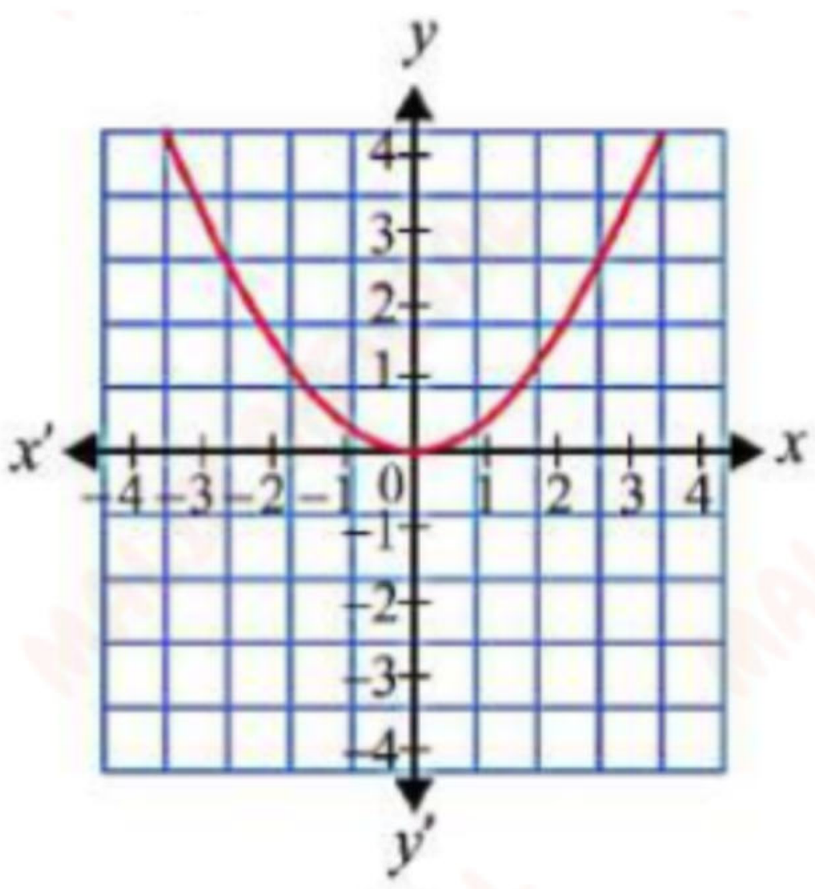
Exercise 2.1

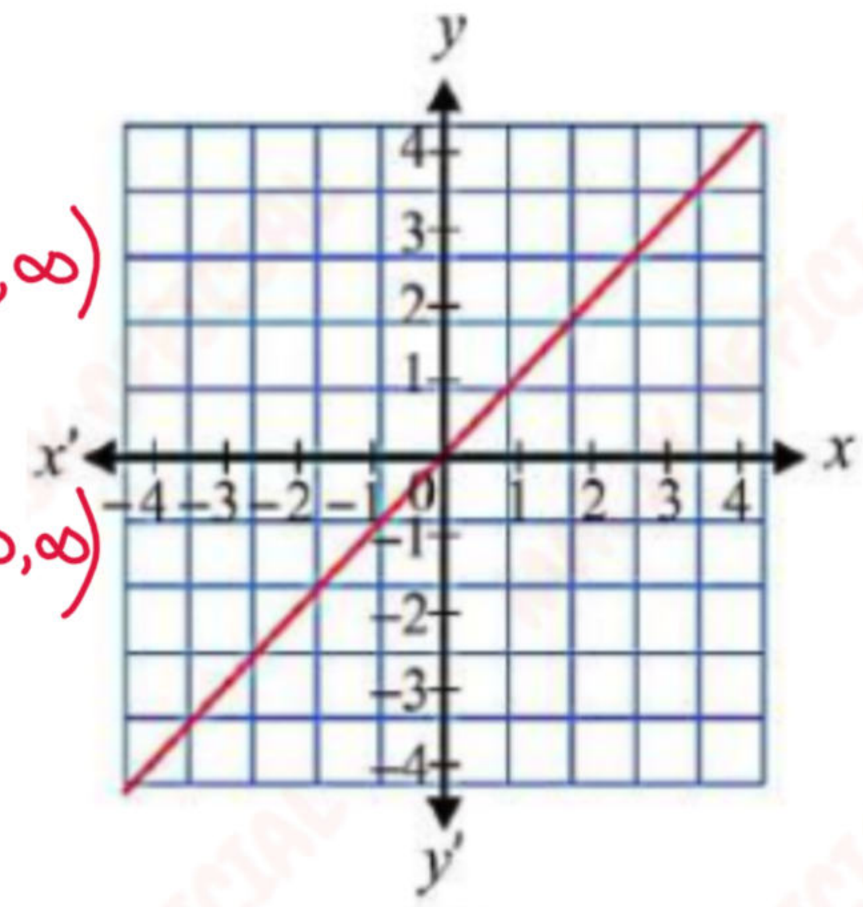
1. Identify the domain and range of the functions through following graph.



(i)

$$\text{Domain} = \{x \mid x \in \mathbb{R}\} = \mathbb{R} = (-\infty, \infty)$$

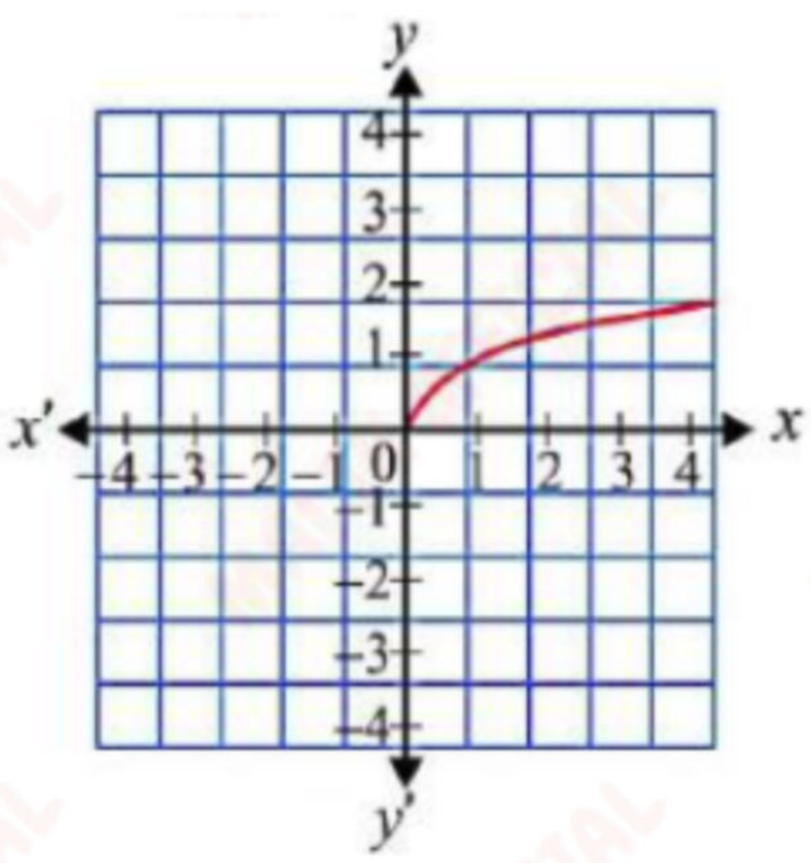
$$\text{Range} = \{y \mid y \geq 0\} = [0, \infty)$$



(ii)

$$\text{Domain} = \{x \mid x \in \mathbb{R}\} = \mathbb{R} = (-\infty, \infty)$$

$$\text{Range} = \{y \mid y \in \mathbb{R}\} = \mathbb{R} = (-\infty, \infty)$$

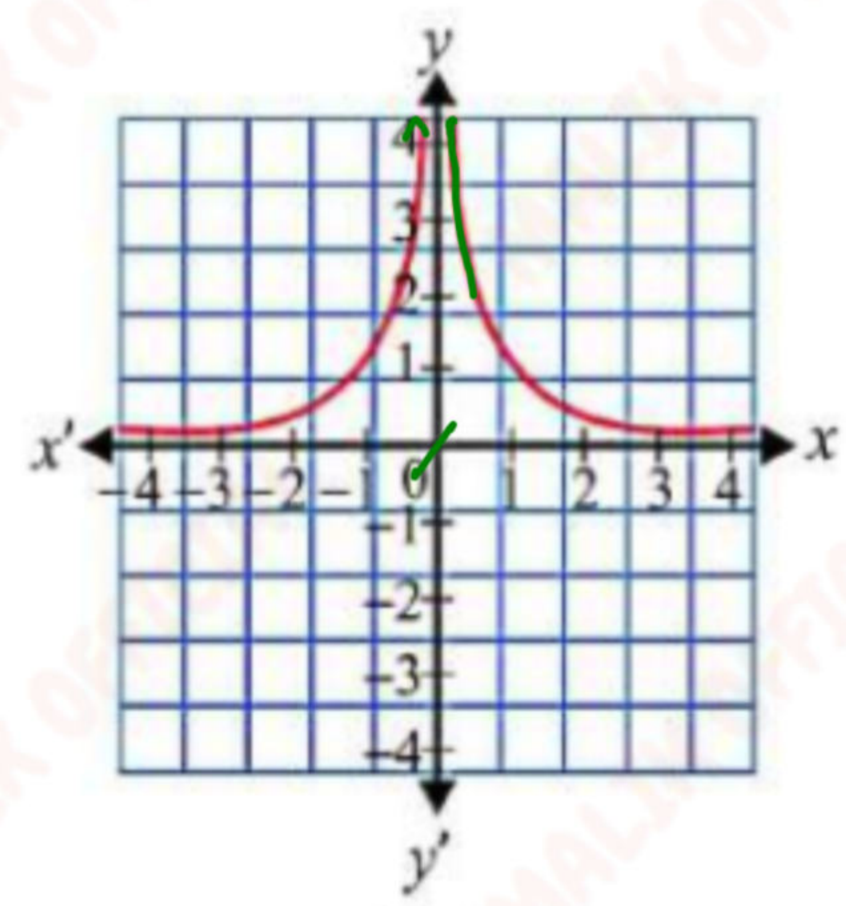


(iii)

$$\text{Domain} = \{x \mid x \in \mathbb{R} \wedge x \geq 0\}$$

$$= \{x \mid x \geq 0\} = [0, \infty)$$

$$\text{Range} = \{y \mid y \in \mathbb{R} \wedge y \geq 0\} = [0, \infty)$$



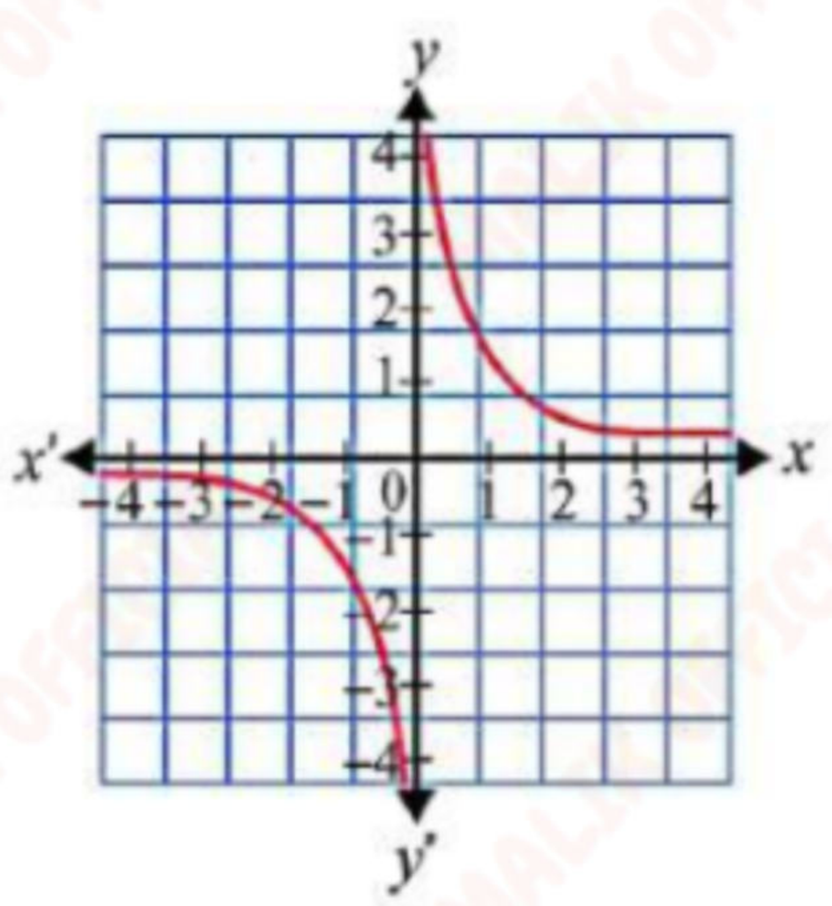
(iv)

$$\text{Domain} = \{x \mid x \in \mathbb{R} \wedge x \neq 0\}$$

$$= \mathbb{R} - \{0\} = (-\infty, 0) \cup (0, \infty)$$

$$\text{Range} = \{y \mid y \in \mathbb{R} \wedge y > 0\}$$

$$= (0, \infty)$$



(v)

$$\text{Domain} = \{x \mid x \in \mathbb{R} \wedge x \neq 0\} = \mathbb{R} - \{0\}$$

$$= (-\infty, 0) \cup (0, \infty).$$

$$\text{Range} = \{y \mid y \in \mathbb{R} \wedge y \neq 0\} = \mathbb{R} - \{0\}$$

$$= (-\infty, 0) \cup (0, \infty).$$

2. If $f(x) = 5x + 2$ and $g(x) = 2x^2 - 3$, then find

(i) fog (ii) gof (iii) fof (iv) gog

(i) $fog(x) = f\left(g(x)\right) = 5g(x) + 2 = 5\left(2x^2 - 3\right) + 2$

$= 10x^2 - 15 + 2$

$= 10x^2 - 13$

(ii) $gof(x) = g\left(f(x)\right) = 2\left(f(x)\right)^2 - 3 = 2\left(5x + 2\right)^2 - 3$

$= 2\left(25x^2 + 4 + 20x\right) - 3$

$= 50x^2 + 8 + 40x - 3$

$= 50x^2 + 40x + 5$

(iii) $fof(x) = f\left(f(x)\right) = 5f(x) + 2$

$= 5\left(5x + 2\right) + 2$

$= 25x + 10 + 2 = 25x + 12$

(iv) $gog(x) = g\left(g(x)\right) = 2g(x)^2 - 3$

$= 2\left(2x^2 - 3\right)^2 - 3$

$= 2\left(4x^4 + 9 - 12x^2\right) - 3$

$= 8x^4 + 18 - 24x^2 - 3$

$= 8x^4 - 24x^2 + 15$

3. If $f(x) = 2x$ and $g(x) = x + 1$, then find $fog(x)$ for $x = -5$.

$$fog(x) = f(g(x)) = 2\,g(x)$$
$$= 2(x+1)$$
$$= 2x + 2$$

$$fog(-5) = 2(-5) + 2$$
$$= -10 + 2$$
$$= -8$$

4. If $f(x) = x + 3$ and $g(x) = x^2$, then find $gof(x)$ for $x = 1$.

$$gof(x) = g(f(x)) = (f(x))^2$$
$$= (x+3)^2$$
$$= x^2 + 9 + 6x$$
$$gof(x) = x^2 + 6x + 9$$

$$gof(1) = (1)^2 + 6(1) + 9$$
$$= 1 + 6 + 9 = 16$$

5. If $c(x) = \cos x$ and $p(x) = x^3 + 1$ then find $poc(x)$.

$$poc(x) = p(c(x)) = (c(x))^3 + 1$$
$$= (\cos x)^3 + 1$$
$$= \cos^3 x + 1.$$

6. Given that $f(x) = x + 2$ and $g(x) = 3x - 2$ are two given functions then find $(fog)^{-1}$ and $(gof)^{-1}$ also show that $(gof)^{-1} = f^{-1}og^{-1}$.

$fog(x) = f(g(x)) = g(x) + 2 = (3x - 2) + 2 = 3x - \cancel{2} + \cancel{2} = 3x$

Let $h(x) = fog(x)$.

$h(x) = 3x$

Suppose $y = h(x)$

$\Rightarrow h^{-1}(y) = x$

$y = 3x$

$\frac{y}{3} = x$

$x = \frac{y}{3}$

$h^{-1}(y) = \frac{y}{3}$

Replace y with x

$h^{-1}(x) = \frac{x}{3} \Rightarrow (fog)^{-1} = \frac{x}{3}$.

$gof(x) = g(f(x)) = 3f(x) - 2 = 3(x+2) - 2 = 3x + 6 - 2 = 3x + 4$

Let $h_1(x) = (g \circ f)(x)$

$h_1(x) = 3x + 4$

Suppose $y = h_1(x)$

$\Rightarrow h_1^{-1}(y) = x$

$y = 3x + 4$

$3x = y - 4$

$x = \frac{y-4}{3}$

$h_1^{-1}(y) = \frac{y-4}{3}$

Replace y with x

$h_1^{-1}(x) = \frac{x-4}{3} \Rightarrow (g \circ f)^{-1} = \frac{x-4}{3}$.

$f(x) = x+2$,

Let $y = f(x) \Rightarrow f^{-1}(y) = x$

$y = x+2$

$y - 2 = x$

$x = y - 2$

$f^{-1}(y) = y - 2$

Replace y with x

$f^{-1}(x) = x - 2$

$g(x) = 3x - 2$

Let $y = g(x) \Rightarrow g^{-1}(y) = x$

$y = 3x - 2$

$y + 2 = 3x$

$x = \frac{y+2}{3}$

$g^{-1}(y) = \frac{y+2}{3}$

Replace y with x

$g^{-1}(x) = \frac{x+2}{3}$

$$(f^{-1} \circ g^{-1})(x) = f^{-1}(g^{-1}(x)) = g^{-1}(x) - 2$$

$$= \frac{x+2}{3} - 2 = \frac{x+2-6}{3}$$

$$= \frac{x-4}{3}$$

Hence proved that

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}.$$

7. Given that $h(x) = x - 3$ and $k(x) = 2x + 5$ are two functions then verify that:
(i) $hok \neq koh$ (ii) $(hok)^{-1} = k^{-1}oh^{-1}$ (iii) $(koh)^{-1} = h^{-1}ok^{-1}$

L.H.S

(i) $h \circ k(x) = h(k(x)) = k(x) - 3 = 2x + 5 - 3 = 2x + 2$

R.H.S

$koh(x) = k(h(x)) = 2h(x) + 5 = 2(x-3) + 5 = 2x - 6 + 5 = 2x - 1$

So $hok \neq koh.$

(ii) L.H.S $hok(x) = h(k(x)) = 2x + 2$

Let $f(x) = hok(x)$

$f(x) = 2x + 2$ Suppose $y = f(x) \Rightarrow f^{-1}(y) = x$

$y = 2x + 2$

$y - 2 = 2x \Rightarrow x = \dfrac{y-2}{2} \Rightarrow f^{-1}(y) = \dfrac{y-2}{2}$

$f^{-1}(x) = \dfrac{x-2}{2}$

$\Rightarrow (hok)^{-1} = \dfrac{x-2}{2}$

R.H.S

$k(x) = 2x + 5$ $h(x) = x - 3$

$k^{-1}(x) = \dfrac{x-5}{2}$ $h^{-1}(x) = x + 3$

$$k^{-1} o h^{-1}(x) = k^{-1}(h^{-1}(x)) = \frac{h^{-1}(x) - 5}{2} = \frac{x + 3 - 5}{2}$$

$$= \frac{x-2}{2}$$

So $(hok)^{-1} = k^{-1} o h^{-1}.$

(iii) L.H.S

$$k \circ h(x) = 2x - 1$$

$$(k \circ h)^{-1}(x) = \frac{x+1}{2}$$

R.H.S

$h(x) = x - 3$, $k(x) = 2x + 5$

$h^{-1}(x) = x + 3$ $\quad$ $k^{-1}(x) = \frac{x-5}{2}$

$$h^{-1} \circ k^{-1}(x) = h^{-1}\left(k^{-1}(x)\right)$$

$$= k^{-1}(x) + 3 = \frac{x-5}{2} + 3$$

$$= \frac{x-5+6}{2} = \frac{x+1}{2}$$

Hence proved.

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YouTube Suppose Math.