TOPOLOGY

CHAPTER ONE

SETS,ELEMENTS

Collection of well-defined and distinct objects

repetition of objects not allowed.

finite or infinite

e.g $\mathbb{N}=\{1,2,3\ldots\}$

$W=\{0,1,2\ldots\}$

$\mathbb{Z}=\{0,\pm1,\pm2\ldots\}$

Denoted by a, b, x, y ..

$X=\{a,b,c\}$

$a \in X$

$f \notin X$

only one ele. called singleton set.

Denoted by A, B, X, Y ..

enclosed by $\{\}$

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If no. of el. in subset is less than no. of el. in set → then it is our proper subset

$\boxed{A \subseteq B}$

B contains A

SUBSETS,SUPERSETS

part of set!

no. of subset of set is 2^n

contains the elements of set

$B \supset A$

If A is subset of B

$A \subset B$

$x \in A$

then as $A \subset B$

$\Rightarrow x \in B$

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CLASSES,COLLECTION AND SPACES

we use
class word
for set
of sets

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PRODUCT SETS

consists of oedeled pairs (a, b)
where $a \in A$ and $b \in B$

cartesian product

written as $\boxed{A \times B}$

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$A = \{1, 2\}$, $B = \{2, 3\}$

$\Rightarrow A \times B = \{(1,2)(2,2)(1,3)(2,3)\}$

$R = \{(a,b), a \in D_R, b \in R_R\}$

$\hookrightarrow R = \{(1,2)\}$

RELATIONS

$R \subseteq$ cartesian product

subset of $A \times B$

$R^* \subseteq A \times B$

$R = \{(a,b) : aRb\}$

if you relate a to b then writes as aRb

Domain and Range:

$R_1 = \{(1,2)(2,2)\}$

$D_R = \{1,2\}$ $R_R = \{2,2\}$

if $a \not R b$

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$R = \{(1,1)\ (2,2)\ (2,3)\ (3,2)\}$

$\boxed{\text{EQUIVALENCE RELATIONS}}$ → Type of Relation

Three properties

① Reflexive property ✓

② Symmetric property ✓

③ Transitive property ✓

$(a,a) \in R$

If $(a,b) \in R$ then $(b,a) \in R$

If $(a,b), (b,c) \in R$ then $(a,c) \in R$

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Prove that $A = \{2, 3, 4, 5\}$ is not a subset of $B = \{x : x \text{ is even}\}$.

$A \not\subset B$

$3, 5 \in A$

while

$3, 5 \notin x$

$3, 5 \notin B$

$\Rightarrow A \not\subset B$

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Prove Theorem 1.1 (iii): If $A \subset B$ and $B \subset C$ then $A \subset C$.

Let $x \in A$

then $A \subset B$

$\Rightarrow x \in B$

As $B \subset C$

$\Rightarrow x \in C$

$\Rightarrow A \subseteq C$

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Prove: If A is a subset of the null set $\emptyset$, then $A = \emptyset$.

Empty set is a subset of every set.

It means

$$\phi \subset A \quad \text{— (i)}$$

As

$$A \subset \phi \quad \text{— (ii)}$$

from (i) and (ii), we can write

$$\boxed{A = \phi}$$

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Find the power set $\mathcal{P}(S)$ of the set $S = \{1,2,3\}$.

class of all subsets

\# $2^n = 2^3 = 8$

$\mathcal{P}(S) = \{ \phi, S, \{1\}, \{2\}, \{3\}, \{1,2\}$
$\{2,3\} \{1,3\} \}$

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Let $U = \{1, 2, \ldots, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$.
Find: (i) A^c, (ii) $(A \cap C)^c$, (iii) $B \setminus C$, (iv) $(A \cup B)^c$.

Operations on a set!

(i) $A^c = U - A = \{1, 2, \ldots 9\} - \{1, 2, 3, 4\}$

$A^c = \{5, 6, 7, 8, 9\}$

(ii) $(A \cap C)^c \Rightarrow (A \cap C) = \{3, 4\}$

$(A \cap C)^c = U - (A \cap C)^c = \{1, 2, 5, 6, 7, 8, 9\}$

(iii) $B \setminus C = \{2, 8\}$

(iv) $A \cup B = \{1, 2, 3, 4, 6, 8\}$

$(A \cup B)^c = U - (A \cup B) = \{5, 7, 9\}$

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Prove De Morgan's Law: $(A \cup B)^c = A^c \cap B^c$.

LHS:

$$(A \cup B)^c = \{x : x \in (A \cup B)^c\}$$
$$= \{x : x \notin A \cup B\}$$
$$= \{x : x \notin A \text{ or } x \notin B\}$$
$$= \{x : x \in A^c \text{ and } x \in B^c\}$$
$$= \{x : x \in A^c \cap B^c\}$$
$$= A^c \cap B^c$$

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Let $A = \{a, b\}$, $B = \{2, 3\}$ and $C = \{3, 4\}$. Find: (i) $A \times (B \cup C)$,

$B \cup C = \{2, 3, 4\}$

$A \times (B \cup C) = \{a, b\} \times \{2, 3, 4\}$

$= \{(a, 2)(a, 3)(a, 4)(b, 2)(b, 3)$

$(b, 4)\}$

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Consider the relation $R = \{\langle 1,1\rangle, \langle 2,3\rangle, \langle 3,2\rangle\}$ in $X = \{1,2,3\}$. Determine whether or not R is (i) reflexive, (ii) symmetric, (iii) transitive.

$(1,1) \in R$ so, Reflexive Not
but $(2,2) \notin R$

$(2,3) \in R$ $(3,2) \in R$ so, symmetric holds

$(2,3) \in R$ but $(2,2) \notin R$ so, Not
$(3,2) \in R$
Transitive