1. Uses synthetic division to find the quotient and the remainder, when

(i) $(x^2+7x-1)\div(x+1)$

Solution:

$P(x)=x^2+7x-1$

$x+1=x-(-1)$

⇨ $a=-1$

	1	7	-1
-1	↓	-1	-6
	1	6	-7

$Quotient=Q(x)=x+6$

$Remainder=-7$

(ii) $(4x^3)-5x+15)\div(x-3)$

Solution:

$P(x)=4x^3-5x+15)\div(x+3)$

$(x+3)=x-(-3)\Rightarrow a=-3$

	4	0	-5	15
-3	↓	-12	36	-93
	1	-12	31	-78

$Quotient=4x^2-12x+31$

$Remainder=-78$

(iii) $(x^3+x^2-3x+2)\div(x-2)$

Solution:

$P(x)=(x^3+x^2-3x+2)$

$x-2=x-(2)\Rightarrow a=2$

	1	1	-3	2
2	↓	2	6	6
	1	3	3	8

$Quotient=x^2+3x+3$

$Remainder=8$

2. Find the value of h using synthetic division, if

(i) $3\ is\ the\ zero\ of\ the\ polynomial\ 2x^3-3hx^2+9$

Solution:

$P(x)=2x^3-3hx^2+9\quad and\ its\ root\ is\ 3$

	2	$-3h$	0	9
3	↓	6	$3(6-3h)$	$9(6-3h)$
	1	$(6-3h)$	$3(6-3h)$	$9+9(6-3h)$

$$Quotient=Q(x)=2x^2+(6-3h)x+3(6-3h)$$
$$Remainder=9+9(6-3h)$$

$$9+9(6-3h)=0$$
$$9+9(6-3h)=0$$
$$9+54-27h=0$$
$$63-27h=0$$
$$63-27h=0$$
$$-27h=-63$$
$$h=\frac{63}{27}$$
$$h=\frac{7}{3}$$

(iii) $1\ is\ the\ zero\ of\ the\ polynomial\ 2x^3-2hx^2+11$

Solution:

$P(x)=2x^3-2hx^2+11\quad and\ its\ root\ is\ 1$

	1	$-2h$	0	11
1	↓	1	$(1-2h)$	$(1-2h)$
	1	$(1-2h)$	$(1-2h)$	$11+(1-2h)$

$$Quotient=Q(x)=x^2+(1-2h)x+(1-2h)$$
$$Remainder=11+(1-2h)$$

$$11+(1-2h)=0$$
$$11+1-2h=0$$
$$12-2h=0$$
$$-2h=-12$$
$$h=6$$

(iv) $-1\ is\ the\ zero\ of\ the\ polynomial\ 2x^3+5hx-23$

Solution:

$P(x)=2x^3+5hx-23$ and $its\ root\ is-1$

	2	0	$5h$	-23
-1	↓	-2	2	$-(5h+2)$
	2	-2	$(5h+2)$	$-23-(5h+2)$

$$Quotient=Q(x)=2x^2-2x+(5h+2)$$
$$Remainder=-23-(5h+2)$$

$$-23-(5h+2)=0$$
$$-23-5h-2=0$$
$$-23-2-5h=0$$
$$-25-5h=0$$
$$-5h=25$$
$$h=-5$$

3. $uses\ synthetic\ division\ to\ find\ the\ valves\ of\ l\ and\ m$

(i) $(x+3) and\ (x-2) are\ the\ factors\ of\ the\ polynomial\ x^3-4x^2+2lx+m$

Solution:

$x=-3\ and\ x=2\ are\ two\ roots\ for\ x=-3$

$Q(x)=x^3-4x^2+2lx+m$

	1	4	$2l$	m
-3	↓	-3	-3	$-3(2l-3)$
	1	-2	$(2l-3)$	$m-3(2l-3)$

$$Quotient=Q(x)=x^2+x+(2l-3)$$
$$Remainder=m-3(2l-3)$$

$m-3(2l-3)=0$
$m-6l+9=0 \rightarrow (i)$
For $x=2$

	1	4	$2l$	m
2	↓	2	12	$2(2l+12)$
	1	6	$(2l-3)$	$m+2(2l+12)$

$Quotient = Q(x) = x^2+6x+(2l+12)$
$Remainder = m+2(2l+12)$

$$m+2(2l+12)=0$$
$$m+2l+24=0 \rightarrow (ii)$$

$eq(i)-eq(ii)$

$$\begin{array}{c} m-6l+9=0 \\ -m \pm 4l \pm 24 = 0 \\ \hline -10l-15=0 \end{array}$$
$$-10l=15$$
$$l=\frac{15}{-10}$$
$$l=-\frac{3}{2}$$

put $l=-\frac{3}{2}$ put in eq (i)

$$m-6l+9=0$$
$$m-6\left(-\frac{3}{2}\right)+9=0$$
$$m-9+9=0$$
$$m+18=0$$
$$m=-18$$

$$\boxed{l=-\frac{3}{2}, m=-18}$$

(ii)
$(x-1) and\ x+1)$ *are the factors of the polynomial* $x^3-3lx^2+2mx+6$
Solution:
$P(x)=x^3-3lx^2+2mx+6$
are two roots $(x-1)$ *and* $(x+1)$
for $x=1$

	1	$-3l$	$2m$	6
1	↓	1	$1-3l$	$2m+(1-2l)$
	1	$(1-3l)$	$2m+(l-3l)$	$6+2m+(1-3l)$

$Quotient = Q(x) = x^2+(1-3l)x+2m+(1-3l)$
$Remainder = 6+2m+(1-3l)$

$6+2m+(1-3l)=0$
$6+2m+1-3l=0$
$7+2m-3l=0 \rightarrow (i)$
for $x=-1$

	1	$-3l$	$2m$	6
-1	↓	-1	$-(-3l-1)$	$-(2m-(3l-1)$
	1	$(-3l-1)$	$(1-2h)$	$6-(2m-(-3L-1))$

$Quotient = Q(x) = x^2-(3l+1)x+2m+(3l+1)$
$Remainder = 6-2m-(-3l-1)$

$$6-(2m+3l+1)=0$$
$$6-2m-(3l+1)=0$$
$$6-2m-3l-1=0$$
$$5-2m-3l=0 \rightarrow (ii)$$

$eq(i)+eq(ii) we\ get$

$$\begin{array}{c} 7+2m-3l=0 \\ 5-2m-3l=0 \\ \hline 12-6l=0 \end{array}$$
$$-6l=-12$$
$$l=2$$

$l=2\ put\ in\ eq(i)$

$$7+2m-3l=0$$
$$7+2m-3(2)=0$$
$$7+2m-6=0$$
$$1+2m=0$$
$$2m=-1$$
$$m=\frac{-1}{2}$$

$$\boxed{l=2, m=\frac{-1}{2}}$$

4. Solve by using synthetic division, if
(i) 2 *is the root of the equation* $x^3-28x+48=0$
Solution:

$$P(x)=x^3-28x+48=0$$

	1	0	-28	48
2	↓	2	4	-48
	1	6	-24	0

$Quotient = Q(x) = x^2+2x-24$
the depressed equation is $x^2+2x-24=0$
$x^2+2x-24=0$
$x^2+6x-4x-24=0$
$x(x+6)-4(x+6)=0$
$(x-4)(x+6)=0$
$(x-4)=0 \qquad (x+6)=0$
⇨ $x=4 \qquad x=-6$

Hence2, 4, -6 *are the roots of the given equation.*

(ii)

3 *is the root of the equation* $2x^3-3x^2-11x+6=0$

Solution:

$P(x)=2x^3-3x^2-11x+6=0$

		2	-3	-11	6
3		↓	6	9	-6
		1	3	-2	0

$Quotient=Q(x)=2x^2+3x-2=0$

the depressed equation is $2x^2+3x-2=0$

$$2x^2+3x-2=0$$
$$2x^2+4x-x-2=0$$
$$2x(x+2)-1(x+2)=0$$
$$(2x-1)(x+2)=0$$
$$(2x-1)=0 \quad (x+2)=0$$
$$2x=1 \qquad x=-2$$
$$x=\frac{1}{2} \qquad x=-2$$

hence$3,\frac{1}{2},-2$ *are the roots of the given eqution.*

(iii)

-1 *is the root of the equation* $4x^3-x^2-11x-6=0$

solution:

$$P(x)=4x^3-x^2-11x-6=0$$

the depressed equation is $4x^3-x^2-11x-6=0$

		4	-1	-11	-6
-1		↓	-4	5	6
		4	-5	-6	0

$Quotient=Q(x)=4x^2-5x-6$

the depressed equation is$4x^2-5x-6=0$

$4x^2-5x-6=0$

$4x^2-8x+3x-6=0$

$$4x(x-2)+3(x-2)=0$$
$$(4x+3)(x-2)=0$$
$$4x+3=0 \qquad x-2=0$$
$$4x=-3 \qquad x=2$$
$$x=\frac{-3}{4} \quad , \quad x=2$$

hence $-1,-\frac{3}{4},2$ *are the roots of the given equation.*

5.

(i) 1 *na* 3 *are the roots of the eqution* $x^4-10x^2+9=0$

Solution:

$x^4-10x^2+9=0$

$P(x)=x^4-10x^2+9=0$

		1	0	-10	0	9
1		↓	1	1	-9	-9
		1	1	-9	-9	0
3		↓	3	12	9	
		1	4	3	0	

$Quotient=Q(x)=x^2+4x+3$

$Remainder=0$

the depresses equation is$x^2+4x+3=0$

$$x^2+4x+3=0$$
$$x^2+3x+x+3=0$$
$$x(x+3)+1(x+3)=0$$
$$(x+1)(x+3)=0$$
$$(x+3)=0 \qquad (x+1)=0$$
$$x=-3 \qquad x=-1$$

hence $1,3,-1,-3$ *ar the roots of the given equation.*

ii)

3 *and* -4 *are the roots of the equation*

$$x^4+2x^3-13x^2-14x+24=0$$

Solution:

$$x^4+2x^3-13x^2-14x+24=0$$
$$P(x)=x^4+2x^3-13x^2-14x+24$$

		1	2	-13	-14	24
3		↓	3	15	6	-24
		1	5	2	-8	0
-4		↓	-4	-4	8	
		1	1	-2	0	

$Quotient=Q(x)=x^2+x-2$

$Remainder=0$

The depressed eqution is $x^2+x-2=0$

$x^2+x-2=0$

$x^2+2x-x-2=0$

$x(x+2)-1(x+2)=0$

$(x-1)(x+2)=0$

$$x-1=0 \qquad x+2=0$$
$$x=1 \qquad x=-2$$

Hence $3,-4,1,-2$ *are the roots of the given equation.*

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