Exercise 2.1 (Solutions)
Mathematics (Science Group): 10th

Written by Amir Shahzad, Version: 1.0

1. Find the discriminant of the following given quadratic equation.

(i) $2x^2+3x-1$

Solution:

$$2x^2+3-1=0$$
$$compare\ it\ with$$
$$ax^2+bx+c=0$$

⇨ $a=2,\ \ b=3\ \ ,c=-1$

$$Disc.=b^2-4ac$$
$$=(3)^2-4(2)(-1)$$
$$=9+8$$
$$=17$$

(ii) $6x^3-8x+3=0$

Solution:

$$6x^3-8x+3=0$$
$$Compare\ it\ with$$
$$ax^2+bx+c=0$$

⇨ $a=6,\ b=-8\ \ c=3$

$$Disc.=b^2-4ac$$
$$=(-8)^2-4(6)(3)$$
$$=64-72$$
$$=-8$$

(iii) $9x^2-30x+25=0$

Solution:

$$9x^2-30x+25=0$$
$$Compare\ it\ with$$
$$ax^2+bx+c=0$$

⇨ $a=9, b=-30, c=25$

$$Disc.=b^2-4ac$$
$$=(-30)^2-4(9)(25)$$
$$=900-900$$
$$=0$$

(iv) $4x^2-7x-2=0$

Solution:

$$4x^2-7x-2=0$$
$$Compare\ it\ with$$
$$ax^2+bx+c=0$$

⇨ $a=4, b=-7, c=-2$

$$Disc.=b^2-4ac$$
$$=(-7)^2-4(4)(-2)$$
$$=49+32$$
$$=81$$

2. Find the nature of the roots of the follow given quadratic and verify the result by solving equations:

(i)$x^2+23x+120=0$

Solution:

$$x^2+23x+120=0$$
$$Compare\ it\ with$$
$$=49$$
$$=(7)^2>0$$

As the disc.is possible and is perfect square. Therefore the roots are rational (real) and unequal, verification by solving the equation.

$$x^2-23x+120=0$$
$$x^2-15x-8x+120=0$$
$$x(x-15)-8(x-15)=0$$
$$(x-15)(x-8)=0$$
$$Either\quad x-8=0\quad or\ x-15=0$$
$$x=8\qquad x=15$$

Thus, the roots are rational (real) and unequal.

(ii) $2x^2+3x+7=0$

Solution:

$$2x^2+3x+7=0$$
$$Compare\ it\ with$$

⇨ $a=2, b=3, c=7$

$$Disc.=b^2-4ac$$
$$=(3)^2-4(2)(7)$$
$$=9-56$$
$$=-47<0$$
$$As\ the\ Disc.\ is\ negative.$$

Therefore the roots are imaginary and unequal.
Verification by solving the equation.

$$2x^2+3x+7=0$$
$$Using\ quadratic\ formula$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-3\pm\sqrt{(3)^2-4(2)(7)}}{2a}$$
$$=\frac{-3\pm\sqrt{9-56}}{4}$$
$$Thus, the\ roots\ are\ imaginary\ and\ unequal$$

(iii) $16x^2-24x+9=0$

Solution:

$$16x^2-24x+9=0$$
$$Compare\ it\ with$$
$$ax^2+bx+c=0$$

⇨ $a=16, b=-24, c=9$

$$Disc.=b^2-4ac$$
$$=(-24)^2-4(16)(9)$$
$$=576-576$$
$$=0$$

$As\ the\ Disc.\ is\ zero$

Therefore the roots of the equation are real and equal.
Verification by solving the equations.

$$16x^2-24x+9=0$$
$$using\ quadratic\ formula$$

$$ax^2+bx+c=0$$
⇨ $a=1, b=-23, c=120$
$$Disc.=b^2-4ac$$
$$=(-23)^2-4(1)(120)$$
$$=529-480$$

$$=\frac{24\pm\sqrt{576-576}}{32}$$
$$=\frac{24\pm\sqrt{0}}{32}$$
$$=\frac{24}{32}=\frac{3}{4}$$

Thus the roots are real and unequal.

iv) $3x^2+7x-13=0$

Solution:

$$3x^2+7x-13=0$$
$$Compare\ it\ with$$
$$ax^2+bx+c=0$$
⇨ $a=3,\ b=7,\ c=-13$
$$Disc.=b^2-4ac$$
$$=(7)^2-4(3)(-13)$$
$$=49+156$$
$$=205>0$$

As the Disc. Is positive and is not perfect square.
Therefore the roots are irrational (real) and unequal.
Verification by solving the equation.

$$3x^2+7x-13=0$$

Using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-7\pm\sqrt{(7)^2-4(3)(-13)}}{2a}$$
$$=\frac{-7\pm\sqrt{49+156}}{6}$$
$$=\frac{-7\pm\sqrt{205}}{6}$$

Thus, the roots are irrational (real) and unequal.

3. **For what value of A, the expression $k^2x^2+2(k+1)x+4\ is\ square$.**

Solution:

$$k^2x^2+2(k+1)x+4=0$$
$$Compare\ it\ with$$
$$ax^2+bx+c=0$$
⇨ $a=k^2,\ b=2(k-1), c=4$
$$Disc.=b^2-4ac$$
$$=\left(2(k-1)\right)^2-4(k^2)(4)$$
$$=4(k^2-2k+1)-16k^2$$
$$=4k^2-8k+4-16k^2$$
$$=-12k^2-8k+4=0$$

As the disc. Of the given expression is a perfect square.
Therefore the roots are rational and equal.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-24)\pm\sqrt{(-24)^2-4(16)(9)}}{2(16)}$$
$$(12k+4k)(k-1)=0$$
$$Either\quad 12k+4=0\quad or\ k-1=0$$
$$12k=-4\quad or\ \ k=1$$
$$k=-\frac{4}{12}$$
$$k=-\frac{1}{3}$$

4. **Find the value of k, if the roots of the following equations are equal.**

(i) $(2k+1)x^2+3Kx+3=0$

⇨ $a=2k+1, b=3k,\ c=3$

$As\ the\ roots\ are\ equal, So$
$$Disc.=0$$
$$b^2-4ac=0$$
$$(3k^2)-4(2k+1)(3)=0$$
$$9k^2-12(2k+1)=0$$
$$9k^2-24k-12=0$$
$$3(3k^2-8k-4)=0$$
$$⇨\ 3k^2-8k-4=0$$
$$3k^2-6k-2k+4=0$$
$$3k(k-2)-2(k-2)=0$$
$$(3k-2)(k-2)=0$$
$$Either\quad 3k-2=0\quad or\quad k-2=0$$
$$3k=2\quad or\ k=2$$
$$k=\frac{2}{3}\quad or\quad k=2$$

(ii) $x^3+2(k+2)x+(3k+4)=0$

Solution:

$$x^3+2(k+2)x+(3k+4)=0$$
⇨ $a=1\ \ b=2(k+2)\ \ c=3k+4$

As the roots are equal
$$Disc.=0$$
$$b^2-4ac=0$$
$$[2(k+2)]^2-4(1)(3k+4)=0$$
$$4(k+2)^2-4(3k+4)=0$$
$$4(k^2+4k+4)-12k-16=0$$
$$4k^2+4k+4-12k-16=0$$
$$4k^2+4k=0$$
$$4k(k+1)=0$$
$$Either\quad 4k=0\quad (k+1)=0$$
$$k=0\quad or\quad k=-1$$

(iii) $(3k+2)x^2-5(k+1)x+(2k+3)=0$

Solution:

$$(3k+2)x^2-5(k+1)x+(2k+3)=0$$
⇨ $a=3k+2,\quad b=-5(k+1),\quad c=(2k+3)$

$As\ the\ roots\ are\ equal, So$
$$Disc.=0$$
$$b^2-4ac=0$$
$$[-5(k+1)]^2-4(3k+2)(2k+3)=0$$

So, $Disc. = 0$

$$-12k^2 + 8k + 4 = 0$$
$$-(12k^2 + 8k + 4) = 0$$
$$\Rightarrow\ 12k^2 - 8k - 4 = 0$$
$$12k^2 - 12k + 4k - 4 = 0$$
$$12k(k-1) + 4(k-1) = 0$$

5. **Show that the equation $x^2 + (mx+c)^2 = a^2$ has**

Equal roots,

$$\boldsymbol{if\ c^2 = a62\ (1+m^2)}$$

Solution:

$$x^2 + (mx+c)^2 = a^2$$
$$x^2 + m^2x^2 + 2mcx + c^2 = a^2$$
$$(1+m^2)x62 + 2mcx + c^2 - a^2 = 0$$
$$a = 1 + m^2, \quad b = 2mc, \ \ c = c^2 = a^2$$
$$As\ the\ roots\ are\ equal, So$$
$$Disc. = 0$$
$$b^2 - 4ac = 0$$
$$(2mc)^2 - 4(1+m^2)(c^2 - a^2) = 0$$
$$4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - a^2m^2) = 0$$
$$4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4a^2m^2 = 0$$
$$-4c^2 + 4a^2 + 4ac^2 + 4a^2m^2 = 0$$
$$-4(c^2 - a^2 - a^2m^2) = 0$$
$$c^2 - a^2 - a62\ m^2 = 0$$
$$c^2 = a^2 + a^2m^2$$
$$c^2 = a^2(a + m^2)$$

Hence proved.

6. **Find the condition that the roots of the equation $(my+c)^2 - 4ax = 0$ *are equal*.**

Solution:

$(my+c)^2 - 4ax = 0$

$m^2x^2 + 2mcx + c^2 - 4ax = 0$

$m^2x^2 + 2mcx - 4ax + c^2 = 0$

$m^2x^2 + 2(mc - 2a)x + c^2 = 0$

⇨ $a = m^2,\ b = 2(mc - 2a),\ c = c^2$

As the roots are equal $\quad Disc. = 0$

$b^2 - 4ac = 0$

$[2(mc - 2a)]^2 - 4(m^2)(c^2) = 0$

$4(mc - 2a)^2 - 4m^2c^2 = 0$

$4(m^2c^2 - 4amc + 4a^2) - 4m^2c^2 = 0$

$4(\ m^2c^2 - amc + a^2 - m^2c^2) = 0$

⇨ $4a^2 - 4amc = 0$

⇨ 4a(a-mc)=0

⇨ $a - mc = 0$

⇨ $a = mc$

Which is required condition.

7. **If the roots of the equation**

$$\boldsymbol{(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^3 - ac) = 0\ are\ equal}$$

$\boldsymbol{, then\ a = 0\ or\ a^3 + b^3 + c^3 = 3abc}$

Solution:

$$25(k^2 + 2k + 1) - 4(6k^2 + 13k + 6) = 0$$
$$25k^2 + 50k + 25 - 24k^2 - 52k - 24 = 0$$
$$k^2 - 2k + 1 = 0$$
$$(k-1)^2 = 0$$
$$k - 1 = 0 \Rightarrow k=1$$

$$a(a^3 + b^3 + c^3 - 3abc) = 0$$

Either $a = 0 \quad or \quad a^3 + b^3 + c^3 - 3abc = 0$

Hence proved $\quad a^3 + b^3 + c^3 - 3abc$

8. **Show that the roots of the following equations are rational.**

(i) $\quad \boldsymbol{a(b-c)x^2 + b(c-a)x + c(a-b)} = 0$

Solution:

⇨ $a = a(b-c),\ b = b(c-a),\ \ c = c(a-b)$

$$Disc. = b^2 - 4ac$$
$$= [b(c-a)]^2 - 4[a(b-c)][c(a-b)]$$
$$= b^2(c-a)^2 - 4ac(b-c)(a-b)$$
$$= b^2(c^2 - 2ac + a^2) - 4ac(ab - b^2 - ac + bc)$$
$$= b^2c^2 - 2ab^2a^2 - 4a^2cb + 4acb^2 + 4a^2c^2 - 4abc^2$$
$$= a^2b^2 + b^2c^2 + 44a^2c^2 + 2ab^2c - 4a^2bc - 4abc^2$$
$$= (ab)^2 + (bc)62 + (-2ac)^2 + 2(ab)(bc) + 2(bc)(-2ac) + 2(-2ac)(ab)$$
$$= (ab + bc - 2ac)^2$$

Hence the roots are rational.

(ii) $\quad \boldsymbol{(a+2b)x^2 + 2(a+b+c)x + (a+2c) = 0}$

Solution:

⇨ $a = (a+2b),\ b = 2(a+b+c)\ ,\ c = (a+2c)$

$$Disc. = b^2 - 4ac$$
$$= [2(a+b+c)]^2 - 4(a+2b)(a+2c)$$
$$= 4(a+b+c)^2 - 4(a^2 + 2ac + 2ab + 4bc)$$
$$= 4[a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - a^2 - 2ac - 4bc]$$
$$= 4(b-c)^2$$

Hence the roots are rational.

9. **For all valves of k, prove that the roots of the equation.**

$$x^2 - 2\left(k + \frac{1}{k}\right)x + 3 = 0$$

Solution:

⇨ $a = 1,\ b = -2\left(k + \frac{1}{k}\right),\ c = 3$

$$Disc. = b^2 - 4ac$$
$$= \left[-2\left(k + \frac{1}{k}\right)\right]^2 - 4(1)(3)$$
$$= 4\left(k + \frac{1}{k}\right)^2 - 12$$
$$= 4\left[\left(k + \frac{1}{k}\right)^2 - 3\right]$$
$$= \left[k^2 + \frac{1}{k^2} + 2 - 3\right]$$
$$= \left[k^2 + \frac{1}{k^2} - 1\right] > 0$$

Hence the roots are real.

$(c^2-ab)x^2-2(a^2-bc)x+(b^3-ac)=0$

⇨ $a=c^3-ab, b=-2(a^2-bc), c=b^2-ac$

As the roots are equal so

$$Disc.=0$$
$$b^2-4ac=0$$
$$[-2(a^2-bc)]^2-4(c^2-ab)(b^2-ac)=0$$
$$4(a^2-bc)^2-4(c^2-ab)(b^2-ac)=0$$
$$4[a^4-2a^2bc+b^2c^2)-(b^2c^2-ac^3+ab^3+a^2bc)]=0$$
$$a^4-2a^2bc+b^2c^2-b^2c^2+ac^3+ab^3+a^2bc=0$$

⇨ $a^4+ab^3+ac^3-3a^2bc=0$

$$=c^2-2ac+a^2-ab+4b^2+4ac-4bc$$

$$=a^2+4b^2+c^2-4ab-4bc+2ac$$
$$=(a)^2+(-2b)^2+(c)^2+2(a)(-2b)+2(-2b)(c)+2(a)(c)$$
$$=(a-2b+c)^2>0$$

hence the roots of the equation are real.

10. Show that the roots of the equation.

$$(b-c)x^2+(c-a)x+(a-b)^2=0$$

Solution:

⇨ $a=(b-c),\ b=(c-a),\ c=(a-b)$

$$Disc.=b^2-4ac$$
$$=(c-a)^2-4(b-c)(a-b)$$
$$=(c^2-2ac+a^2)-4(ab-b^2-ac+bc)$$
$$=a^2+4b^2+c^2-4ab-4bc+2ac$$
$$=(a)^2+(-2b)^2+(c)^2+2(a)(-2b)+2(-2b)(c)+2(a)(c)$$
$$=(a-2b+c)^2>0$$

hence the roots of the equation are real

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