cos\theta$	b	$uv\cos\theta$	c	$\lvert\underline{u}\rvert\lvert\underline{v}\rvert\cos\theta$	d	Both b and c
29	If $\underline{u} = a_1\underline{i} + b_1\underline{j}$and $\underline{v} = a_2\underline{i} + b_2\underline{j}$.Then their dot product $\underline{u}.\underline{v}$ is defined by						
a	$a_1a_2 + b_1b_2$	b	$a_1b_1 + a_2b_2$	c	$a_1b_2 + a_2b_1$	d	$a_1a_2 - b_1b_2$
30	Dot product is also called						
a	Scalar product	b	Inner product	c	Cross product	d	Both a and b
31	$\underline{i}.\underline{i} =$ __						
a	0	b	-1	c	1	d	2
32	$\underline{k}.\underline{j} =$ __						
a	0	b	-1	c	1	d	2
33	Two non-zero vectors $\underline{u}$ and $\underline{v}$ are perpendicular iff						
a	$\underline{u} \times \underline{v} = 0$	b	$\underline{u}.\underline{v} = 0$	c	$\underline{u}.\underline{v} = 1$	d	None
34	If $2\underline{i} + \alpha\underline{j} + 5\underline{k}$ and $3\underline{i} + \underline{j} + \alpha\underline{k}$ are perpendicular then value of α is						
a	0	b	3	c	1	d	-1
35	Projection of $\underline{v}$ along $\underline{u}$ is define as						
a	$\frac{\underline{u}.\underline{v}}{\lvert\underline{u}\rvert}$	b	$\frac{\underline{u}.\underline{v}}{\lvert\underline{v}\rvert}$	c	$\frac{\underline{u}.\underline{v}}{\underline{u}}$	d	None
36	The Cross or Vector product of $\underline{u}$ and $\underline{v}$, written as $\underline{u} \times \underline{v}$, is defined by						
a	$\lvert\underline{u}\rvert\lvert\underline{v}\rvert\cos\theta$	b	$uv\cos\theta$	c	$\lvert\underline{u}\rvert\lvert\underline{v}\rvert\sin\theta\,\hat{n}$	d	None
37	$\underline{i} \times \underline{i} = \underline{j} \times \underline{j} = \underline{k} \times \underline{k} =$ __						
a	0	b	-1	c	1	d	$\underline{i}$
38	$\underline{i} \times \underline{j} =$ __						
a	$\underline{i}$	b	$\underline{j}$	c	$\underline{k}$	d	0
39	$\underline{j} \times \underline{i} =$ __						
a	$\underline{k}$	b	$\underline{j}$	c	$-\underline{k}$	d	0

40	If $\underline{u}$ and $\underline{v}$ be non-zero vectors then $\underline{u} \times \underline{v} =$ __						
a	$\underline{u}$	b	$\underline{v} \times \underline{u}$	c	$-\underline{v} \times \underline{u}$	d	None
41	If $\underline{u}$ be non-zero vectors then $\underline{u} \times \underline{u} =$ __						
a	$\underline{u}$	b	0	c	1	d	-1
42	Let $\underline{u}$ and $\underline{v}$ be two non-zero vectors. $\lvert\underline{u}\rvert$ and $\lvert\underline{v}\rvert$ represents the length of adjacent sides of a parallelogram, then area of parallelogram is						
a	$\underline{u} \times \underline{v}$	b	$\frac{1}{2}\lvert\underline{u} \times \underline{v}\rvert$	c	$\lvert\underline{u}.\underline{v}\rvert$	d	$\lvert\underline{u} \times \underline{v}\rvert$
43	Area of Triangle = __						
a	$\underline{u} \times \underline{v}$	b	$\frac{1}{2}\lvert\underline{u} \times \underline{v}\rvert$	c	$\lvert\underline{u}.\underline{v}\rvert$	d	$\lvert\underline{u} \times \underline{v}\rvert$
44	If $\underline{u}$, $\underline{v}$ and $\underline{w}$ are vectors then $\underline{u}.(\underline{v} \times \underline{w})$ is equal						
a	$\underline{u} \times (\underline{v} \times \underline{w})$	b	$[\underline{u} \quad \underline{v} \quad \underline{w}]$	c	$[\underline{u} \quad \underline{v} \quad \underline{v}]$	d	None
45	Magnitude of vector $\underline{v} = \underline{i} - \underline{j} - \underline{k}$ is:						
a	$\sqrt{1}$	b	$\sqrt{2}$	c	$\sqrt{3}$	d	$\sqrt{5}$
46	If $\underline{u} = 2\underline{i} + 7\underline{j} + 9\underline{k}$ then $\underline{u} \times \underline{u} =$						
a	$7\underline{j}$	b	0	c	-1	d	$3\underline{i} + 5\underline{j} + 19\underline{k}$
47	Volume of Parallelepiped = ___						
a	$\underline{u}.(\underline{v} \times \underline{w})$	b	$\underline{u} \times (\underline{v} \times \underline{w})$	c	$\frac{1}{6}[\underline{u} \quad \underline{v} \quad \underline{w}]$	d	$\lvert\underline{u} \times \underline{v}\rvert$
48	Volume of Tetrahedron = ____						
a	$\underline{u}.(\underline{v} \times \underline{w})$	b	$\underline{u} \times (\underline{v} \times \underline{w})$	c	$\frac{1}{6}[\underline{u} \quad \underline{v} \quad \underline{w}]$	d	$\lvert\underline{u} \times \underline{v}\rvert$
49	Zero vector is both parallel and perpendicular to						
a	Every vector	b	Only to unit vector	c	Position vector	d	Parallel vector
50	If a constant force $\underline{F}$ displaces the body A to B then work done is						
a	$\overline{OP} \times \underline{F}$	b	$\underline{F}.\overline{AB}$	c	$\underline{F} \times \overline{AB}$	d	None
51	Moment about O of a force $\underline{F}$ acting at a point P is						
a	$\overline{OP} \times \underline{F}$	b	$\underline{F}.\overline{AB}$	c	$\underline{F} \times \overline{AB}$	d	None
52	If $\lvert\alpha\underline{i} + (\alpha+1)\underline{j} + 2\underline{k}\rvert = 3$ then $\alpha =$ _						
a	1,2	b	$-1,-2$	c	$-1,2$	d	$1,-2$

53	A vector perpendicular to $\underline{a} = 2\underline{i} - \underline{j} + \underline{k}$ *and* $\underline{b} = 4\underline{i} + 2\underline{j} - \underline{k}$						
a	$-\underline{i} + 6\underline{j} + 8\underline{k}$	b	$-\underline{i} - 6\underline{j} + 8\underline{k}$	c	$2\underline{i} - \underline{j} + \underline{k}$	d	None
54	If $P = (2,3)$ and $Q = (6,-2)$ then $\overrightarrow{PQ} =$ __						
a	$4\underline{i} + 5\underline{j}$	b	$-4\underline{i} + 5\underline{j}$	c	$4\underline{i} - 5\underline{j}$	d	$5\underline{i} - 4\underline{j}$
55	The position vector of a point $P(-1,2,3)$ is:						
a	$\underline{i} + 2\underline{j} + 3\underline{k}$	b	$\underline{i} - 2\underline{j} - 3\underline{k}$	c	$-\underline{i} + 2\underline{j} + 3\underline{k}$	d	$-\underline{i} - 2\underline{j} - 3\underline{k}$
56	If $\underline{v} = -\frac{\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j}$, then $\lvert\underline{v}\rvert =$ __:						
a	1	b	0	c	$\frac{1}{2}$	d	4
57	$\lvert\cos\alpha\,\underline{i} + \sin\alpha\,\underline{j} + 0\underline{k}\rvert = ?$						
a	1	b	-1	c	0	d	2
58	If $\underline{i} - 3\underline{j} + \underline{k}$ and $\lambda\underline{i} + 6\underline{j} - 2\underline{k}$ are parallel then $\lambda = ?$						
a	-2	b	2	c	3	d	-3
59	If $P = (2,3)$ and $Q = (6,-2)$ then $\lvert\overrightarrow{PQ}\rvert =$ __						
a	$\sqrt{40}$	b	$\sqrt{41}$	c	$\sqrt{42}$	d	$\sqrt{43}$
60	Magnitude of the vector $2\underline{i} + 3\underline{j} + 4\underline{k}$ is:						
a	29	b	$\sqrt{29}$	c	28	d	$\sqrt{28}$
61	Magnitude of the vector $2\underline{i} - 3\underline{j} + \underline{k}$ is:						
a	$\sqrt{16}$	b	$\sqrt{15}$	c	$\sqrt{14}$	d	$\sqrt{13}$
62	Length of the vector $-\underline{i} + 2\underline{j} + 2\underline{k}$ is:						
a	3	b	4	c	5	d	6
63	Which is not unit vector.						
a	[1,0,0]	b	[0,1,0]	c	[0,0,1]	d	[1,1,1]
64	For a vector $\underline{v} = 2\underline{i} + 3\underline{j} - 6\underline{k}$, then $\cos\beta =$ __:						
a	$\frac{3}{7}$	b	$-\frac{3}{7}$	c	$\frac{2}{7}$	d	$-\frac{6}{7}$
65	Projection of $\underline{a} = \underline{i} - \underline{k}$ along $\underline{b} = \underline{j} + \underline{k}$ is:						
a	$-\frac{1}{\sqrt{2}}$	b	$\frac{3}{\sqrt{2}}$	c	$\frac{1}{\sqrt{2}}$	d	$\frac{1}{2}$
66	If the vectors $\underline{u} = 2\alpha\underline{i} + \underline{j} - \underline{k}$ and $\underline{v} = \underline{i} + \alpha\underline{j} + 4\underline{k}$ are perpendicular then $\alpha = ?$						

a	$\frac{1}{3}$	b	$\frac{2}{3}$	c	$\frac{4}{3}$	d	3
67	Projection of $\underline{u}$ along $\underline{v}$ is define as						
a	$\frac{\underline{u}.\underline{v}}{\lvert\underline{u}\rvert}$	b	$\frac{\underline{u}.\underline{v}}{\lvert\underline{v}\rvert}$	c	$\frac{\underline{u}.\underline{v}}{\underline{v}}$	d	None
68	Cosine of the angle between two non-zero vectors $\underline{a}$ and $\underline{b}$ is:						
a	$\frac{\underline{a}.\underline{b}}{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}$	b	$\frac{\underline{a}.\underline{b}}{\lvert\underline{b}\rvert}$	c	$\frac{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}{\underline{a}.\underline{b}}$	d	$\frac{\underline{a}\times\underline{b}}{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}$
69	Sine of the angle between two non-zero vectors $\underline{a}$ and $\underline{b}$ is:						
a	$\frac{\underline{a}.\underline{b}}{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}$	b	$\frac{\underline{a}.\underline{b}}{\lvert\underline{b}\rvert}$	c	$\frac{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}{\underline{a}.\underline{b}}$	d	$\frac{\underline{a}\times\underline{b}}{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}$
70	If two vectors $\underline{i}-\underline{j}+\alpha\underline{k}$ and $\underline{i}-2\underline{j}-3\underline{k}$ are perpendicular then value of α is						
a	-2	b	3	c	-1	d	1
71	The angle between the vectors $2\underline{i}+3\underline{j}+\underline{k}$ and $2\underline{i}-\underline{j}-\underline{k}$ is:						
a	30°	b	45°	c	60°	d	90°
72	The angle between the vectors $\underline{i}+\underline{j}$ and $\underline{i}-\underline{j}$ is:						
a	0°	b	$\frac{\pi}{4}$	c	π	d	$\frac{\pi}{2}$
73	If the vectors $\underline{u}=2\underline{i}+4\underline{j}-7\underline{k}$ and $\underline{v}=2\underline{i}+6\underline{j}+x\underline{k}$ are perpendicular then $x=?$						
a	-4	b	4	c	28	d	0
74	The angle between the vectors $4\underline{i}+2\underline{j}-\underline{k}$ and $-\underline{i}+\underline{j}-2\underline{k}$ is						
a	30°	b	45°	c	60°	d	90°
75	If the vectors $\underline{u}=\underline{i}+\alpha\underline{j}-\underline{k}$ and $\underline{v}=2\underline{i}+\underline{j}-\underline{k}$ are perpendicular then $\alpha=?$						
a	3	b	0	c	-3	d	1
76	If the vectors $\underline{u}=3\underline{i}-2\underline{j}+\underline{k}$ and $\underline{v}=2\underline{i}-\underline{j}+x\underline{k}$ are perpendicular then $x=?$						
a	-8	b	5	c	-1	d	1
77	Vector product of two vector is a:						
a	Scalar quantity	b	Unit vector	c	Vector quantity	d	Null vector
78	Scalar product of two vector is a:						
a	Scalar quantity	b	Unit vector	c	Vector quantity	d	Null vector
79	$[\underline{k}\quad\underline{i}\quad\underline{j}]=$ ____						
a	3	b	0	c	-1	d	1

80	$2\underline{i} \times 2\underline{j}.\underline{k} =$ ___						
a	2	b	4	c	-2	d	-4
81	$2\underline{i} \times \underline{j}.\underline{k} =$ ___						
a	2	b	0	c	-1	d	1
82	$[\underline{i} \quad \underline{i} \quad \underline{k}] = \underline{i}.\underline{i} \times \underline{k} =$ ___						
a	2	b	0	c	-1	d	1
83	If $\underline{u}$, $\underline{v}$ and $\underline{w}$ are coplanar then $\underline{u}.(\underline{v} \times \underline{w}) =$ __						
a	2	b	0	c	-1	d	1
84	$\underline{i}.\underline{k} \times \underline{j} =$ ___						
a	2	b	0	c	-1	d	1
85	$[\underline{i} \quad \underline{j} \quad \underline{k}] = \underline{i}.\underline{j} \times \underline{k} =$ ___						
a	2	b	0	c	-1	d	1
86	If $\underline{F} = 4\underline{i} + 3\underline{j} + 5\underline{k}$ and $\underline{d} = -\underline{i} + 3\underline{j} + 5\underline{k}$, then work done is:						
a	30 units	b	45 units	c	53 units	d	47 units
87	Length of the vector $2\underline{i} - \underline{j} - 2\underline{k}$ is:						
a	2	b	3	c	4	d	5
88	The non-zero vectors $\underline{a}$ and $\underline{b}$ are parallel iff $\underline{a} \times \underline{b} =$						
a	2	b	0	c	-1	d	1
89	If $\underline{a}$ and $\underline{b}$ are two non-zero vectors then angle between $\underline{a}$ and $\underline{a} \times \underline{b}$ is always:						
a	30°	b	45°	c	60°	d	90°
90	If any two vectors of scalar triple product are equal, then its value is:						
a	0	b	180	c	-1	d	1
91	$\underline{i} \times \underline{k} =$ ___						
a	$\underline{i}$	b	$\underline{j}$	c	$-\underline{j}$	d	1
92	Area of a triangle whose adjacent sides are $3\underline{i} + 4\underline{j}$ and $12\underline{i} + 9\underline{j}$ is:						
a	$\frac{45}{2}$	b	$\frac{55}{2}$	c	$\frac{21}{2}$	d	$\frac{75}{2}$
93	$[2\underline{k} \quad \underline{j} \quad \underline{i}]$ is equal to:						
a	-2	b	0	c	2	d	1
94	$(\underline{i} \times \underline{k}) \times \underline{j} =$ __						

a	0	b	1	c	$\underline{j}$	d	$-\underline{j}$
95	$2\underline{j}.(2\underline{k} \times \underline{i}) =$ ___						
a	**4**	b	1	c	2	d	-4

MCQ's Answers Key

1	2	3	4	5	6	7	8	9	10
c	a	c	a	c	d	c	d	c	d
11	12	13	14	15	16	17	18	19	20
d	a	a	c	b	a	a	b	c	b
21	22	23	24	25	26	27	28	29	30
d	c	a	b	c	d	b	d	a	d
31	32	33	34	35	36	37	38	39	40
c	a	b	d	a	c	a	c	c	c
41	42	43	44	45	46	47	48	49	50
b	d	b	b	c	b	a	c	a	b
51	52	53	54	55	56	57	58	59	60
a	d	a	c	c	a	a	a	b	b
61	62	63	64	65	66	67	68	69	70
c	a	d	a	a	c	b	a	d	d
71	72	73	74	75	76	77	78	79	80
d	d	b	d	c	a	c	a	d	b
81	82	83	84	85	86	87	88	89	90
a	b	b	c	d	a	b	b	d	a
91	92	93	94	95	96	97	98	99	100
c	c	a	a	a					

IMPORTANT SHORT QUESTIONS

1. Define Unit Vector.
2. Define Equal Vectors.
3. Define Position Vector.
4. Find the unit vector in the same direction as the vector $\underline{v} = [3, -4]$.
5. Find a unit vector in the direction of vector $\underline{v} = 2\underline{i} + 6\underline{j}$
6. Find a unit vector in the direction of vector $\underline{v} = [-2, 4]$.
7. Write the vector $\overrightarrow{PQ}$ in the form $x\underline{i} + y\underline{j}$. Where $P(2,3), Q(6, -2)$
8. Write the vector $\overrightarrow{PQ}$ in the form $x\underline{i} + y\underline{j}$. Where $P(0,5), Q(-1, -6)$
9. Find the magnitude of the vector $\underline{u} = \underline{i} + \underline{j}$.
10. Find the magnitude of the vector $\underline{u} = [3, -4]$.
11. If $\underline{u} = 2\underline{i} - 7\underline{j}$, $\underline{v} = \underline{i} - 6\underline{j}$ and $\underline{w} = -\underline{i} + \underline{j}$. Find $2\underline{u} - 3\underline{v} + 4\underline{w}$.
12. If $\underline{u} = 2\underline{i} - 7\underline{j}$, $\underline{v} = \underline{i} - 6\underline{j}$ and $\underline{w} = -\underline{i} + \underline{j}$. Find $\frac{1}{2}\underline{u} + \frac{1}{2}\underline{v} + \frac{1}{2}\underline{w}$.
13. Find the sum of the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$, given the four points $A(1, -1), B(2, 0), C(-1, 3)$ and $D(-2, 2)$.
14. Find the vector from the point A to the origin where $\overrightarrow{AB} = 4\underline{i} - 2\underline{j}$ and B is the point $(-2, 5)$.
15. Find a unit vector in the direction of vector $\underline{v} = 2\underline{i} - \underline{j}$.
16. Find a unit vector in the direction of vector $\underline{v} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$
17. Find a unit vector in the direction of vector $\underline{v} = -\frac{\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j}$.
18. If O is the origin and $\overrightarrow{OP} = \overrightarrow{AB}$ find the point P when A and B are $(-3, 7)$ and $(1, 0)$ respectively.
19. If $\overrightarrow{AB} = \overrightarrow{CD}$ find the coordinates of the point A when points B, C, D are $(1, 2), (-2, 5), (4, 11)$ respectively.
20. Find the position vectors of the point of division of the line segments joining the following pair of points, in the given ratio: Point C with position vector $2\underline{i} - \underline{3j}$. and point D with position vector $3\underline{i} + 2\underline{j}$.in the ratio $4 : 3$.
21. If $\underline{u} = 2\underline{i} + 3\underline{j} + \underline{k}$, $\underline{v} = 4\underline{i} + 6\underline{j} + 2\underline{k}$. Then find $\underline{u} + 2\underline{v}$ and $|\underline{u} + 2\underline{v}|$.
22. Define Direction Angles.
23. Define Direction Cosines of a Vector. Find direction cosines of $\vec{v} = x\underline{i} + y\underline{j} + \underline{k}$.
24. Prove that $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$.
25. Let $A = (2, 5), B = (-1,1)$ and $C = (2, -6)$. Find *(i)* $\overrightarrow{AB}$ *(ii)* $2\overrightarrow{CB} - 2\overrightarrow{CA}$
26. If $\underline{u} = \underline{i} + 2\underline{j} - \underline{k}$, $\underline{v} = 3\underline{i} - 2\underline{j} + 2\underline{k}$ and $\underline{w} = 5\underline{i} - \underline{j} + 3\underline{k}$. Then find

    (i) $|\underline{u} + 2\underline{v}|$ (ii) $\underline{v} - 3\underline{w}$ (iii) $|3\underline{v} + \underline{w}|$
27. Find magnitude and direction cosines of $\underline{v} = 2\underline{i} + 3\underline{j} + 4\underline{k}$.
28. Find magnitude and direction cosines of $\underline{v} = \underline{i} - \underline{j} - \underline{k}$.

29. Find magnitude and direction cosines of $\underline{v} = 4\underline{i} - 5\underline{j}$.

30. Find α, so that $\left|\alpha\underline{i} + (\alpha+1)\underline{j} + 2\underline{k}\right| = 3$.

31. Find a unit vector in the direction of vector $\underline{v} = \underline{i} + 2\underline{j} - \underline{k}$.

32. Find a vector whose magnitude is 4 and is parallel to $2\underline{i} - 3\underline{j} + 6\underline{k}$.

33. Find a vector whose magnitude is 2 and is parallel to $-\underline{i} + \underline{j} + \underline{k}$.

34. Find the constant α, so that the vectors $\underline{v} = \underline{i} - 3\underline{j} + 4\underline{k}$ and $\underline{w} = \alpha\underline{i} + 9\underline{j} - 12\underline{k}$ are parallel.

35. Find a vector of length 5 in the direction opposite that of $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$.

36. Find a and b, so that the vectors $3\underline{i} - \underline{j} + 4\underline{k}$ and $a\underline{i} + b\underline{j} - 2\underline{k}$ are parallel.

37. Find the direction cosines of $\underline{v} = 3\underline{i} - \underline{j} + 2\underline{k}$.

38. Find the direction cosines of $\underline{v} = 6\underline{i} - 2\underline{j} + \underline{k}$.

39. Find the direction cosines of $\overrightarrow{PQ}$, where $P = (2,1,5)$ and $Q = (1,3,1)$.

40. Is $45°, 45°, 60°$ can be the direction angles of a single vector.

41. Define Scalar Product of two vectors.

42. Define Perpendicular (Orthogonal) Vectors.

43. Write properties of Dot Product.

44. Find the angle between the vectors $\underline{u} = 2\underline{i} - \underline{j} + \underline{k}$ and $\underline{v} = -\underline{i} + \underline{j}$.

45. Find a scalar α so that the vectors $2\underline{i} + \alpha\underline{j} + 5\underline{k}$ and $3\underline{i} + \underline{j} + \alpha\underline{k}$ are perpendicular.

46. Find cosine of the angle between $\underline{u} = 3\underline{i} + \underline{j} - \underline{k}$ and $\underline{v} = 2\underline{i} - \underline{j} + \underline{k}$.

47. Find cosine of the angle between $\underline{u} = \underline{i} - 3\underline{j} + 4\underline{k}$ and $\underline{v} = 4\underline{i} - \underline{j} + 3\underline{k}$.

48. Find cosine of the angle between $\underline{u} = [-3,5]$ and $\underline{v} = [6,-2]$.

49. Find cosine of the angle between $\underline{u} = [2,-3,1]$ and $\underline{v} = [2,4,1]$.

50. Calculate the projection of $\underline{a}$ along $\underline{b}$ and projection of $\underline{b}$ along $\underline{a}$ when

$$\underline{a} = \underline{i} - \underline{k}\,, \quad \underline{b} = \underline{j} + \underline{k}$$

51. Calculate the projection of $\underline{a}$ along $\underline{b}$ and projection of $\underline{b}$ along $\underline{a}$ when

$$\underline{a} = 3\underline{i} + \underline{j} - \underline{k}\,, \quad \underline{b} = -2\underline{i} - \underline{j} + \underline{k}$$

52. Find a scalar α so that the vectors $\underline{u} = 2\alpha\underline{i} + \underline{j} - \underline{k}$ and $\underline{v} = \underline{i} + \alpha\underline{j} + 4\underline{k}$ are perpendicular.

53. Find a scalar α so that the vectors $\underline{u} = \alpha\underline{i} + 2\alpha\underline{j} - \underline{k}$ and $\underline{v} = \underline{i} + \alpha\underline{j} + 3\underline{k}$ are perpendicular.

54. If $\underline{v}$ is a vector for which $\underline{v}.\underline{i} = 0$, $\underline{v}.\underline{j} = 0$, $\underline{v}.\underline{k} = 0$, find $\underline{v}$.

55. Show that the vectors $3\underline{i} - 2\underline{j} + \underline{k}$, $\underline{i} - 3\underline{j} + 5\underline{k}$ and $2\underline{i} + \underline{j} - 4\underline{k}$ from a right angle.

56. Show that the set of points $P = (1,3,2), Q(4,1,4)$ and $R = (6,5,5)$ from a right triangle.

57. Define Cross Product of two vectors.

58. Write properties of Cross Product.

59. Find a vector perpendicular to each of the vectors $\underline{a} = 2\underline{i} - \underline{j} + \underline{k}$ *and* $\underline{b} = 4\underline{i} + 2\underline{j} - \underline{k}$.

60. Find area of the parallelogram whose vertices are $P(0,0,0), Q(-1,2,4), R(2,-1,4)$ and $S(1,1,8)$.

61. Compute the cross product $\underline{a} \times \underline{b}$ and $\underline{b} \times \underline{a}$. If $\underline{a} = 2\underline{i} + \underline{j} - \underline{k}\,, \underline{b} = \underline{i} - \underline{j} + \underline{k}$.

62. Compute the cross product $\underline{a}\times\underline{b}$ and $\underline{b}\times\underline{a}$. If $\underline{a}=\underline{i}+\underline{j}, \underline{b}=\underline{i}-\underline{j}$.

63. Compute the cross product $\underline{a}\times\underline{b}$ and $\underline{b}\times\underline{a}$. If $\underline{a}=3\underline{i}-2\underline{j}+\underline{k}, \underline{b}=\underline{i}+\underline{j}$.

64. Compute the cross product $\underline{a}\times\underline{b}$ and $\underline{b}\times\underline{a}$. If $\underline{a}=-4\underline{i}+\underline{j}-2\underline{k}, \underline{b}=2\underline{i}+\underline{j}+\underline{k}$.

65. Find a unit vector perpendicular to the plane containing $\underline{a}$ and $\underline{b}$. If $\underline{a}=\underline{i}+\underline{j}, \underline{b}=\underline{i}-\underline{j}$.

66. Find a unit vector perpendicular to the plane containing $\underline{a}$ and $\underline{b}$.
If $\underline{a}=-\underline{i}-\underline{j}-\underline{k}, \underline{b}=2\underline{i}-3\underline{j}+4\underline{k}$.

67. Find a unit vector perpendicular to the plane containing $\underline{a}$ and $\underline{b}$.
If $\underline{a}=2\underline{i}-2\underline{j}+4\underline{k}, \underline{b}=-\underline{i}+\underline{j}-2\underline{k}$.

68. Find area of the triangle PQR with vertices $P(0,0,0), Q(2,3,2), R(-1,1,4)$.

69. Find area of parallelogram whose vertices are $A(0,0,0), B(1,2,3), C(2,-1,1), D(3,1,4)$

70. Find area of parallelogram whose vertices are

$$A(1,2,-1), B(4,2,-3), C(6,-5,2), D(9,-5,0)$$

71. Find area of parallelogram whose vertices are

$$A(-1,1,1), B(-1,2,2), C(-3,4,-5), D(-3,5,-4)$$

72. Which vectors, if any, are perpendicular or parallel

$$\underline{u}=5\underline{i}-\underline{j}+\underline{k}, \quad \underline{v}=\underline{j}-5\underline{k}, \quad \underline{w}=-15\underline{i}+3\underline{j}-3\underline{k}$$

73. Prove that $\underline{a}\times(\underline{b}+\underline{c})+\underline{b}\times(\underline{c}+\underline{a})+\underline{c}\times(\underline{a}+\underline{b})=0$.

74. If $\underline{a}+\underline{b}+\underline{c}=0$, then prove that $\underline{a}\times\underline{b}=\underline{b}\times\underline{c}=\underline{c}\times\underline{a}$.

75. If $\underline{a}\times\underline{b}=0$ and $\underline{a}\cdot\underline{b}=0$, what conclusion can be drawn from $\underline{a}$ or $\underline{b}$?

76. Define Scalar Triple Product.

77. Write formula to find volume of parallelepiped and tetrahedron.

78. Find volume of the parallelepiped determined by

$$\underline{u}=\underline{i}+2\underline{j}-\underline{k}, \quad \underline{v}=\underline{i}-2\underline{j}+3\underline{k}, \quad \underline{w}=\underline{i}-7\underline{j}-4\underline{k}$$

79. Find the value of α so that the vectors $\alpha\underline{i}+\underline{j}$, $\underline{i}+\underline{j}+3\underline{k}$ and $2\underline{i}+\underline{j}-2\underline{k}$ are coplanar.

80. Find the work done by a constant force $F = 2\underline{i}+4\underline{j}$, if its points of application to a body moves it from $A(1,1)$ to $B(4,6)$.

81. Find volume of the parallelepiped with the given vectors

$$\underline{u}=3\underline{i}+2\underline{k}, \quad \underline{v}=\underline{i}+2\underline{j}+\underline{k}, \quad \underline{w}=-\underline{j}+4\underline{k}$$

82. Find volume of the parallelepiped with the given vectors

$$\underline{u}=\underline{i}-4\underline{j}-\underline{k}, \quad \underline{v}=\underline{i}-\underline{j}-2\underline{k}, \quad \underline{w}=2\underline{i}-3\underline{j}+\underline{k}$$

83. Find volume of the parallelepiped with the given vectors

$$\underline{u}=\underline{i}-2\underline{j}+3\underline{k}, \quad \underline{v}=2\underline{i}-\underline{j}-\underline{k}, \quad \underline{w}=\underline{j}+\underline{k}$$

84. Prove that the vectors $\underline{i}-2\underline{j}+3\underline{k}$, $-2\underline{i}+3\underline{j}-4\underline{k}$ and $\underline{i}-3\underline{j}+5\underline{k}$ are coplanar.

85. Find the constant α so that the vectors $\underline{i}-\underline{j}+\underline{k}$, $\underline{i}-2\underline{j}-3\underline{k}$ and $3\underline{i}-\alpha\underline{j}+5\underline{k}$ are coplanar.

86. Find the constant α so that the vectors $\underline{i}-2\alpha\underline{j}-\underline{k}$, $\underline{i}-\underline{j}+2\underline{k}$ and $\alpha\underline{i}-2\underline{j}+\underline{k}$ are coplanar.

87. Find the value of $2\underline{i}\times 2\underline{j}.\underline{k}$.

88. Find the value of $3\underline{j}.\underline{k}\times\underline{i}$.

89. Find the value of $[\underline{k} \quad \underline{i} \quad \underline{j}]$.

90. Find the value of $[\underline{i} \quad \underline{i} \quad \underline{k}]$.

91. Prove that $\underline{u}.(\underline{v} \times \underline{w}) + \underline{v}.(\underline{w} \times \underline{u}) + \underline{w}.(\underline{u} \times \underline{v}) = 3\underline{u}.(\underline{v} \times \underline{w})$.

92. Find the work done, if the point at which the constant force $\underline{F} = 4\underline{i} + 3\underline{j} + 5\underline{k}$ is applied to an object, moves from $P_1(3,1,-2)$ to $P_2(2,4,6)$.

93. A force $\underline{F} = 3\underline{i} + 2\underline{j} - 4\underline{k}$ is applied at the point $(1,-1,2)$. Find the moment of the force about the point $(2,-1,3)$.

94. A force $\underline{F} = 4\underline{i} - 3\underline{k}$, passes through the point $A(2,-2,5)$. Find the moment of F about the point $B(1,-3,1)$.

95. Given a force $\underline{F} = 2\underline{i} + \underline{j} - 3\underline{k}$ acting at a point $A(1,-2,1)$. Find the moment of F about the point $B(2,0,-2)$.

96. A force $\underline{F} = 7\underline{i} + 4\underline{j} - 3\underline{k}$ is applied at $P(1,-2,3)$. Find its moment about the point $Q(2,1,1)$.

IMPORTANT LONG QUESTIONS

1. Use vectors, to prove that the diagonals of a parallelogram bisect each other.
2. Prove that the line segment joining the mid points of two sides of a triangle is parallel to the third side and half as long.
3. Prove that the line segments joining the mid points of the sides of a quadrilateral taken in order form a parallelogram.
4. The position vectors of the points A, B, C and D are $2\underline{i} - \underline{j} + \underline{k}$, $3\underline{i} + \underline{j}$, $2\underline{i} + 4\underline{j} - 2\underline{k}$ and $-\underline{i} - 2\underline{j} + \underline{k}$ respectively. Show that $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$.
5. Using Vectors, prove that in any triangle ABC, $a^2 = b^2 + c^2 - 2bc\cos A$.
6. Using Vectors, prove that in any triangle ABC, $a = b\cos C + c\cos B$.
7. Using Vectors, prove that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
8. Show that midpoint of hypotenuse a right triangle is equidistant from its vertices.
9. Prove that perpendicular bisectors of the sides of a triangle are concurrent.
10. Prove that the altitudes of a triangle are concurrent.
11. Prove that the angle in a semi-circle is a right angle.
12. Using Vectors, prove that $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
13. Using Vectors, prove that in any triangle ABC, $c = a\cos B + b\cos A$.
14. Using Vectors, prove that in any triangle ABC, $b^2 = c^2 + a^2 - 2ca\cos B$.
15. Using Vectors, prove that in any triangle ABC, $c^2 = a^2 + b^2 - 2ab\cos C$.
16. If $\underline{a} = 4\underline{i} + 3\underline{j} + \underline{k}$ and $\underline{b} = 2\underline{i} - \underline{j} + 2\underline{k}$. Find a unit vector perpendicular to both $\underline{a}$ and $\underline{b}$. Also find the sine of the angle between the vectors $\underline{a}$ and $\underline{b}$.
17. Using Vectors, prove that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
18. Using Vectors, in any triangle ABC, prove that
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
19. Find the area of the triangle with vertices $A(1, -1, 1), B(2, 1, -1)$ and $C(-1, 1, 2)$. Also find a unit vector perpendicular to the plane ABC.
20. Find a unit vector perpendicular to the plane containing $\underline{a}$ and $\underline{b}$. Also find the sine of the angle between the vectors $\underline{a}$ and $\underline{b}$. If $\underline{a} = 2\underline{i} - 6\underline{j} - 3\underline{k}$ and $\underline{b} = 4\underline{i} + 3\underline{j} - \underline{k}$.
21. Which vectors, if any, are perpendicular or parallel
$$\underline{u} = \underline{i} + 2\underline{j} - \underline{k},\ \underline{v} = -\underline{i} + \underline{j} + \underline{k},\ \underline{w} = -\frac{\pi}{2}\underline{i} - \pi\underline{j} + \frac{\pi}{2}\underline{k}$$
22. Using Vectors, prove that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
23. Prove that four points $A(-3, 5, -4), B(-1, 1, 1), C(-1, 2, 2)$ and $D(-3, 4, -5)$ are coplanar.
24. Find the volume of the tetrahedron whose vertices are $A(2, 1, 8), B(3, 2, 9), C(2, 1, 4)$ and $D(3, 3, 0)$.
25. Prove that the points whose position vectors are $A\left(-6\underline{i} + 3\underline{j} + 2\underline{k}\right), B\left(3\underline{i} - 2\underline{j} + 4\underline{k}\right)$, $C\left(5\underline{i} + 7\underline{j} + 3\underline{k}\right), D(-13\underline{i} + 17\underline{j} - \underline{k})$ are coplanar.
26. Find volume of the Tetrahedron with the vertices $(0, 1, 2), (3, 2, 1), (1, 2, 1)$ and $(5, 5, 6)$.
27. Find volume of the Tetrahedron with the vertices $(2, 1, 8), (3, 2, 9), (2, 1, 4)$ and $(3, 3, 10)$.

28. A particle, acted by constant forces $4\underline{i}+\underline{j}-3\underline{k}$ and $3\underline{i}-\underline{j}-\underline{k}$, is displaced from $A(1,2,3)$ to $B(5,4,1)$. Find the work done.

29. A force of magnitude 6 units acting parallel to $2\underline{i}-2\underline{j}+\underline{k}$ displaces, the point of application from $(1,2,3)$ to $(5,3,7)$. Find the work done.

……THE END……

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