# Chapter # 1

# REAL NUMBERS

## Exercise # 1.2

**Question # 1**: **Rationalize the denominator of the following:**

**(i).** $\frac{13}{4+\sqrt{3}}$

$= \frac{13}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}}$ $\quad \because a^2-b^2=(a+b)(a-b)$

$= \frac{13(4-\sqrt{3})}{(4)^2-(\sqrt{3})^2}$

$= \frac{13(4-\sqrt{3})}{16-3}$

$= \frac{13(4-\sqrt{3})}{13}$

$= 4-\sqrt{3}$ (Answer)

**(ii).** $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

$= \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{(\sqrt{2\times3}+\sqrt{5\times3})}{(\sqrt{3})^2}$

$= \frac{(\sqrt{6}+\sqrt{15})}{3}$ (Answer)

**(iii).** $\frac{\sqrt{2}-1}{\sqrt{5}}$

$= \frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{(\sqrt{2\times5}-\sqrt{5})}{(\sqrt{5})^2}$

$= \frac{(\sqrt{10}-\sqrt{5})}{5}$ (Answer)

**(iv).** $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

$= \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}}$

$= \frac{(6-4\sqrt{2})^2}{(6)^2-(4\sqrt{2})^2}$ $\quad \because (a-b)^2=a^2+b^2-2ab$, $a^2-b^2=(a+b)(a-b)$

$= \frac{(6)^2+(4\sqrt{2})^2-2(6)(4\sqrt{2})}{36-(16\times2)}$

$= \frac{36+(16\times2)-48\sqrt{2}}{36-32}$

$= \frac{36+32-48\sqrt{2}}{4}$

$= \frac{68-48\sqrt{2}}{4}$

$= \frac{4(17-12\sqrt{2})}{4}$

$= 17-12\sqrt{2}$ (Answer)

**(v).** $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$ $\quad \because (a-b)^2=a^2+b^2-2ab$, $a^2-b^2=(a+b)(a-b)$

$= \frac{(\sqrt{3})^2+(\sqrt{2})^2-2(\sqrt{3})(\sqrt{2})}{3-2}$

$= \frac{3+2-2\sqrt{3\times2}}{1}$

$= 5-2\sqrt{6}$ (Answer)

**(vi).** $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

$= \frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

$= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{(\sqrt{7})^2-(\sqrt{5})^2}$ $\quad \because a^2-b^2=(a+b)(a-b)$

$= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{7-5}$

$= \frac{{}^{2}4\sqrt{3}(\sqrt{7}-\sqrt{5})}{2}$

$= 2\sqrt{3}(\sqrt{7}-\sqrt{5})$ (Answer)

**Question # 2: Simplify the following:**

**(i).** $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

$= \left(\frac{16}{81}\right)^{\frac{3}{4}}$

$= \frac{2^{\cancel{4}\times\frac{3}{\cancel{4}}}}{3^{\cancel{4}\times\frac{3}{\cancel{4}}}}$

$= \frac{2^3}{3^3}$

$= \frac{8}{27}$ (Answer)

| | |
|---|---|
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

| | |
|---|---|
| 3 | 81 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

**(ii).** $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$

$= \left(\frac{4}{3}\right)^{2} \div \frac{4^3}{3^{2\times3}} \times \frac{4^2}{3^3}$

$= \frac{4^2}{3^2} \times \frac{3^6}{4^3} \times \frac{4^2}{3^3}$

$= 4^{2+2-3} \times 3^{6-2-3}$

$= 4 \times 3$

$= 12$ (Answer)

| | |
|---|---|
| 3 | 81 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

**(iii).** $(0.027)^{-\frac{1}{3}}$

$= \left(\frac{27}{1000}\right)^{-\frac{1}{3}}$

$= \left(\frac{1000}{27}\right)^{\frac{1}{3}}$

$= \left(\frac{10^3}{3^3}\right)^{\frac{1}{3}}$

$= \frac{10^{\cancel{3}\times\frac{1}{\cancel{3}}}}{3^{\cancel{3}\times\frac{1}{\cancel{3}}}}$

$= \frac{10}{3}$ (Answer)

| | |
|---|---|
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

**(iv).** $\sqrt[7]{\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}z^{7}}}$

$= (x^{14}y^{21-14}z^{35-7})^{\frac{1}{7}}$

$= (x^{14}y^{7}z^{28})^{\frac{1}{7}}$

$= x^{\overset{2}{\cancel{14}}\times\frac{1}{\cancel{7}}}\, y^{\cancel{7}\times\frac{1}{\cancel{7}}}\, z^{\overset{4}{\cancel{28}}\times\frac{1}{\cancel{7}}}$

$= x^2yz^4$ (Answer)

**(v).** $\frac{\mathbf{5.(25)^{n+1}-25.(5)^{2n}}}{\mathbf{5.(5)^{2n+3}-(25)^{n+1}}}$

$= \frac{5.5^{2(n+1)}-5^2.5^{2n}}{5.5^{2n+3}-5^{2(n+1)}}$

$= \frac{5.5^{2n+2}-5^{2n+2}}{5.5^{2n+2}.5-5^{2n+2}}$

$= \frac{\cancel{5^{2n+2}}(5-1)}{\cancel{5^{2n+2}}(5.5-1)}$

$= \frac{4}{25-1}$

$= \frac{4}{24}$

$= \frac{1}{6}$ (Answer)

**(vi).** $\frac{\mathbf{(16)^{x+1}+20\,(4^{2x})}}{\mathbf{(2)^{x-3}\times 8^{x+2}}}$

$= \frac{(2^4)^{x+1}+20(2^{2\times2x})}{2^{x-3}\times2^{3(x+2)}}$

$= \frac{2^{4x+4}+20(2^{4x})}{2^{x-3}\times2^{3x+6}}$

$= \frac{2^{4x}.2^4+20(2^{4x})}{2^{x-3+3x+6}}$

$= \frac{2^{4x}(2^4+20)}{2^{4x+3}}$

$= \frac{\cancel{2^{4x}}(16+20)}{\cancel{2^{4x}}.2^3}$

$= \frac{36}{8}$

$= \frac{9}{2}$ (Answer)

**(vii).** $(64)^{\frac{-2}{3}} \div (9)^{\frac{-3}{2}}$

$= (4^{\cancel{3}})^{\frac{-2}{\cancel{3}}} \div (3^{\cancel{2}})^{\frac{-3}{\cancel{2}}}$

$= 4^{-2} \div 3^{-3}$

$= \frac{4^{-2}}{3^{-3}}$

$= \frac{3^3}{4^2}$

$= \frac{27}{16}$ (Answer)

**(viii).** $\frac{3^n\times9^{n+1}}{3^{n-1}\times9^{n-1}}$

$= \frac{\cancel{3^n}\times3^{2(n+1)}}{\cancel{3^n}\times3^{-1}\times3^{2(n-1)}}$

$= \frac{3^{2n+2}}{3^{-1}\times3^{2n-2}}$

$= \frac{3^{2n} \times 3^2 \times 3^1}{3^{2n} \times 3^{-2}}$

$= 3^{2+1+2}$

$= 3^5 = 243$ (Answer)

**(ix).** $\frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 4 \times 5^n}$

$= \frac{5^n \times 5^3 - 6 \times 5^n \times 5^1}{5^n(9 - 2^2)}$

$= \frac{5^n(5^3 - 6 \times 5)}{5^n(9 - 4)}$

$= \frac{125 - 30}{5}$

$= \frac{95}{5}$

$= 19$ (Answer)

**Question # 3: If $x = 3 + \sqrt{8}$ then find the value of:**

$\frac{x}{1} = \frac{3 + \sqrt{8}}{1}$

$\frac{1}{x} = \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}$

$= \frac{3 - \sqrt{8}}{3^2 - (\sqrt{8})^2}$

$= \frac{3 - \sqrt{8}}{9 - 8}$

$= \frac{3 - \sqrt{8}}{1}$

$\frac{1}{x} = 3 - \sqrt{8}$

**(i)** $x + \frac{1}{x}$

$= 3 + \sqrt{8} + 3 - \sqrt{8}$

$= 6$ (Answer)

**(ii)** $x - \frac{1}{x}$

$= 3 + \sqrt{8} - (3 - \sqrt{8})$

$= 3 + \sqrt{8} - 3 + \sqrt{8}$

$= 2\sqrt{8}$ (Answer)

**(iii)** $x^2 + \frac{1}{x^2}$

$\because x + \frac{1}{x} = 6$

Taking square on both sides

$\left(x + \frac{1}{x}\right)^2 = 6^2$

$(x)^2 + \left(\frac{1}{x}\right)^2 + 2(x)\left(\frac{1}{x}\right) = 36$

$x^2 + \frac{1}{x^2} + 2 = 36$

$x^2 + \frac{1}{x^2} = 36 - 2$

$x^2 + \frac{1}{x^2} = 34$ (Answer)

**(iv)** $x^2 - \frac{1}{x^2}$

$x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$= (6)(2\sqrt{8})$

$= 12\sqrt{8}$ (Answer)

**(v)** $x^4 + \frac{1}{x^4}$

$\because x^2 + \frac{1}{x^2} = 34$

Taking square on both sides

$\left(x^2 + \frac{1}{x^2}\right)^2 = (34)^2$

$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(x^2)\left(\frac{1}{x^2}\right) = 1156$

$x^4 + \frac{1}{x^4} + 2 = 1156$

$x^4 + \frac{1}{x^4} = 1156 - 2$

$x^4 + \frac{1}{x^4} = 1154$ (Answer)

**(vi)** $\left(x - \frac{1}{x}\right)^2$

$= (2\sqrt{8})^2$

$= 4 \times 8$

$= 32$ (Answer)

**Question # 4: Find the rational number *P* and *Q* such that: $\frac{8 - 3\sqrt{2}}{4 + 3\sqrt{2}} = p + q\sqrt{2}$**

$\frac{8 - 3\sqrt{2}}{4 + 3\sqrt{2}} \times \frac{4 - 3\sqrt{2}}{4 - 3\sqrt{2}} = p + q\sqrt{2}$

$\frac{8(4 - 3\sqrt{2}) - 3\sqrt{2}(4 - 3\sqrt{2})}{4^2 - (3\sqrt{2})^2} = p + q\sqrt{2}$

$\frac{32 - 24\sqrt{2} - 12\sqrt{2} + (3\sqrt{2})^2}{16 - (9 \times 2)} = p + q\sqrt{2}$

$\frac{32 - 36\sqrt{2} + (9 \times 2)}{16 - 18} = p + q\sqrt{2}$

$\frac{32 - 36\sqrt{2} + 18}{-2} = p + q\sqrt{2}$

$$\frac{50-36\sqrt{2}}{-2}=p+q\sqrt{2}$$

$$\frac{50}{-2}-\frac{36\sqrt{2}}{-2}=p+q\sqrt{2}$$

$$-25+18\sqrt{2}=p+q\sqrt{2}$$

By comparing we get,

$p=-25\ and\ q=18$

**Question # 5: Simplify the following:**

**(i).** $\dfrac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$

| | |
|---|---|
| 3 | 243 |
| 3 | 81 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

| | |
|---|---|
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

$=\dfrac{(5^2)^{\frac{3}{2}}\times(3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}}\times(2^3)^{\frac{4}{3}}}$

$=\dfrac{5^3\times 2^3}{2^5\times 2^4}$

$=\dfrac{125\times 27}{32\times 16}$

$=\dfrac{3375}{512}$ (Answer)

**(ii).** $\dfrac{54\times\sqrt[3]{(27)^{2x}}}{9^{x+1}+216(3^{2x-1})}$

$=\dfrac{54\times(3^3)^{\frac{2x}{3}}}{3^{2(x+1)}+216(3^{2x}\times 3^{-1})}$

$=\dfrac{54\times 3^{2x}}{3^{2x+2}+\frac{^{72}216(3^{2x})}{3}}$

$=\dfrac{54\times 3^{2x}}{3^{2x}\times 3^2+72(3^{2x})}$

$=\dfrac{54\times 3^{2x}}{3^{2x}(9+72)}$

$=\dfrac{54}{81}$

$=\dfrac{2}{3}$ (Answer)

**(iii).** $\sqrt{\dfrac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

| | |
|---|---|
| 2 | 216 |
| 2 | 108 |
| 2 | 54 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

$=\sqrt{\dfrac{(2^3\times 3^3)^{\frac{2}{3}}\times(5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{-3}{2}}}}$

$=\sqrt{\dfrac{(2^3)^{\frac{2}{3}}\times(3^3)^{\frac{2}{3}}\times 5^1}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}}$

$=\sqrt{\dfrac{2^2\times 3^2\times 5}{(25)^{\frac{3}{2}}}}$

$=\sqrt{\dfrac{2^2\times 3^2\times 5}{(5^2)^{\frac{3}{2}}}}$

$=\sqrt{\dfrac{2^2\times 3^2\times 5}{5^3}}$

$=\sqrt{\dfrac{2^2\times 3^2}{5^{3-1}}}$

$=\sqrt{\dfrac{2^2\times 3^2}{5^2}}$

$=\dfrac{2\times 3}{5}$

$=\dfrac{6}{5}$ (Answer)

**(iv).** $\left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)$

$\because\ a^3+b^3=(a+b)(a^2-ab+b^2)$

$=\left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\left(\left(a^{\frac{1}{3}}\right)^2-a^{\frac{1}{3}}b^{\frac{2}{3}}+\left(b^{\frac{2}{3}}\right)^2\right)$

$=\left(a^{\frac{1}{3}}\right)^3+\left(b^{\frac{2}{3}}\right)^3$

$=a+b^2$ (Answer)

Exercise 1.2 (Solutions)
Mathematics 9: PCTB
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