Unit 3 Sets and Functions

EXERCISE 3.1

1. Write the following sets in set builder notation:

(i) $\{1, 4, 9, 16, 25, 36, \ldots, 484\}$ (ii) $\{2, 4, 8, 16, 32, 64, \ldots, 150\}$

(iii) $\{0, \pm 1, \pm 2, \ldots, \pm 1000\}$ (iv) $\{6, 12, 18, \ldots, 120\}$

(v) $\{100, 102, 104, \ldots, 400\}$ (vi) $\{1, 3, 9, 27, 81, \ldots\}$

(vii) $\{1, 2, 4, 5, 10, 20, 25, 50, 100\}$ (viii) $\{5, 10, 15, \ldots, 100\}$

(ix) The set of all integers between – 100 and 1000

Solution

(i) $\{x|x = n^2, n \in N \wedge 1 \leq x < 500\}$ **(ii)** $\{x|x = 2^n, n \in N \wedge 2 \leq x \leq 150\}$

(iii) $\{x|x \in Z \wedge 0 \leq x \leq 1000\}$ **(iv)** $\{x|x = 6n, n \in N \wedge 1 \leq n \leq 20\}$

(v) $\{x|x = 100 + 2n, n \in W \wedge 1 \leq n \leq 150\}$ **(vi)** $\{x|x = 3^n, n \in W\}$

(vii) $\{x|x \text{ is a divisor of } 100\}$ **(viii)** $\{x|x = 5n, n \in N \wedge 1 \leq n \leq 20\}$

(ix) $\{x|x \in Z \wedge -100 < x < 1000\}$

2. Write each of the following sets in tabular forms:

(i) $\{x|x \text{ is a multiple of } 3 \wedge x \leq 35\}$ (ii) $\{x|x \in R \wedge 2x+1=0\}$

(iii) $\{x|x \in P \wedge x < 12\}$ (iv) $\{x|x \text{ is a divisor of } 128\}$

(v) $\{x|x = 2^n, n \in N \wedge n < 8\}$ (vi) $\{x|x \in N \wedge x+4=0\}$

(vii) $\{x|x \in N \wedge x = x\}$ (viii) $\{x|x \in Z \wedge 3x + 1 = 0\}$

Solution

(i) $\{3, 6, 9, \ldots, 35\}$ (ii) $\left\{-\frac{1}{2}\right\}$

(iii) $\{2, 3, 5, 7, 11\}$ (iv) $\{1, 2, 4, 8, 16, 32, 64, 128\}$ (v) $\{2,4,8,16,32,64,128\}$

(vi) $\{\ \}$ (vii) $\{1, 2, 3, 4, 5, \ldots\}$ (viii) $\{\ \}$

3. Write two proper subsets of each of the following sets:

(i) $\{a, b, c\}$ (ii) $\{0, 1\}$ (iii) N (iv) Z

(v) Q (vi) R (vii) $\{x \mid x \in Q \wedge 0 < x \leq 2\}$

Solution

i. The Proper subsets of {a, b, c} are {a}, {b}.

ii. The Proper subsets of {0,1} are { 0 }, { 1 }.

iii. The Proper subsets of N = {1,2,3,… } are {1}, {2}.

iv. The Proper subsets of Z = $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ are {1}, {2}.

v. The Proper subsets of Q are {1}, {2}.

vi. The Proper subsets of R are {1}, {2}.

vii. The Proper subsets of $\{x|x \in Q \wedge 0 < x \leq 2\}$ are {1}, {2}.

4. Is there any set which has no proper subset? If so, name that set.

Solution

Yes, { } or φ

5. What is the difference between $\{a, b\}$ and $\{\{a,b\}\}$?

Solution

{a,b} is a set containing two elements a and b while {{a,b}} is a set containing one element {a,b}.

6. What is the number of elements of the power set of each of the following sets?

(i) { } (ii) $\{0, 1\}$ (iii) $\{1, 2, 3, 4, 5, 6, 7\}$

(iv) $\{0, 1, 2, 3, 4, 5, 6, 7\}$ (v) $\{a, \{b, c\}\}$

(vi) $\{\{a, b\}, \{b, c\}, \{d, e\}\}$

Solution

(i) 1 (ii) 4 (iii) 128 (iv) 256 (v) 4 (vi) 8

7. Write down the power set of each of the following sets:

(i) $\{9, 11\}$ (ii) $\{+, -, \times, \div\}$ (iii) $\{\phi\}$ (iv) $\{a, \{b,c\}\}$

Solution

i. The Power set of {9,11} is { φ, {9}, {11},{9,11}}.

ii. The Power set of $\{+, -, \times, \div\}$ is

$\{\varphi, \{+\}, \{-\}, \{\times\}, \{\div\}, \{+, -\}, \{+\ \times\}, \{+, \div\}, \{-, \times\}, \{-, \div\}, \{\times, \div\}, \{+, -, \times\},$
$\{+, -, \div\}, \{+, \times, \div\}, \{-, \times, \div\}, \{+, -, \times, \div\}\}.$

iii. The Power set of {φ} is { φ, {φ}}.

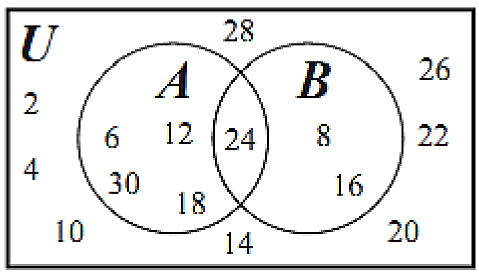
iv. The Power set of $\{a, \{b, c\}\}$ is $\{\varphi, \{a\}. \{\{b, c\}\}, \{a, \{b, c\}\}\}$.

Exercise 3.2

1. Consider the universal set $U = \{x : x \text{ is multiple of 2 and } 0 < x \leq 30\}$,
   $A = \{x : x \text{ is a multiple of } 6\}$ and $B = \{x : x \text{ is a multiple of } 8\}$
   (i) List all elements of sets A and B in tabular form
   (ii) Find $A \cap B$ (iii) Draw a Venn diagram

Solution

(i) $A = \{6, 12, 18, 24, 30\}, B = \{8, 16, 24\}$ (ii) $A \cap B = \{24\}$



2. Let, $U = \{x : x \text{ is an integer and } 0 < x \leq 150\}$,
   $G = \{x : x = 2^m \text{ for integer } m \text{ and } 0 \leq m \leq 7\}$ and
   $H = \{x : x \text{ is a square}\}$
   (i) List all elements of sets G and H in tabular form
   (ii) Find $G \cup H$ (iii) Find $G \cap H$

Solution

(i) $G = \{1, 2, 4, 8, 16, 32, 64, 128\}$,
$H = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144\}$

(ii) $G \cup H = \{1, 2, 4, 8, 9, 16, 25, 32, 36, 49, 64, 81, 100, 121, 128, 144\}$

(iii) $G \cap H = \{1, 4, 16, 64\}$

3. Consider the sets $P = \{x : x \text{ is a prime number and } 0 < x \leq 20\}$ and
   $Q = \{x : x \text{ is a divisor of 210 and } 0 < x \leq 20\}$
   (i) Find $P \cap Q$ (ii) Find $P \cup Q$

Solution

(i) $P \cap Q = \{2,3,5,7,9,11,13,17,19,20\} \cap \{1, 2, 3, 5, 6, 7, 10, 14, 15\} = \{2,3,5,7\}$

(ii) $P \cup Q = \{2,3,5,7,9,11,13,17,19,20\} \cup \{1, 2, 3, 5, 6, 7, 10, 14, 15\}$

$P \cup Q = \{1, 2, 3, 5, 6, 7, 10,11,13, 14, 15,17,19,20\}$

4. Verify the commutative properties of union and intersection for the following pairs of sets:

(i) $A = \{1, 2, 3, 4, 5\}$, $B = \{4, 6, 8, 10\}$ (ii) N, Z

(iii) $A = \{x \mid x \in R \wedge x \geq 0\}$, $B = R$.

Solution

4.(i) $\boldsymbol{A \cup B = B \cup A}$ **also** $\boldsymbol{A \cap B = B \cap A}$

$A \cup B = \{1,2,3,4,5\} \cup \{4,6,8,10\} = \{1,2,3,4,5,6,8,10\}$

$B \cup A = \{4,6,8,10\} \cup \{1,2,3,4,5\} = \{1,2,3,4,5,6,8,10\}$

Hence $A \cup B = B \cup A$

$A \cap B = \{1,2,3,4,5\} \cap \{4,6,8,10\} = \{4\}$

$B \cap A = \{4,6,8,10\} \cap \{1,2,3,4,5\} = \{4\}$

Hence $A \cap B = B \cap A$

4.(ii) $\boldsymbol{N \cup Z = Z \cup N}$ **also** $\boldsymbol{N \cap Z = Z \cap N}$

$N \cup Z = \{1,2,3, \dots\} \cup \{0, \pm 1, \pm 2, \pm 3, \dots\} = \{0, \pm 1, \pm 2, \pm 3, \dots\}$

$Z \cup N = \{0, \pm 1, \pm 2, \pm 3, \dots\} \cup \{1,2,3, \dots\} = \{0, \pm 1, \pm 2, \pm 3, \dots\}$

Hence $N \cup Z = Z \cup N$

$N \cap Z = \{1,2,3, \dots\} \cap \{0, \pm 1, \pm 2, \pm 3, \dots\} = \{1,2,3, \dots\}$

$Z \cap N = \{0, \pm 1, \pm 2, \pm 3, \dots\} \cap \{1,2,3, \dots\} = \{1,2,3, \dots\}$

Hence $N \cap Z = Z \cap N$

4.(iii) $\boldsymbol{A \cup B = B \cup A}$ **also** $\boldsymbol{A \cap B = B \cap A}$

$A \cup B = \{0,1,2,3,4,5\} \cup R = R$

$B \cup A = R \cup \{0,1,2,3,4,5\} = R$

Hence $A \cup B = B \cup A$

$A \cap B = \{0,1,2,3,4,5\} \cap R = \{0,1,2,3,4,5\}$

$B \cap A = R \cap \{0,1,2,3,4,5\} = \{0,1,2,3,4,5\}$

Hence $A \cap B = B \cap A$

5. Let $U = \{a, b, c, d, e, f, g, h, i, j\}$

$A = \{a, b, c, d, g, h\}$, $B = \{c, d, e, f, j\}$,

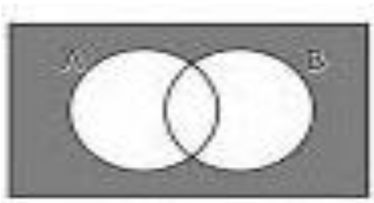
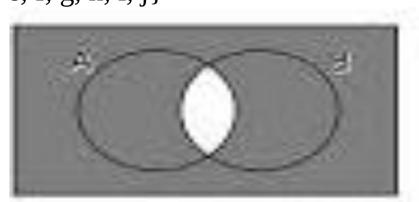
Verify De Morgan's Laws for these sets. Draw Venn diagram

Solution

We have to verify

$(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

$(\mathbf{A} \cup \mathbf{B})' = \mathbf{A}' \cap \mathbf{B}'$	$(\mathbf{A} \cap \mathbf{B})' = \mathbf{A}' \cup \mathbf{B}'$
$A = \{a, b, c, d, g, h\}$ $B = \{c, d, e, f, j\}$ $U = \{a, b, c, d, e, f, g, h, i, j\}$ $A' = U - A$ $= \{a, b, c, d, e, f, g, h, i, j\} - \{a, b, c, d, g, h\}$ $= \{e, f, i, j\}$ $B' = U - B$ $= \{a, b, c, d, e, f, g, h, i, j\} - \{c, d, e, f, j\}$ $= \{a, b, g, h, i\}$ $A \cup B = \{a, b, c, d, e, f, g, h, j\}$ $(A \cup B)' = U - (A \cup B)$ $= \{a, b, c, d, e, f, g, h, i, j\} - \{a, b, c, d, e, f, g, h, j\}$ $= \{i\}$ $A' \cap B' = \{e, f, i, j\} \cap \{a, b, g, h, i\}$ $= \{i\}$ 	$A = \{a, b, c, d, g, h\}$ $B = \{c, d, e, f, j\}$ $U = \{a, b, c, d, e, f, g, h, i, j\}$ $A' = U - A$ $= \{a, b, c, d, e, f, g, h, i, j\} - \{a, b, c, d, g, h\}$ $= \{e, f, i, j\}$ $B' = U - B$ $= \{a, b, c, d, e, f, g, h, i, j\} - \{c, d, e, f, j\}$ $= \{a, b, g, h, i\}$ $A \cap B = \{c, d\}$ $(A \cap B)' = U - (A \cap B)$ $= \{a, b, c, d, e, f, g, h, i, j\} - \{c, d\}$ $= \{a, b, e, f, g, h, i, j\}$ $A' \cup B' = \{e, f, i, j\} \cup \{a, b, g, h, i\}$ $= \{a, b, e, f, g, h, i, j\}$ 

6. If $U = \{1, 2, 3, \ldots, 20\}$ and $A = \{1, 3, 5, \ldots, 19\}$, verify the following:

(i) $A \cup A' = U$ (ii) $A \cap U = A$ (iii) $A \cap A' = \phi$

Solution

$U = \{1,2,3, \ldots ,20\}$ and $A = \{1,3,5, \ldots ,19\}$

$A' = U - A = \{1,2,3, \ldots ,20\} - \{1,3,5, \ldots ,19\} = \{2,4,6, \ldots ,20\}$

(i) $A \cup A' = \{1,3,5, \ldots ,19\} \cup \{2,4,6, \ldots ,20\} = \{1,2,3, \ldots ,20\} = U$

(ii) $A \cap U = \{1,3,5, \ldots ,19\} \cap \{1,2,3, \ldots ,20\} = \{1,3,5, \ldots ,19\} = A$

(iii) $A \cap A' = \{1,3,5, \ldots ,19\} \cap \{2,4,6, \ldots ,20\} = \varphi$

7. In a class of 55 students, 34 like to play cricket and 30 like to play hockey. Also each student likes to play at least one of the two games. How many students like to play both games?

Solution

$n(C) = 34$; $n(H) = 30$; $n(U) = 55$; $n(C \cup H) = 55$

$n(C \cup H) = n(C) + n(H) - n(C \cap H)$

$55 = 34 + 30 - n(C \cap H) \Rightarrow 55 = 64 - n(C \cap H)$

$\Rightarrow n(C \cap H) = 64 - 55$

$\Rightarrow \mathbf{n(C \cap H) = 9}$.

8. In a group of 500 employees, 250 can speak Urdu, 150 can speak English, 50 can speak Punjabi, 40 can speak Urdu and English, 30 can speak both English and Punjabi, and 10 can speak Urdu and Punjabi. How many can speak all three languages?

Solution

$n(U \cup E \cup P) = 500$; $n(U) = 250$; $n(E) = 150$; $n(P) = 50$

$n(U \cap E) = 40$; $n(E \cap P) = 30$; $n(U \cap P) = 10$

$n(U \cap E \cap P) = ???$

$n(U \cup E \cup P) = n(U) + n(E) + n(P) - n(U \cap E) - n(E \cap P) - n(U \cap P) + n(U \cap E \cap P)$

$500 = 250 + 150 + 50 - 40 - 30 - 10 + n(U \cap E \cap P)$

$500 = 450 - 80 + n(U \cap E \cap P)$

$500 = 370 + n(U \cap E \cap P)$

$\mathbf{n(U \cap E \cap P) = 130}$

9. In sports events, 19 people wear blue shirts, 15 wear green shirts, 3 wear blue and green shirts, 4 wear a cap and blue shirts, and 2 wear a cap and green shirts. The total number of people with either a blue or green shirt or cap is 25. How many people are wearing caps?

Solution

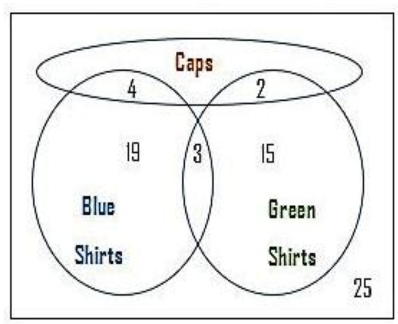
$n(B) = 19$; $n(G) = 15$; $n(C) = ?$; $n(B \cap G) = 3$; $n(B \cap C) = 4$

$n(G \cap C) = 2$; $n(B \cup G \cup C) = 25$

$n(B \cup G \cup C) = n(B) + n(G) + n(C) - n(B \cap G) - n(B \cap C) - n(G \cap C) + n(B \cap G \cap C)$

$25 = 19 + 15 + n(C) - 3 - 4 - 2 + n(B \cap G \cap C)$

$0 = n(C) + n(B \cap G \cap C)$



As number of element in any set can be zero or positive, which concludes that Sum of number of elements of two sets can only be zero, if both sets are empty.

Hence, **n(C) = 0** or number of players wearing only caps are zero.

10. In a training session,17 participants have laptops, 11 have tablets, 9 have laptops and tablets, 6 have laptops and books, and 4 have both tablets and books. Eight participants have all three items. The total number of participants with laptops, tablets, or books is 35. How many participants have books?

Solution

$n(L) = 17$; $n(T) = 11$; $n(L \cap T) = 9$; $n(L \cap B) = 6$; $n(T \cap B) = 4$

$n(L \cap T \cap B) = 8$; $n(L \cup T \cup B) = 35$

$n(L \cup T \cup B) = n(L) + n(T) + n(B) - n(L \cap T) - n(L \cap B) - n(T \cap B) + n(L \cap T \cap B)$

$35 = 17 + 11 + n(B) - 9 - 6 - 4 + 8$

$35 = 17 + n(B)$

$n(B) = 18$

11. A shopping mall has150 employees labelled 1 to 150, representing the Universal set U. The employees fall into the following categories:
- Set A: 40 employees with a salary range of 30k-45k, labelled from 50 to 89.
- Set B: 50 employees with a salary range of 50k-80k, labelled from 101 to 150.
- Set C: 60 employees with a salary range of 100k-150k, labelled from 1 to 49 and 90 to 100.

(a) Find $(A' \cup B') \cap C$ (a) Find $n\{A \cap (B^c \cap C^C)\}$

Solution

$U = \{1,2,3,\dots,150\}$; $n(U) = 150$

$A = \{50,51,52,\dots,89\}$; $n(A) = 40$

$B = \{101,102,\dots,150\}$; $n(B) = 50$

$C = \{1,2,3,\dots,49,90,91,\dots,100\}$; $n(C) = 60$

$A' = U - A = \{1,2,3,\dots,150\} - \{50,51,52,\dots,89\} = \{1,2,3,\dots,49,90,91,\dots,100\}$
$B' = U - B = \{1,2,3,\dots,150\} - \{101,102,\dots,150\} = \{1,2,3,\dots,100\}$
$C' = U - C = \{1,2,3,\dots,150\} - \{1,2,3,\dots,49,90,91,\dots,100\} = \{50,51,52,\dots,89\}$
(i) $(\mathbf{A}' \cup \mathbf{B}') \cap \mathbf{C} = ???$
$A' \cup B' = \{1,2,3,\dots,49,90,91,\dots,100\} \cup \{1,2,3,\dots,100\} = \{1,2,3,\dots,100\}$
$(A' \cup B') \cap C = \{1,2,3,\dots,100\} \cap \{1,2,3,\dots,49,90,91,\dots,100\}$
$(A' \cup B') \cap C = \{1,2,3,\dots,49,90,91,\dots,100\}$

(ii) $\boldsymbol{n}\{\mathbf{A} \cap (\mathbf{B}' \cap \boldsymbol{C}')\} = ??$

$B' \cap C' = \{1,2,3,\ldots,100\} \cap \{50,51,52,\ldots,89\} = \{50,51,52,\ldots,89\}$

$A \cap (B' \cap C') = \{50,51,52,\ldots,89\} \cap \{50,51,52,\ldots,89\} = \{50,51,52,\ldots,89\}$

$\boldsymbol{n}\{\mathbf{A} \cap (\mathbf{B}' \cap \boldsymbol{C}')\} = \mathbf{40}$

12. In a secondary school with 125 students participate in at least one of the following sports: cricket, football, or hockey.
    - 60 students play cricket.
    - 70 students play football.
    - 40 students play hockey.
    - 25 students play both cricket and football.
    - 15 students play both football and hockey.
    - 10 students play both cricket and hockey.
        - (a) How many students play all three sports?
        - (b) Draw a Venn diagram showing the distribution of sports participation in all the games.

Solution

$n(C \cup F \cup H) = 125$; $n(C) = 60$; $n(F) = 70$; $n(H) = 40$

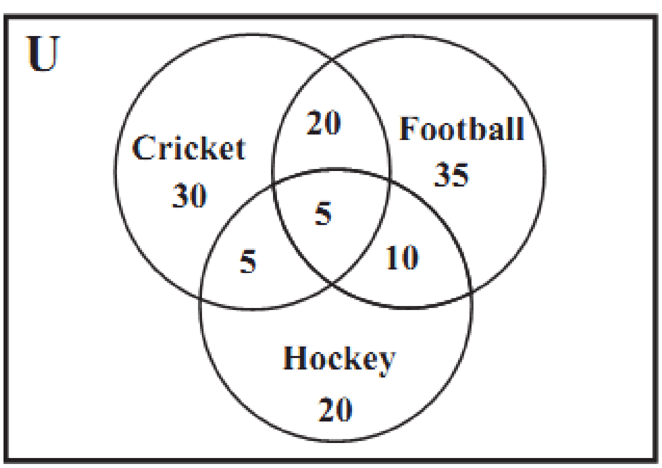
$n(C \cap F) = 25$; $n(F \cap H) = 15$; $n(C \cap H) = 10$; $n(C \cap F \cap H) = ???$

$n(C \cup F \cup H) = n(C) + n(F) + n(H) - n(C \cap F) - n(F \cap H) - n(C \cap H) + n(C \cap F \cap H)$

$125 = 60 + 70 + 40 - 25 - 15 - 10 + n(C \cap F \cap H)$

$n(C \cap F \cap H) = 125 - 60 - 70 - 40 + 25 + 15 + 10$

$\mathbf{n(C \cap F \cap H) = 5}$



13. A survey was conducted in which 130 people were asked about their favourite foods. The survey results showed the following information:

- 40 people said they liked nihari
- 65 people said they liked biryani
- 50 people said they liked korma
- 20 people said they liked nihari and biryani
- 35 people said they liked biryani and korma
- 27 people said they liked nihari and korma
- 12 people said they liked all three foods nihari, biryani, and korma

(a) At least how many people like nihari, biryani or korma?

(b) How many people did not like nihari, biryani, or korma?

(c) How many people like only one of the following foods: nihari, biryani, or korma?

(d) Draw a Venn diagram.

Solution

$n(N \cup B \cup K) = ???$; $n(N) = 40$; $n(B) = 65$; $n(K) = 50$

$n(N \cap B) = 20$; $n(B \cap K) = 35$; $n(N \cap K) = 27$; $n(N \cap B \cap K) = 12$

a) At least how many people like Nihari, Biryani or Korma:

$n(N \cup B \cup K) = n(N) + n(B) + n(K) - n(N \cap B) - n(B \cap K) - n(N \cap K) + n(N \cap B \cap K)$

$n(N \cup B \cup K) = 40 + 65 + 50 - 20 - 35 - 27 + 12$

$n(N \cup B \cup K) = \mathbf{85}$

b) How many people did not like Nihari, Biryani or Korma:

Total people = 130

People who like nihari, biryani, or korma = 85

People who did not like nihari, biryani, or korma = 130 – 85 = **45**

c) How many people like only one of the Nihari, Biryani or Korma:

People who like only nihari = $n(N) - n(N \cap B) - n(N \cap K) + n(N \cap B \cap K)$

$= 40 - 20 - 27 + 12 = 5$

People who like only biryani = n(B) – n(N ∩ B) – n(B ∩ K) + n(N ∩ B ∩ K)

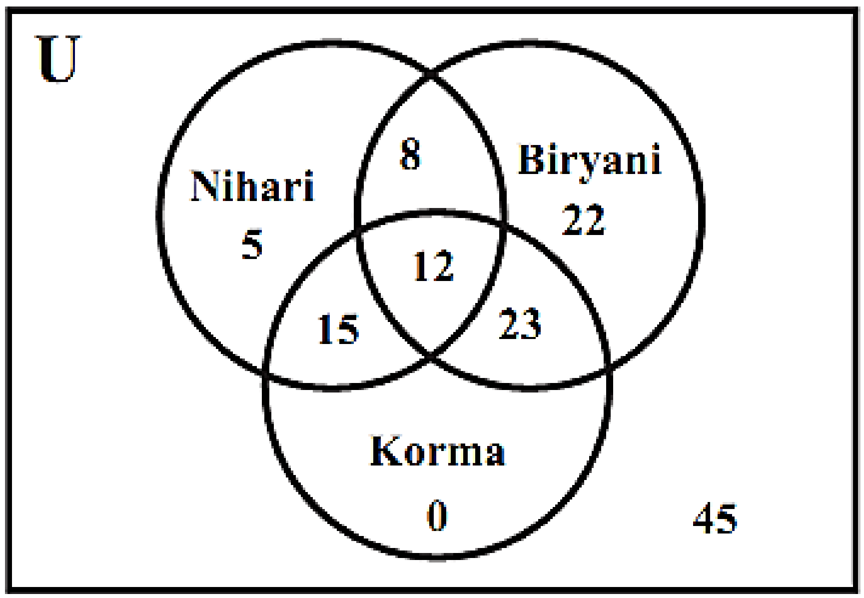
= 65 - 20 - 35 + 12 = 22

People who like only korma = n(K) – n(N ∩ K) – n(B ∩ K) + n(N ∩ B ∩ K)

= 50 - 27 - 35 + 12 = 0

Total people who like only one food = 5 + 22 + 0 = **27**

(d)



EXERCISE 3.3

1. For $A = \{1, 2, 3, 4\}$, find the following relations in A. State the domain and range of each relation.

(i) $\{(x, y) \mid y = x\}$ (ii) $\{(x, y) \mid y + x = 5\}$

(iii) $\{(x, y) \mid x + y < 5\}$ (iv) $\{(x, y) \mid x + y > 5\}$

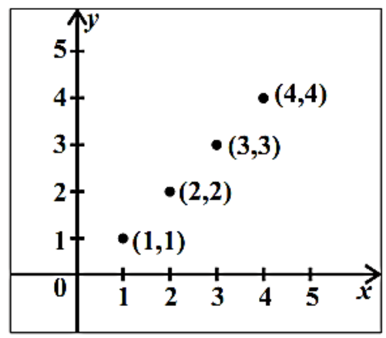
Solution

$A = \{1,2,3,4\}$

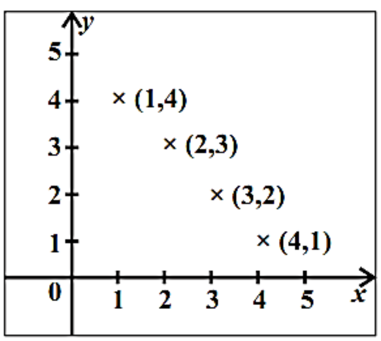
$A \times A = \{1,2,3,4\} \times \{1,2,3,4\} =$

$\{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$

(i) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
Domain of (i) $= \{1, 2, 3, 4\}$
Range of (i) $= \{1, 2, 3, 4\}$

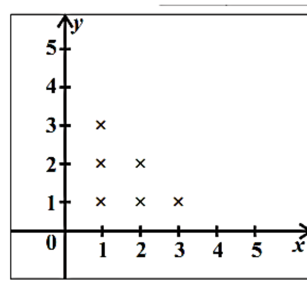


(ii)

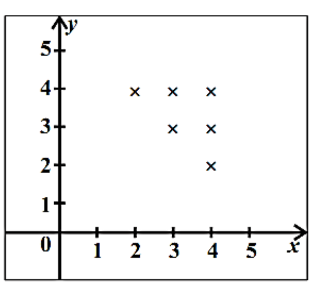


$\{(1, 4), (2, 3), (3, 2), (4, 1)\}$
Domain of (ii) $= \{1, 2, 3, 4\}$
Range of (ii) $= \{1, 2, 3, 4\}$

(iii) $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\}$
Domain of (iii) $= \{1, 2, 3\}$
Range of (iii) $= \{1, 2, 3\}$

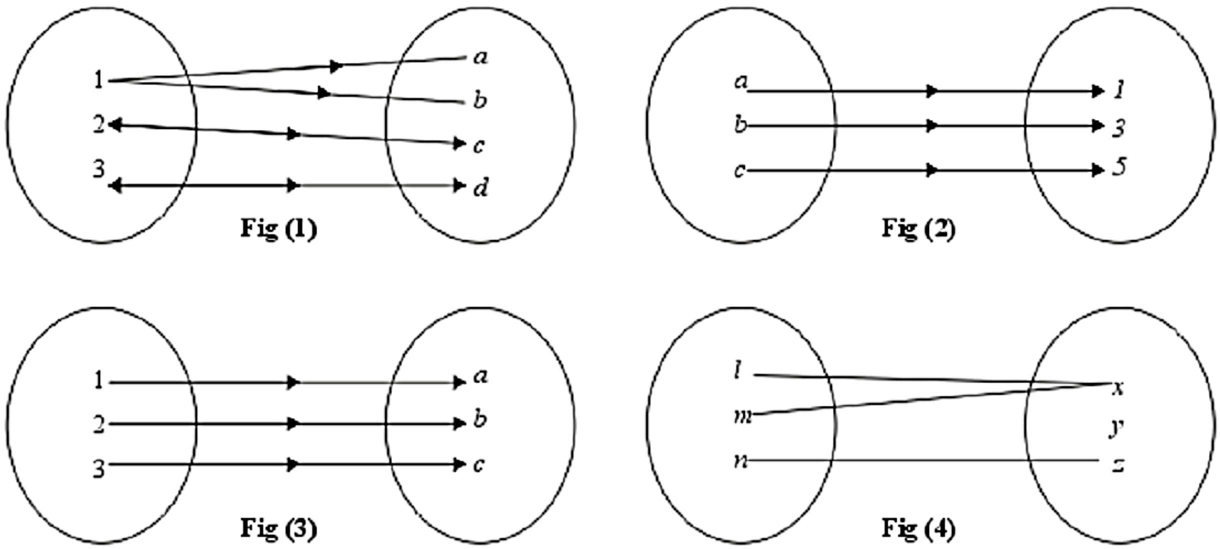


(iv)



$\{(2, 4), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)\}$
Domain of (iv) $= \{2, 3, 4\}$
Range of (iv) $= \{2, 3, 4\}$

2. Which of the following diagrams represent functions and of which type?



Solution

Fig (1) does not represent a function. Fig (2) represents a function, which is a bijective function.
Fig (3) represents a function, which is a bijective function.
Fig (4) represents a function, which is an into function.

3. If $g(x) = 3x + 2$ and $h(x) = x^2 + 1$, then find:

(i) $g(0)$ (ii) $g(-3)$ (iii) $g\left(\frac{2}{3}\right)$

(iv) $h(1)$ (v) $h(-4)$ (vi) $h\left(-\frac{1}{2}\right)$

Solution

i. $g(x) = 3x + 2 \Rightarrow g(0) = 3(0) + 2 \Rightarrow g(0) = 2$

ii. $g(x) = 3x + 2 \Rightarrow g(-3) = 3(-3) + 2 \Rightarrow g(-3) = -9 + 2 \Rightarrow g(-3) = -7$

iii. $g(x) = 3x + 2 \Rightarrow g\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) + 2 \Rightarrow g\left(\frac{2}{3}\right) = 2 + 2 \Rightarrow g\left(\frac{2}{3}\right) = 4$

iv. $h(x) = x^2 + 1 \Rightarrow h(1) = (1)^2 + 1 \Rightarrow h(1) = 1 + 1 \Rightarrow h(1) = 2$

v. $h(x) = x^2 + 1 \Rightarrow h(-4) = (-4)^2 + 1 \Rightarrow h(-4) = 16 + 1 \Rightarrow h(-4) = 17$

vi. $h(x) = x^2 + 1 \Rightarrow h\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + 1 \Rightarrow h\left(-\frac{1}{2}\right) = \frac{1}{4} + 1 \Rightarrow h\left(-\frac{1}{2}\right) = \frac{5}{4}$

4. Given that $f(x) = ax + b + 1$, where a and b are constant numbers. If $f(3) = 8$ and $f(6) = 14$, then find the values of a and b.

Solution

$f(x) = ax + b + 1$

$f(3) = a(3) + b + 1$	$f(6) = a(6) + b + 1$
$8 = 3a + b + 1$	$14 = 6a + b + 1$
$8 - 1 = 3a + b$	$14 - 1 = 6a + b$
$\mathbf{3a + b = 7}$ **……(i)**	$\mathbf{6a + b = 13}$ **……(ii)**

2(i) - (ii)	**(ii) - (i)**
$6a + 2b = 14$	$6a + b = 13$
$-6a \pm b = -13$	$-3a \pm b = -7$
$\mathbf{b = 1}$	$\mathbf{a = 2}$

5. Given that $g(x) = ax + b + 5$, where a and b are constant numbers. If $g(-1) = 0$ and $g(2) = 10$, find the values of a and b.

Solution

$g(x) = ax + b + 5$

$g(-1) = a(-1) + b + 5$	$g(2) = a(2) + b + 5$
$0 = -a + b + 5$	$10 = 2a + b + 5$
$0 - 5 = -a + b$	$10 - 5 = 2a + b$
$\mathbf{-a + b = -5}$ **……(i)**	$\mathbf{2a + b = 5}$ **……(ii)**

(i) - (ii)	**2(i) + (ii)**
$-a + b = -5$	$-2a + 2b = -10$
$-2a \pm b = -5$	$2a + b = 5$
$\mathbf{a = \frac{10}{3}}$	$\mathbf{b = -\frac{5}{3}}$

6. Consider the function defined by $f(x) = 5x + 1$. If $f(x) = 32$, find the x value.

Solution

$f(x) = 5x + 1$	$f(x) = 5x + 1$
$f(x) = 32$	$\boldsymbol{f(x) = 31}$
$5x + 1 = 32$	$5x + 1 = 31$
$5x = 32 - 1$	$5x = 31 - 1$
$x = \frac{31}{5}$ **6 is wrong answer in book**	$x = \frac{30}{5} = \mathbf{6}$ **according to book**

7. Consider the function $f(x) = cx^2 + d$, where c and d are constant numbers. If $f(1) = 6$ and $f(-2) = 10$, then find the values of c and d.

Solution

$f(x) = cx^2 + d$

$f(1) = c(1)^2 + d$	$f(-2) = c(-2)^2 + d$
$\mathbf{c + d = 6}$ **……(i)**	$\mathbf{4c + d = 10}$ **……(ii)**

(i) - (ii)	**4(i) - (ii)**
$c + d = 6$	$4c + 4d = 24$
$-4c \pm d = -10$	$-4c \pm d = -10$
$\mathbf{c = \frac{4}{3}}$	$\mathbf{d = \frac{14}{3}}$

REVIEW EXERCISE 3

1. Four options are given against each statement. Encircle the correct option.

(i) The set builder form of the set $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\right\}$ is:

(a) $\left\{x \mid x = \frac{1}{n},\ n \in W\right\}$ ✓(b) $\left\{x \mid x = \frac{1}{2n+1},\ n \in W\right\}$

(c) $\left\{x \mid x = \frac{1}{n+1},\ n \in W\right\}$ (d) $\{x \mid x = 2n+1,\ n \in W\}$

(ii) If $A = \{\ \}$, then $P(A)$ is:

(a) $\{\ \}$ (b) $\{1\}$ ✓(c) $\{\{\ \}\}$ (d) ϕ

(iii) If $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $U - (A \cap B)$ is:

✓(a) $\{1, 2, 4, 5\}$ (b) $\{2, 3\}$ (c) $\{1, 3, 4, 5\}$ (d) $\{1, 2, 3\}$

(iv) If A and B are overlapping sets, then $n(A - B)$ is equal to

(a) $n(A)$ (b) $n(B)$ (c) $A \cap B$ ✓(d) $n(A) - n(A \cap B)$

(v) If $A \subseteq B$ and $B - A \neq \phi$, then $n(B - A)$ is equal to

(a) 0 (b) $n(B)$ (c) $n(A)$ ✓(d) $n(B) - n(A)$

(vi) If $n(A \cup B) = 50$, $n(A) = 30$ and $n(B) = 35$, then $n(A \cap B) =$:

(a) 23 ✓(b) 15 (c) 9 (d) 40

(vii) If $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$, then cartesian product of A and B contains exactly ________ elements.

(a) 13 ✓(b) 12 (c) 10 (d) 6

(viii) If $f(x) = x^2 - 3x + 2$, then the value of $f(a + 1)$ is equal to:

(a) $a + 1$ (b) $a^2 + 1$ (c) $a^2 + 2a + 1$ ✓(d) $a^2 - a$

(ix) Given that $f(x) = 3x+1$, if $f(x)=28$, then the value of x is:

✓(a) 9 (b) 27 (c) 3 (d) 18

(x) Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$ two non-empty sets and $f : A \to B$ be a a function defined as $f = \{(1, a), (2, b), (3, b)\}$, then which of the following statement is true?

(a) f is injective ✓(b) f is surjective (c) f is bijective (d) f is into only

2. Write each of the following sets in tabular forms:

(i) $\{x \mid x = 2n,\ n \in N\}$ (ii) $\{x \mid x = 2m+1,\ m \in N\}$

(iii) $\{x \mid x = 11n, n \in W \wedge n < 11\}$ (iv) $\{x \mid x \in E \wedge 4 < x < 6\}$

(v) $\{x \mid x \in O \wedge 5 \leq x < 7\}$ (vi) $\{x \mid x \in Q \wedge x^2 = 2\}$

(vii) $\{x \mid x \in Q \wedge x = -x\}$ (viii) $\{x \mid x \in R \wedge x \notin Q'\}$

Solution

(i) $\{2, 4, 6, 8, 10, \ldots\}$ (ii) $\{3, 5, 7, 9, 11, \ldots\}$

(iii) $\{0, 11, 22, 33, 44, 55, 66, 77, 88, 99, 110\}$

(iv) ϕ (v) ϕ (vi) ϕ

(vii) $\{0\}$ (viii) Q

3. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{1, 3, 5, 7, 9\}$

List the members of each of the following sets:

(i) A' (ii) B' (iii) $A \cup B$ (iv) $A - B$

(v) $A \cap C$ (vi) $A' \cup C'$ (vii) $A' \cup C$ (viii) U'

Solution

i. $A' = U - A = \{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\} = \{1,3,5,7,9\}$

ii. $B' = U - B = \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5\} = \{6,7,8,9,10\}$

iii. $A \cup B = \{2,4,6,8,10\} \cup \{1,2,3,4,5\} = \{1,2,3,4,5,6,7,8,9,10\}$

iv. $A - B = \{2,4,6,8,10\} - \{1,2,3,4,5\} = \{6,8,10\}$

v. $A \cap C = \{2,4,6,8,10\} \cap \{1,3,5,7,9\} = \varphi$

vi. $A' \cup C' = \{1,3,5,7,9\} \cup \{2,4,6,8,10\} = \{1,2,3,4,5,6,7,8,9,10\}$

vii. $A' \cup C = \{1,3,5,7,9\} \cup \{1,3,5,7,9\} = \{1,3,5,7,9\}$

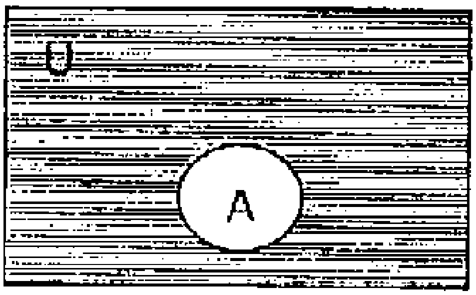
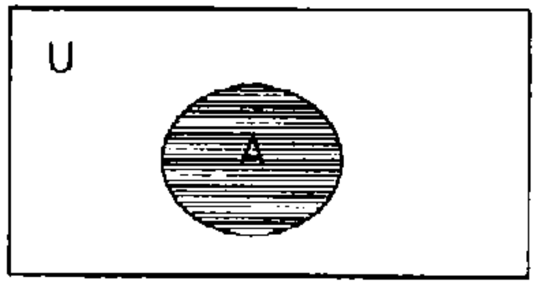
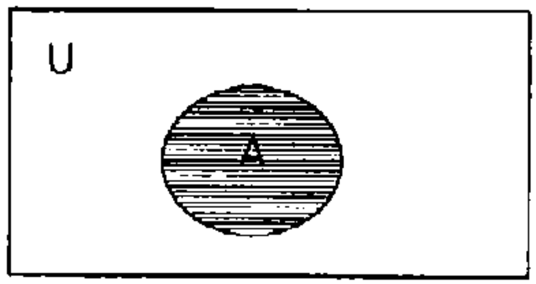
viii. $U' = U - U = \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5,6,7,8,9,10\} = \varphi$

4. Using the Venn diagrams, if necessary, find the single sets equal to the following:

(i) A' (ii) $A \cap U$ (iii) $A \cup U$

(iv) $A \cup \phi$ (v) $\phi \cap \phi$

Solution

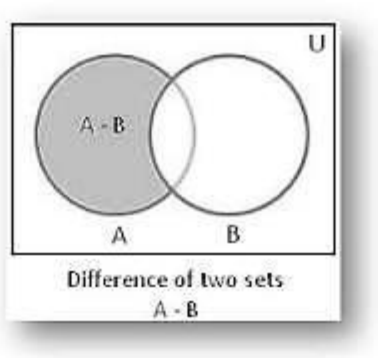
(i) $A' = U - A$  	**(ii) $A \cap U = A$** 
(iii) $A \cup U = U$  	**(iv) $A \cup \varphi = A$** 
(v) $\varphi \cap \varphi = \varphi$ It has no Venn diagram.	

5. Use Venn diagrams to verify the following:

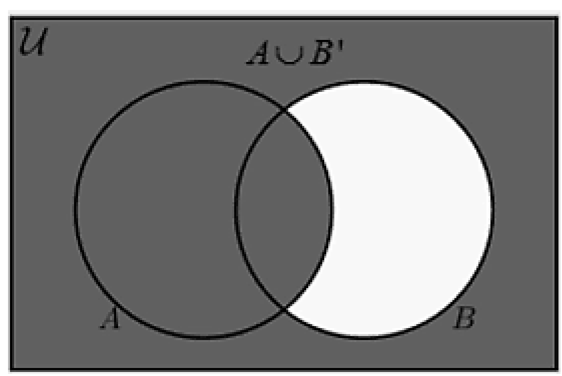
(i) $A - B = A \cup B'$ (ii) $(A - B)' \cap B = B$

Solution

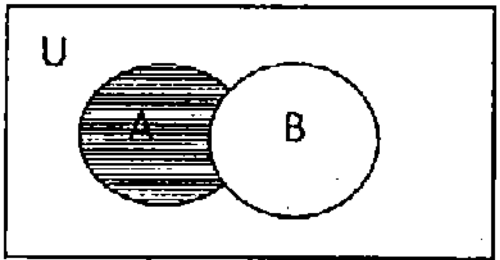
(i) $A - B = A \cup B'$



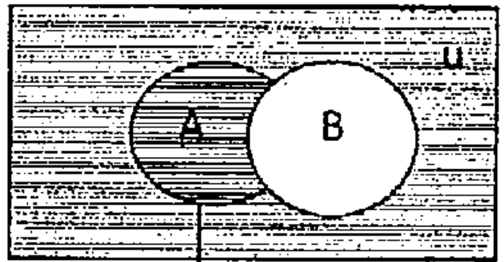
not equals to



(i) $\boldsymbol{A - B = A \cap B'}$



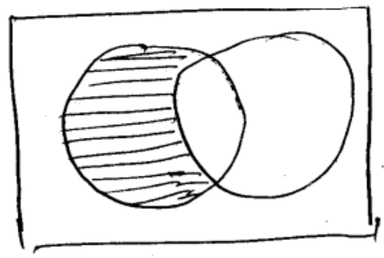
$A - B$



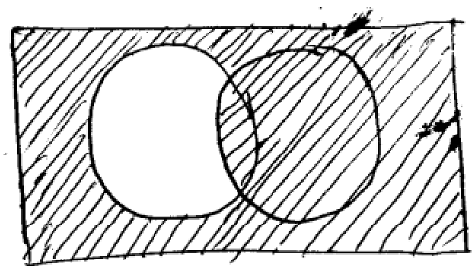
$A \cap B^c$

(ii) $\boldsymbol{(A - B)' \cap B = B}$

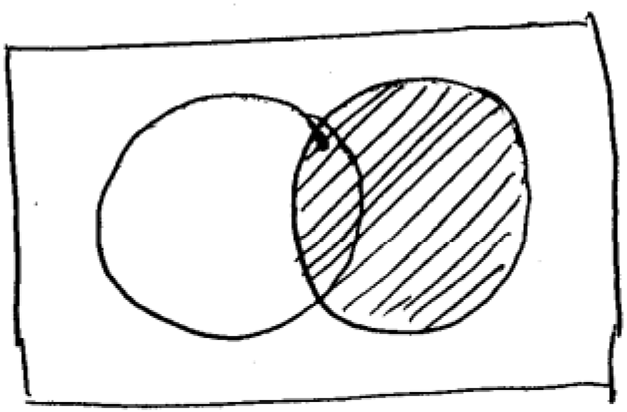
$A - B$



$(A - B)'$



Then $\boldsymbol{(A - B)' \cap B = B}$ **is**



6. Verify the properties for the sets A, B and C given below:

(i) Associativity of Union (ii) Associativity of intersection.

(iii) Distributivity of Union over intersection.

(iv) Distributivity of intersection over union.

(a) $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7, 8\}$, $C = \{5, 6, 7, 9, 10\}$

(b) $A = \phi$, $B = \{0\}$, $C = \{0, 1, 2\}$

(c) $A = N$, $B = Z$, $C = Q$

Solution

(a) $A = \{1,2,3,4\}\,; B = \{3,4,5,6,7,8\}\,; C = \{5,6,7,9,10\}$

i. Associativity of Union: $\mathbf{A \cup (B \cup C) = (A \cup B) \cup C}$

$L.H.S = A \cup (B \cup C) = \{1,2,3,4\} \cup [\{3,4,5,6,7,8\} \cup \{5,6,7,9,10\}]$

$A \cup (B \cup C) = \{1,2,3,4\} \cup \{3,4,5,6,7,8,9,10\} = \{1,2,3,4,5,6,7,8,9,10\}$

$R.H.S = (A \cup B) \cup C = [\{1,2,3,4\} \cup \{3,4,5,6,7,8\}] \cup \{5,6,7,9,10\}$

$(A \cup B) \cup C = \{1,2,3,4,5,6,7,8\} \cup \{5,6,7,9,10\} = \{1,2,3,4,5,6,7,8,9,10\}$

Hence $A \cup (B \cup C) = (A \cup B) \cup C$

ii. Associativity of Intersection: $\mathbf{A \cap (B \cap C) = (A \cap B) \cap C}$

$L.H.S = A \cap (B \cap C) = \{1,2,3,4\} \cap [\{3,4,5,6,7,8\} \cap \{5,6,7,9,10\}]$

$A \cap (B \cap C) = \{1,2,3,4\} \cap \{5,6,7\} = \{\quad\}$

$R.H.S = (A \cap B) \cap C = [\{1,2,3,4\} \cap \{3,4,5,6,7,8\}] \cap \{5,6,7,9,10\}$

$(A \cap B) \cap C = \{3,4\} \cap \{5,6,7,9,10\} = \{\quad\}$

Hence $A \cap (B \cap C) = (A \cap B) \cap C$

iii. Distributivity of Union over Intersection: $\mathbf{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$

$L.H.S = A \cup (B \cap C) = \{1,2,3,4\} \cup [\{3,4,5,6,7,8\} \cap \{5,6,7,9,10\}]$

$A \cup (B \cap C) = \{1,2,3,4\} \cup \{5,6,7\} = \{1,2,3,4,5,6,7\}$

$R.H.S = (A \cup B) \cap (A \cup C) = [\{1,2,3,4\} \cup \{3,4,5,6,7,8\}] \cap [\{1,2,3,4\} \cup \{5,6,7,9,10\}]$

$(A \cup B) \cap (A \cup C) = \{1,2,3,4,5,6,7,8\} \cap \{1,2,3,4,5,6,7,9,10\} = \{1,2,3,4,5,6,7\}$

Hence $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

iv. Distributivity of Intersection over Union: $\mathbf{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

$L.H.S = A \cap (B \cup C) = \{1,2,3,4\} \cap [\{3,4,5,6,7,8\} \cup \{5,6,7,9,10\}]$

$A \cap (B \cup C) = \{1,2,3,4\} \cap \{3,4,5,6,7,8,9,10\} = \{3,4\}$

$R.H.S = (A \cap B) \cup (A \cap C) = [\{1,2,3,4\} \cap \{3,4,5,6,7,8\}] \cup [\{1,2,3,4\} \cap \{5,6,7,9,10\}]$

$(A \cap B) \cup (A \cap C) = \{3,4\} \cup \{\quad\} = \{3,4\}$

Hence $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b) $A = \varphi\ ; B = \{0\}\ ; C = \{0,1,2\}$

i. Associativity of Union: $\mathbf{A \cup (B \cup C) = (A \cup B) \cup C}$

$\text{L.H.S} = A \cup (B \cup C) = \varphi \cup [\{0\} \cup \{0,1,2\}] = \varphi \cup \{0,1,2\} = \{0,1,2\}$

$\text{R.H.S} = (A \cup B) \cup C = [\varphi \cup \{0\}] \cup \{0,1,2\} = \{0\} \cup \{0,1,2\} = \{0,1,2\}$

Hence $A \cup (B \cup C) = (A \cup B) \cup C$

ii. Associativity of Intersection: $\mathbf{A \cap (B \cap C) = (A \cap B) \cap C}$

$\text{L.H.S} = A \cap (B \cap C) = \varphi \cap [\{0\} \cap \{0,1,2\}] = \varphi \cap \{0\} = \varphi$

$\text{R.H.S} = (A \cap B) \cap C = [\varphi \cap \{0\}] \cap \{0,1,2\} = \varphi \cap \{0,1,2\} = \varphi$

Hence $A \cap (B \cap C) = (A \cap B) \cap C$

iii. Distributivity of Union over Intersection: $\mathbf{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$

$\text{L.H.S} = A \cup (B \cap C) = \varphi \cup [\{0\} \cap \{0,1,2\}] = \varphi \cup \{0\} = \{0\}$

$\text{R.H.S} = (A \cup B) \cap (A \cup C) = [\varphi \cup \{0\}] \cap [\varphi \cup \{0,1,2\}] = \{0\} \cap \{0,1,2\} = \{0\}$

Hence $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

iv. Distributivity of Intersection over Union: $\mathbf{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

$\text{L.H.S} = A \cap (B \cup C) = \varphi \cap [\{0\} \cup \{0,1,2\}] = \varphi \cap \{0,1,2\} = \varphi$

$\text{R.H.S} = (A \cap B) \cup (A \cap C) = [\varphi \cap \{0\}] \cup [\varphi \cap \{0,1,2\}] = \varphi \cup \varphi = \varphi$

Hence $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(c) $A = N = \{1,2,3,\dots\}\ ; B = Z = \{0,\pm 1,\pm 2,\pm 3,\dots\}\ ; C = Q\ ;\ \mathbf{N \le Z \le Q}$

i. Associativity of Union: $\mathbf{A \cup (B \cup C) = (A \cup B) \cup C}$

$\text{L.H.S} = A \cup (B \cup C) = N \cup [Z \cup Q] = N \cup Q = Q$

$\text{R.H.S} = (A \cup B) \cup C = [N \cup Z] \cup Q = Z \cup Q = Q$

Hence $A \cup (B \cup C) = (A \cup B) \cup C$

ii. Associativity of Intersection: $\mathbf{A \cap (B \cap C) = (A \cap B) \cap C}$

$\text{L.H.S} = A \cap (B \cap C) = N \cap [Z \cap Q] = N \cap Z = N$

$\text{R.H.S} = (A \cap B) \cap C = [N \cap Z] \cap Q = N \cap Q = N$

Hence $A \cap (B \cap C) = (A \cap B) \cap C$

iii. Distributivity of Union over Intersection: $\mathbf{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$

$\text{L.H.S} = A \cup (B \cap C) = N \cup [Z \cap Q] = N \cup Z = Z$

$\text{R.H.S} = (A \cup B) \cap (A \cup C) = [N \cup Z] \cap [N \cup Q] = Z \cap Q = Z$

Hence $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

iv. Distributivity of Intersection over Union: $\mathbf{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

$\text{L.H.S} = A \cap (B \cup C) = N \cap [Z \cup Q] = N \cap Q = N$

$\text{R.H.S} = (A \cap B) \cup (A \cap C) = [N \cap Z] \cup [N \cap Q] = N \cup N = N$

Hence $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

7. Verify De Morgan's Laws for the following sets:
$U = \{1, 2, 3, ..., 20\}$, $A = \{2, 4, 6, ..., 20\}$ and $B = \{1, 3, 5, ...,19\}$.

Solution

$(A \cup B)' = A' \cap B'$ U = {1, 2, 3, ..., 20} A = {2, 4, 6, ..., 20} B = {1, 3, 5, ..., 19} $A' = U - A$ = {1, 2, 3, ..., 20} - {2, 4, 6, ..., 20} = {1, 3, 5, ..., 19} $B' = U - B$ = {1, 2, 3, ..., 20} - {1, 3, 5, ..., 19} = {2, 4, 6, ..., 20} $A \cup B$ = {1, 2, 3, ..., 20} $(A \cup B)' = U - (A \cup B)$ = {1, 2, 3, ..., 20} - {1, 2, 3, ..., 20} = { } $A' \cap B'$= {1, 3, 5, ..., 19} ∩ {2, 4, 6, ..., 20} = { } Hence $(A \cup B)' = A' \cap B'$	**$(A \cap B)' = A' \cup B'$** U = {1, 2, 3, ..., 20} A = {2, 4, 6, ..., 20} B = {1, 3, 5, ..., 19} $A' = U - A$ = {1, 2, 3, ..., 20} - {2, 4, 6, ..., 20} = {1, 3, 5, ..., 19} $B' = U - B$ = {1, 2, 3, ..., 20} - {1, 3, 5, ..., 19} = {2, 4, 6, ..., 20} $A \cap B$ = { } $(A \cap B)' = U - (A \cap B)$ = {1, 2, 3, ..., 20} - { } = {1, 2, 3, ..., 20} $A' \cup B'$= {1, 3, 5, ..., 19} ∪ {2, 4, 6, ..., 20} = {1, 2, 3, ..., 20} Hence $(A \cap B)' = A' \cup B'$

8. Consider the set $P = \{x \mid x = 5m, m \in N\}$ and $Q = \{x \mid x = 2m, m \in N\}$. Find $P \cap Q$

Solution

P = {x | x = 5m, m ∈ N} = {5, 10, 15, 20, 25, ...}

Q = {x | x = 2m, m ∈ N} = {2, 4, 6, 8, 10, 12, ...}

P ∩ Q = {5, 10, 15, 20, 25, ...} ∩ {2, 4, 6, 8, 10, 12, ...}

P ∩ Q = {10, 20, 30, 40, 50, ...} = {x | x = 10m, m ∈ N}

9. From suitable properties of union and intersection, deduce the following results:
(i) $A \cap (A \cup B) = A \cup (A \cap B)$ (ii) $A \cup (A \cap B) = A \cap (A \cup B)$

Solution

(i) L.H.S. = $A \cap (A \cup B) = (A \cap A) \cup (A \cap B) = A \cup (A \cap B) =$ **R.H.S.**

(i) L.H.S. = $A \cup (A \cap B) = (A \cup A) \cap (A \cup B) = A \cap (A \cup B) =$ **R.H.S.**

10. If $g(x) = 7x - 2$ and $s(x) = 8x^2 - 3$ find:

(i) $g(0)$ (ii) $g(-1)$ (iii) $g\left(-\frac{5}{3}\right)$ (iv) $s(1)$ (v) $s(-9)$ (vi) $s\left(\frac{7}{2}\right)$

Solution

i. $g(x) = 7x - 2 \Rightarrow g(0) = 7(0) - 2 \Rightarrow g(0) = -2$

ii. $g(x) = 7x - 2 \Rightarrow g(-1) = 7(-1) - 2 \Rightarrow g(-1) = -7 - 2 \Rightarrow g(-1) = -9$

iii. $g(x) = 7x - 2 \Rightarrow g\left(-\frac{5}{3}\right) = 7\left(-\frac{5}{3}\right) - 2 \Rightarrow g\left(-\frac{5}{3}\right) = -\frac{35}{3} - 2 \Rightarrow g\left(-\frac{5}{3}\right) = -\frac{41}{3}$

iv. $s(x) = 8x^2 - 3 \Rightarrow s(1) = 8(1)^2 - 3 \Rightarrow s(1) = 8 - 3 \Rightarrow s(1) = 5$

v. $s(x) = 8x^2 - 3 \Rightarrow s(-9) = 8(-9)^2 - 3 \Rightarrow s(-9) = 648 - 3 \Rightarrow s(-9) = 645$

vi. $s(x) = 8x^2 - 3 \Rightarrow s\left(\frac{7}{2}\right) = 8\left(\frac{7}{2}\right)^2 - 3 \Rightarrow s\left(\frac{7}{2}\right) = 98 - 3 \Rightarrow s\left(\frac{7}{2}\right) = 95$

11. Given that $f(x) = ax + b$, where a and b are constant numbers. If $f(-2) = 3$ and $f(4) = 10$, then find the values of a and b.

Solution

$f(x) = ax + b$

$f(-2) = a(-2) + b$	$f(4) = a(4) + b$
$\mathbf{-2a + b = 3}$ **……(i)**	$\mathbf{4a + b = 10}$ **……(ii)**

(i) - (ii)	**2(i) + (ii)**
$-2a + b = 3$	$-4a + 2b = 6$
$-4a \pm b = -10$	$4a + b = 10$
$\mathbf{a = \frac{7}{6}}$	$\mathbf{b = \frac{16}{3}}$

12. Consider the function defined by $k(x) = 7x - 5$. If $k(x) = 100$, find the value of x.

Solution

$k(x) = 7x - 5$

Using $k(x) = 100$

$7x - 5 = 100$

$7x = 100 + 5$

$x = \frac{105}{7}$

13. Consider the function $g(x) = mx^2+n$, where m and n are constant numbers. If $g(4) = 20$ and $g(0) = 5$, find the values of m and n.

Solution

$g(x) = mx^2 + n$

$g(0) = m(0)^2 + n$

$\mathbf{n = 5}$

Now $g(4) = m(4)^2 + n$

$\mathbf{16m + n = 20}$

Using $n = 5$

$16m + 5 = 20$

$\mathbf{m = \frac{15}{16}}$

14. A shopping mall has 100 products from various categories labeled 1 to 100, representing the universal set U. The products are categorized as follows:

- Set A: Electronics, consisting of 30 products labeled from 1 to 30.
- Set B: Clothing comprises 25 products labeled from 31 to 55.
- Set C: Beauty Products, comprising 25 products labeled from 76 to 100.

Write each set in tabular form, and find the union of all three sets.

Solution

U = {1, 2, 3, ... , 100}

A = {1, 2, 3, ... , 30}

B = {31, 32, 33, ... , 55}

C = {76, 77, 78, ... , 100}

A ∪ B ∪ C = {1, 2, 3, ..., 30} ∪ {31, 32, ..., 55} ∪ {76, 77, ..., 100}

A ∪ B ∪ C = {1, 2, 3, ..., 30, 31, 32, ..., 55, 76, 77, ..., 100}

15. Out of the 180 students who appeared in the annual examination, 120 passed the math test, 90 passed the science test, and 60 passed both the math and science tests.
    (a) How many passed either the math or science test?
    (b) How many did not pass either of the two tests?
    (c) How many passed the science test but not the math test?
    (d) How many failed the science test?

Solution

Total students = 180

Passed Math = 120, Passed Science = 90, Passed both Math and Science = 60

1. How many passed either the Math or Science test?

Passed either Math or Science = Passed Math + Passed Science - Passed both

= 120 + 90 – 60 = 150

2. How many did not pass either of the two tests?

Failed both Math and Science = Total students - Passed either Math or Science

= 180 – 150 = 30

3. How many passed the Science test but not the Math test?

Passed Science but not Math = Passed Science - Passed both

= 90 – 60 = 30

4. How many failed the Science test?

Failed Science = Total students - Passed Science

= 180 – 90 = 90

16. In a software house of a city with 300 software developers, a survey was conducted to determine which programming languages are liked more. The survey revealed the following statistics:

- 150 developers like Python.
- 130 developers like Java.
- 120 developers like PHP.
- 70 developers like both Python and Java.
- 60 developers like both Python and PHP.
- 50 developers like both Java and PHP.
- 40 developers like all three languages: Python, Java and PHP.

(a) How many developers use at least one of these languages?

(b) How many developers use only one of these languages?

(c) How many developers do not use any of these languages?

(d) How many developers use only PHP?

Solution

Total developers = 300

$n(P) = 150$, $n(J) = 130$, $n(H) = 120$, $n(P \cap J) = 70$, $n(P \cap H) = 60$, $n(J \cap H) = 50$

$n(P \cap J \cap H) = 40$

1. How many developers use at least one of these languages?

$n(P \cup J \cup H) = n(P) + n(J) + n(H) - n(P \cap J) - n(P \cap H) - n(J \cap H) + n(P \cap J \cap H)$

$= 150 + 130 + 120 - 70 - 60 - 50 + 40 =$ **260**

2. How many developers use only one of these languages?

Developers who use only P $= n(P) - n(P \cap J) - n(P \cap H) + n(P \cap J \cap H)$

$= 150 - 70 - 60 + 40 = 60$

Developers who use only J = n(J) – n(P ∩ J) – n(J ∩ H) + n(P ∩ J ∩ H)

= 130 - 70 - 50 + 40 = 50

Developers who use only H = n(H) – n(P ∩ H) – n(J ∩ H) + n(P ∩ J ∩ H)

= 120 - 60 - 50 + 40 = 50

Total developers who use only one language = 60 + 50 + 50 = **160**

3. How many developers do not use any of these languages?

Developers who do not use any language = Total developers - Developers who use at least one language

= 300 – 260 = **40**

4. How many developers use only PHP?

Developers who use only PHP = n(H) = **50**