

SETS AND FUNCTIONS

Exercise # 3.1

Question # 1: Write the following sets in set builder notation:

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| i. $\{1, 4, 9, 16, 25, 36, \dots, 484\}$
$\{x x = n^2, n \in N \wedge 1 \leq n \leq 22\}$ | ii. $\{2, 4, 8, 16, \dots, 256\}$
$\{x x = 2^n, n \in N \wedge 1 \leq n \leq 8\}$ |
| iii. $\{0, \pm 1, \pm 2, \dots, \pm 1000\}$
$\{x x \in Z \wedge -1000 \leq x \leq 1000\}$ | iv. $\{6, 12, 18, \dots, 120\}$
$\{x x = 6n, n \in N \wedge 1 \leq n \leq 20\}$ |
| v. $\{100, 102, 104, \dots, 400\}$
$\{x x = 100 + 2n, n \in W \wedge 0 \leq n \leq 150\}$ | vi. $\{1, 3, 9, 27, 81, \dots\}$
$\{x x = 3^n, n \in W\}$ |
| vii. $\{1, 2, 4, 5, 10, 20, 25, 50, 100\}$
$\{x x \text{ is divisor of } 100, x \in N \wedge 1 \leq x \leq 100\}$ | viii. $\{5, 10, 15, \dots, 100\}$
$\{x x = 5n, n \in N \wedge 1 \leq n \leq 20\}$ |
| ix. The set of all integers between -100 and 1000
$\{x x \in Z \wedge -100 < n < 1000\}$ | |

Question # 2: Write each of the following sets in tabular form:

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| i. $\{x x \text{ is a multiple of } 3 \wedge x \leq 35\}$
$\{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33\}$ | ii. $\{x x \in R \wedge 2x + 1 = 0\}$
$\{-\frac{1}{2}\}$ |
| iii. $\{x x \in P \wedge x < 12\}$
$\{2, 3, 5, 7, 11\}$ | iv. $\{x x \text{ is a divisor of } 128\}$
$\{1, 2, 4, 8, 16, 32, 64, 128\}$ |
| v. $\{x x = 2^n, n \in N \wedge n < 8\}$
$\{2, 4, 8, 16, 32, 64, 128\}$ | vi. $\{x x \in N \wedge x + 4 = 0\}$
$\{\}$ |
| vii. $\{x x \in N \wedge x = x\}$
$\{1, 2, 3, 4, \dots\}$ | viii. $\{x x \in Z \wedge 3x + 1 = 0\}$
$\{\}$ |

Question # 3: Write two proper subsets of each of the following sets:

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| i. $\{a, b, c\}$
$\{a\}, \{b\}$ | ii. $\{0, 1\}$
$\{0\}, \{1\}$ | iii. N
Prime Numbers
Even Natural Numbers |
| iv. Z
Prime Numbers
Whole Numbers | v. Q
Integers
Natural Numbers | vi. R
Rational Numbers
Irrational Numbers |
| vii. $\{x x \in Q \wedge 0 < x \leq 2\}$
$\{\frac{1}{2}\}, \{\frac{3}{4}\}$ | | |

Question # 4: Is there any set which has no proper subset? If so, name that set.

Ans: Yes, there exist a set which has no proper subset, that is called 'empty set' $\{\}$.

Question # 5: What is the difference between $\{a, b\}$ and $\{\{a, b\}\}$?

Ans: $\{a, b\}$ set has two elements: 'a' and 'b'. While $\{\{a, b\}\}$ set has only one element $\{a, b\}$.

Question # 6: What is the number of elements of the power set of each of the following sets?

i. $\{ \}$
 $n = 0$

$$\begin{aligned} \text{No of elements} &= 2^n \\ &= 2^0 \\ &= 1 \end{aligned}$$

ii. $\{0, 1\}$
 $n = 2$

$$\begin{aligned} \text{No of elements} &= 2^n \\ &= 2^2 \\ &= 4 \end{aligned}$$

iii. $\{1, 2, 3, 4, 5, 6, 7\}$
 $n = 7$

$$\begin{aligned} \text{No of elements} &= 2^n \\ &= 2^7 \\ &= 128 \end{aligned}$$

iv. $\{0, 1, 2, 3, 4, 5, 6, 7\}$
 $n = 8$

$$\begin{aligned} \text{No of elements} &= 2^n \\ &= 2^8 \\ &= 256 \end{aligned}$$

v. $\{a, \{b, c\}\}$
 $n = 2$

$$\begin{aligned} \text{No of elements} &= 2^n \\ &= 2^2 \\ &= 4 \end{aligned}$$

vi. $\{\{a, b\}, \{b, c\}, \{d, e\}\}$
 $n = 3$

$$\begin{aligned} \text{No of elements} &= 2^n \\ &= 2^3 \\ &= 8 \end{aligned}$$

Question # 7: Write down the power set of each of the following sets:

i. $\{9, 11\}$

Let, $A = \{9, 11\}$

$$P(A) = \{\emptyset, \{9\}, \{11\}, \{9, 11\}\}$$

ii. $\{+, -, \times, \div\}$

Let, $A = \{+, -, \times, \div\}$

$$P(A) = \{\emptyset, \{+\}, \{-\}, \{\times\}, \{\div\}, \{+, -\}, \{+, \times\}, \{+, \div\}, \{-, \times\}, \{-, \div\}, \{\times, \div\}, \{+, -, \times\}, \{+, -, \div\}, \{-, \times, \div\}, \{+, \times, \div\}, \{+, -, \times, \div\}\}$$

iii. $\{\emptyset\}$

Let, $A = \{\emptyset\}$

$$P(A) = \{\emptyset, \{\emptyset\}\}$$

iv. $\{a, \{b, c\}\}$

Let, $A = \{a, \{b, c\}\}$

$$P(A) = \{\emptyset, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$$

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