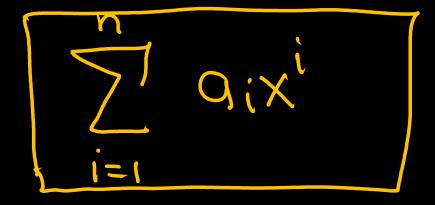
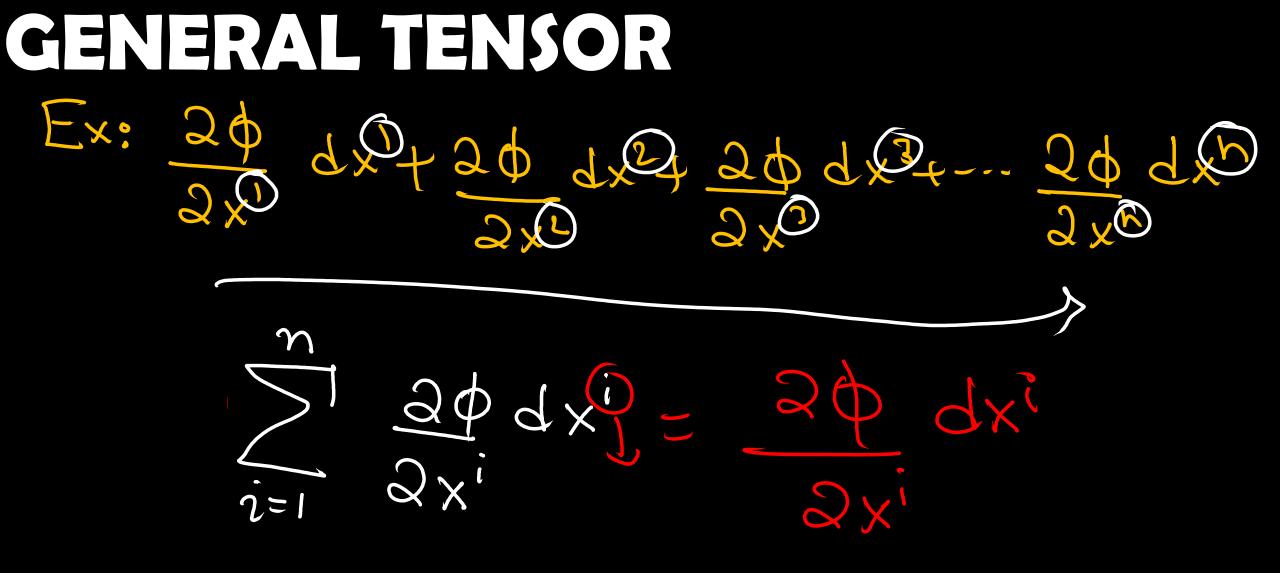
GENERAL TENSOR Einstein's Summation Conventions.







GENERALTENSOR Ex: $(x^2)^2 + (x^2)^2 + (x^2)^2 + \cdots + (x^2)^2$ Waite in summation $(\chi^{i})^{2}$ convention form. ... Super scoupt genceal Tensol.

GENERAL TENSOR





Fre Indexs-

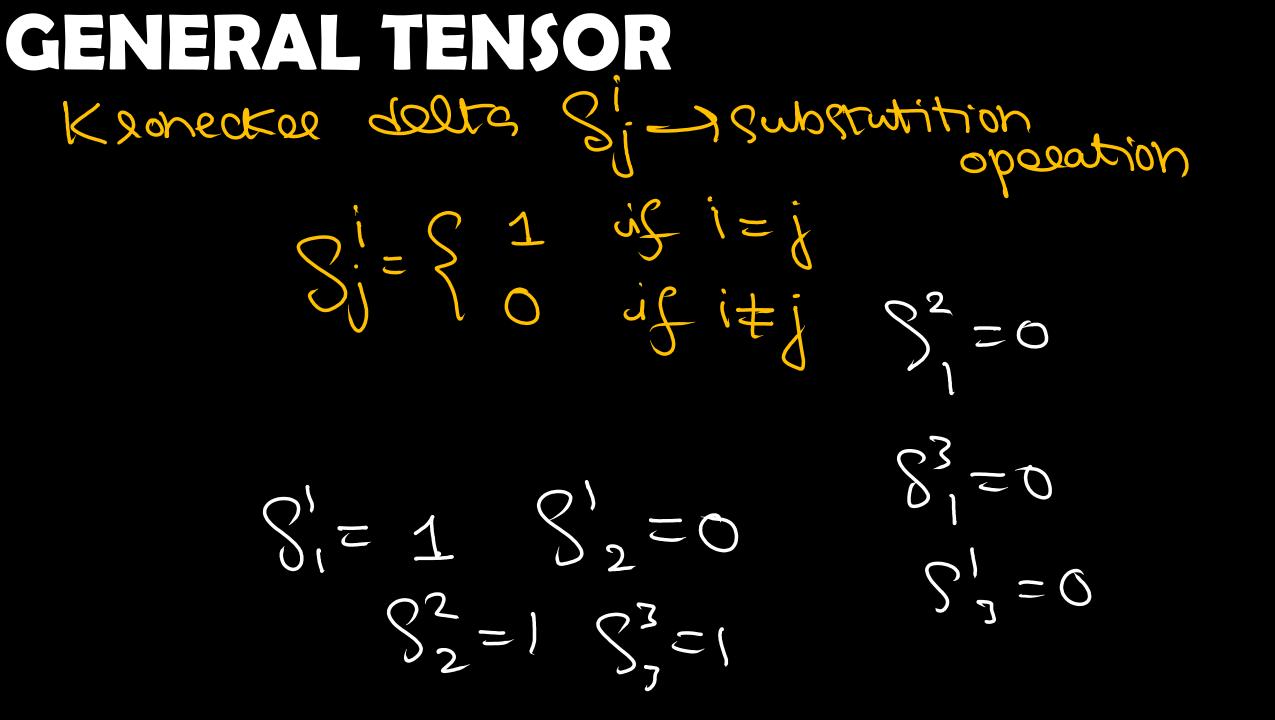
Ci X indices feer index= i Dung=f

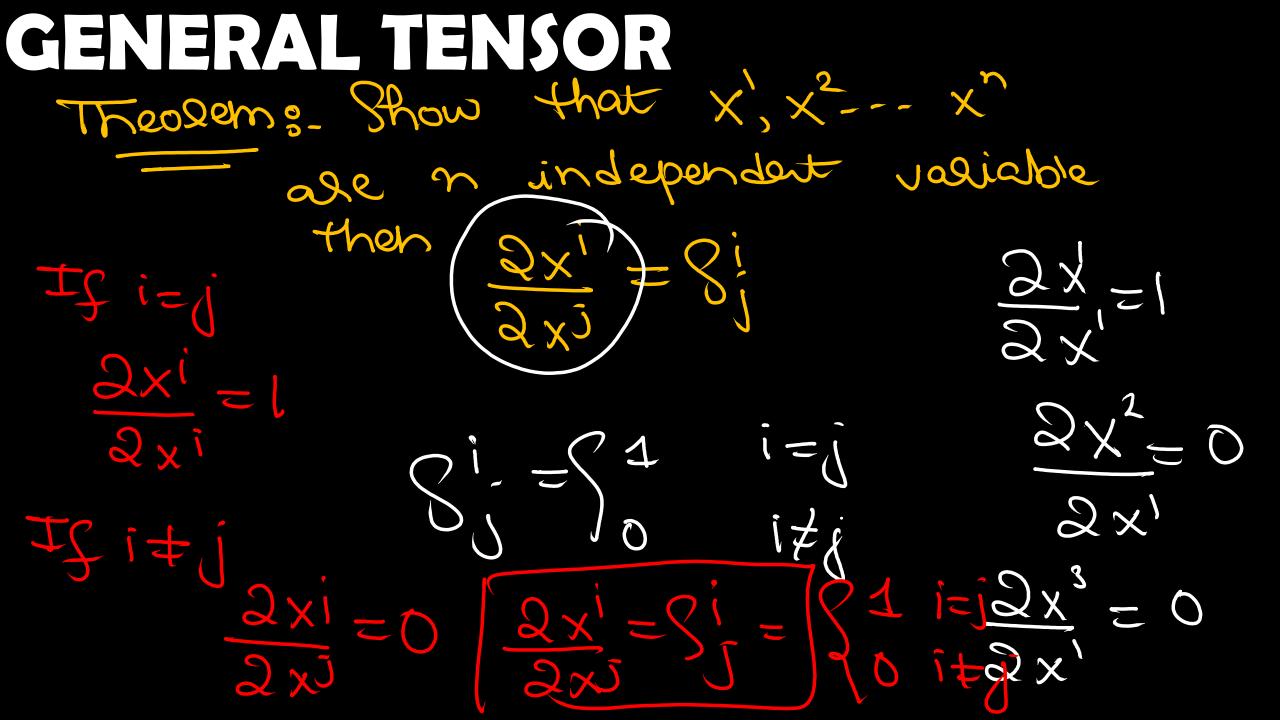
DOUBLE SUM? aixin index (1) $\begin{array}{c} O_{ij} \times i \times j \\ z & \downarrow \\ z & \downarrow \\ z & \downarrow \\ z & \chi' \times j + O_{2j} \times \chi' \times j + - \end{array}$ $\sum_{i=1}^{n} \alpha_i x^i = n$ Team + anj x nx j (aij x x j =) indices (2) somed is $= q_{11} x' x' + q_{12} x' x^{2} + \dots + q_{1n} x' x' + q_{2n} x' + q_{2n}$

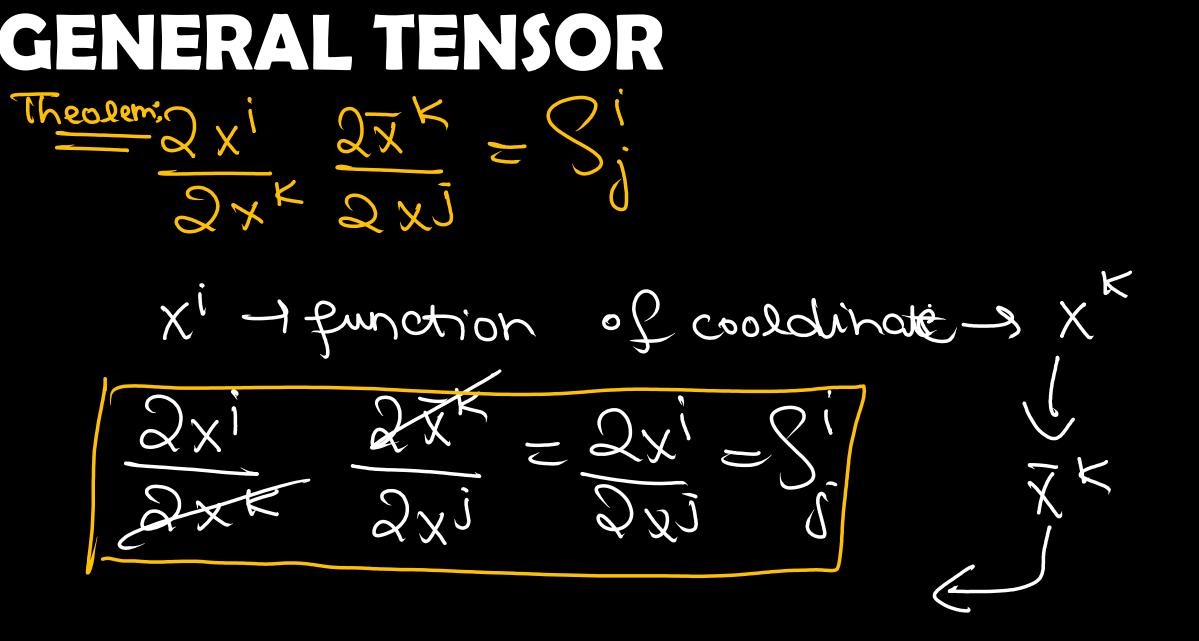
GENERAL TENSOR Exe-If $\overline{x}^i = \alpha_{ij} x^j$ exposes from $G = g_{ij} \overline{x}^i \overline{x}^j$ in terms of x-valiables

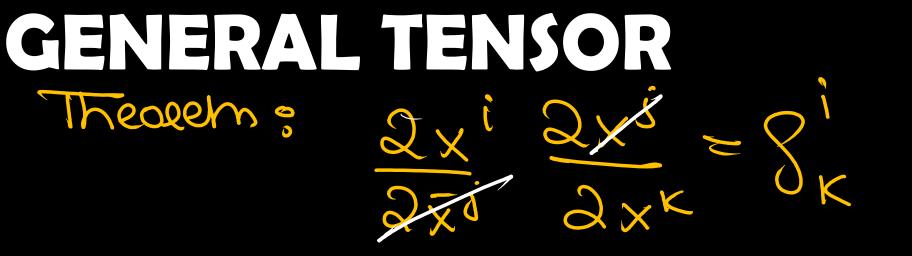
 $\overline{\chi}^{i} = \operatorname{Qig} \chi^{A}$ $\overline{\chi}^{j} = \operatorname{Qig} \chi$

 $\overline{\chi}' = \Omega_{ij} \times \overline{J} = \Omega_{ik} \times^{\mathcal{N}}$ $\overline{\chi}^{\overline{J}} = \Omega_{j} \times \overline{\chi}^{S}$ Q= Jij aig Xª ajs XS = gij aig ajs Xª xS

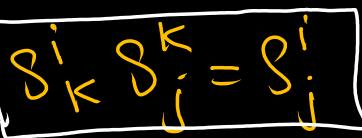








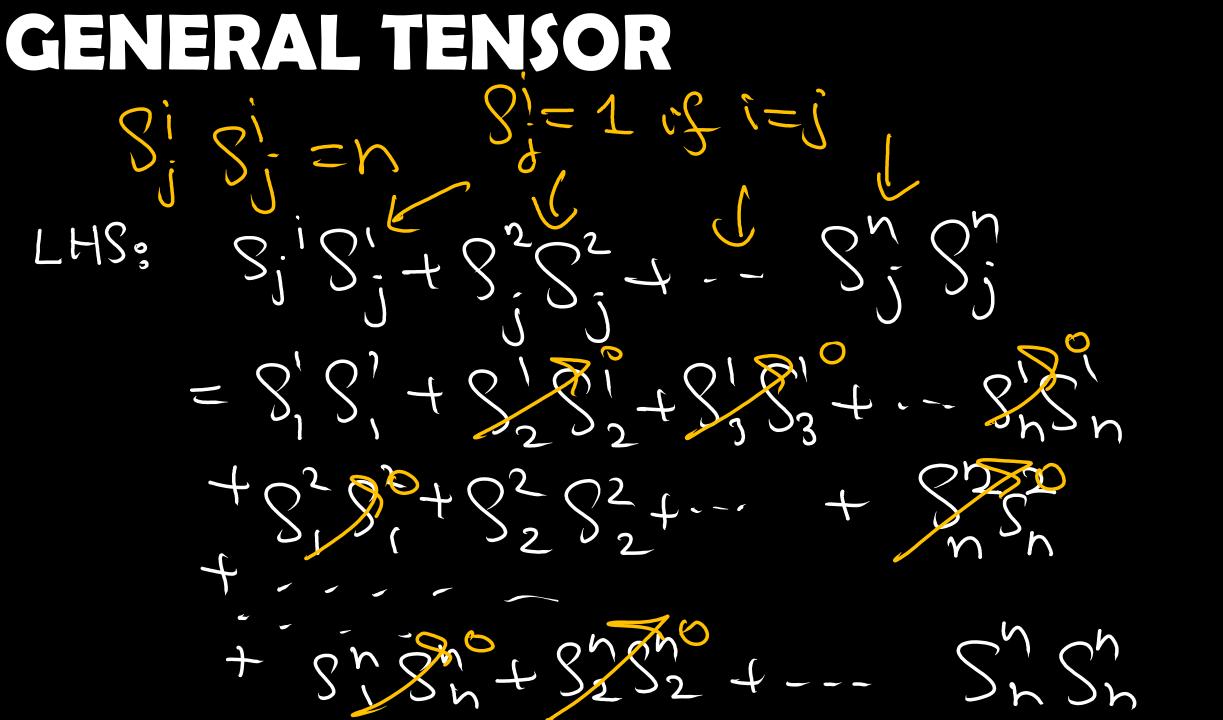




S(K)) dumany semole = Sí VR)

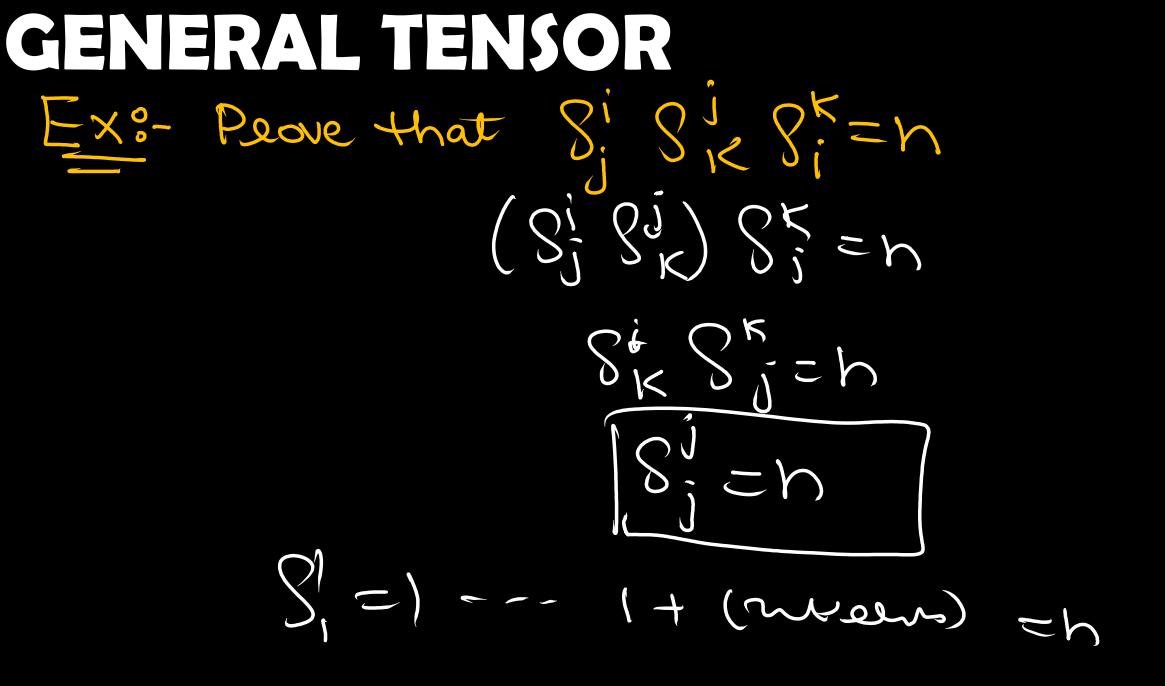
5 Jeep index

GENERAL TENSOR Ex: Peove that S! = n LHS $j_{i} = S'_{i} + S_{2}^{2} + S_{3}^{3} + \dots + S_{n}^{n}$ r using= 1+1+1+ ---1(n teams S'=1ころ

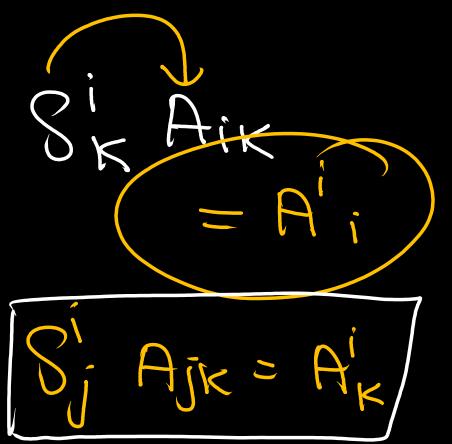


GENERAL TENSOR $S'_1S'_1 + \leftarrow S^2_2S^2_2 + S^2_3S^2_3 + S^n_5S^n_n$ = | + | + | + - + | (n terms)

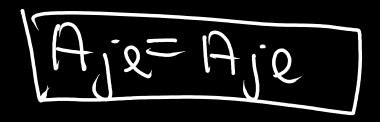




GENERAL TENSOR, Ex: Prove that $S_j^{k} S_{k}^{k} A_{ik}^{k} A_{jk}^{k}$



Sj (Aie) = Aje



GENERAL TENSOR Teansfor TRANSFORMATION: Po lal cooldi Calterian cooldinate $(\mathcal{S}, \mathcal{O})$ $X \rightarrow \overline{X} (X, \overline{Y})$ $\chi' = \chi' Cos(\chi')$ R=9 x24 y2 C = X = X cos B $\chi^2 = \bar{\chi}^2 \sin(\bar{\chi}^2) - \frac{1}{2} \sin \theta$ $0, = ton'(y_{x})$ $\overline{\chi}' = \sqrt{(\chi')^2 + (\chi^2)^2}$ $(2,0) = (\overline{\chi}, \overline{\chi})^2$ $\overline{\chi}^2 = tan^2$ $(X, 7) = (X', (X^2))$

GENERAL TENSOR , Cy Dindai cal Calteriah -X = X co S QX= 1 x2+ y2 y=2sin o and $\Theta = \tan'(3/2)$ Z = Zてこ $(x, 0, z) = (\overline{x}, \overline{x}, \overline{x}^2)$ $(X_9Y_)Z) - (X_5X_3X^3)$ $\chi \rightarrow \bar{\chi}$ $\chi' = \bar{\chi}' \cos \bar{\chi}^2$ $X = \sqrt{(X')^2 + (X^2)^2}$ $\overline{\chi}^2 = \tan'(\chi')$ $\chi^2 = \overline{\chi}' Sih \overline{\chi}^2$ $\chi^3 = \chi^3$ $X^3 = \overline{X}^3$

GENERALTENSOR Cartesjan Spericel Cooldinates $\int X = 2 \sin \theta \cos \phi$ $\int J = 2 \sin \theta \sin \phi$ X=1 x442+22 Oz tan' Jx22 Z = r coso ϕ = tan' $(\frac{7}{3})$ $(X, \gamma)Z) = (X', X^{2}, X^{3})$ $(\mathcal{A}, \Theta, \phi) = (\overline{X}', \overline{\chi}^2, \overline{\chi}^3)$ $\chi' = \overline{\chi}' Sih \overline{\chi}^2 Cos \overline{\chi}^3$ $\frac{2}{3} = \overline{X}$, Sin \overline{X}^2 Sin \overline{X}^3

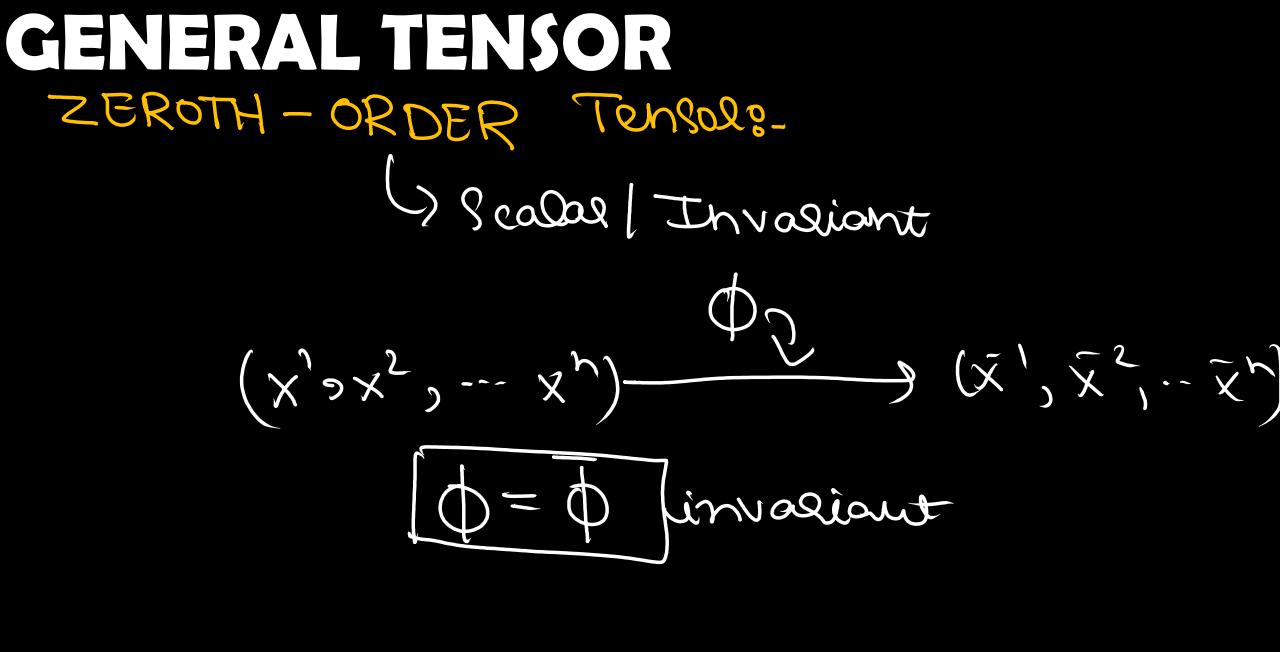
GENERAL TENSOR Tensoe Notation for Matrices: (a) Lower - index mataix $nototiong_ Cui (1221 Giz)$ 911 96 G12. Q13 --. $\left[\begin{array}{c} q_{ij} \\ \eta_{nn} \end{array}\right] =$ am Smz 9ms 9 mn

GENERALTENSOR (b) ^upper index

 $Q'', Q'^2, Q^{22}, Q^{22}, Q^{22}, Q^{21}, Q^{21}, Q^{22}, Q^{22}, Q^{21}, Q^{21}, Q^{22}, Q^{21}, Q$

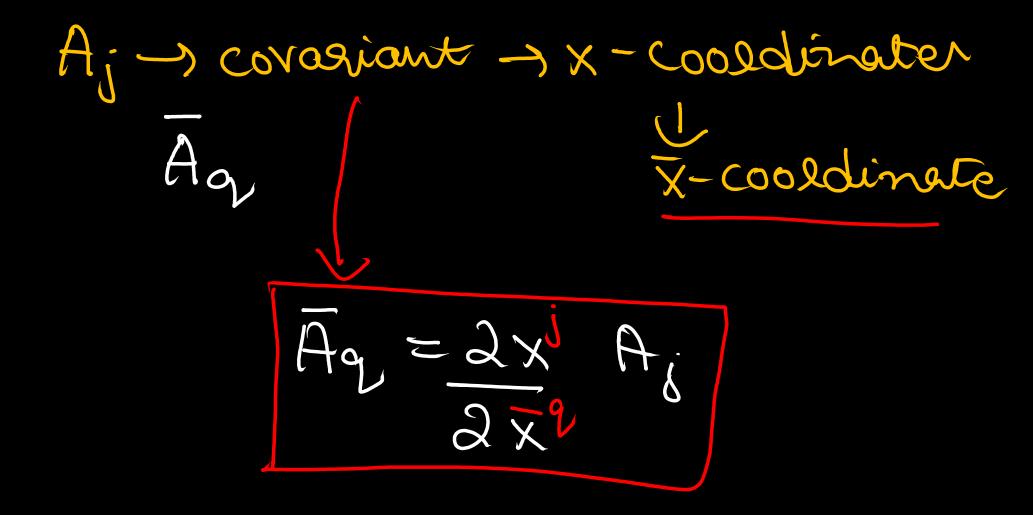
 $\begin{array}{l} \text{los Mixed} \\ \text{Inde} \\ \text{Inde} \\ \text{a', a'_2 + - - a'_n} \\ = \overline{[a_{ij}]mn} \end{array}$

GENERAL TENSOR Scalal - quartity - mulsel & constant Vector Aquantity) Tensor gzelo order grealal ORDER (RANK TENSOL - Same Baat () suffices -) v sed.



GENERALTENSOR first Order Tensors. 1 1 suffix. A'-gyantity stensoe contravagiant (1), X', X' --- Xn) - X-cooldier Sy ster and and P' Transfornation of solinate Psidges

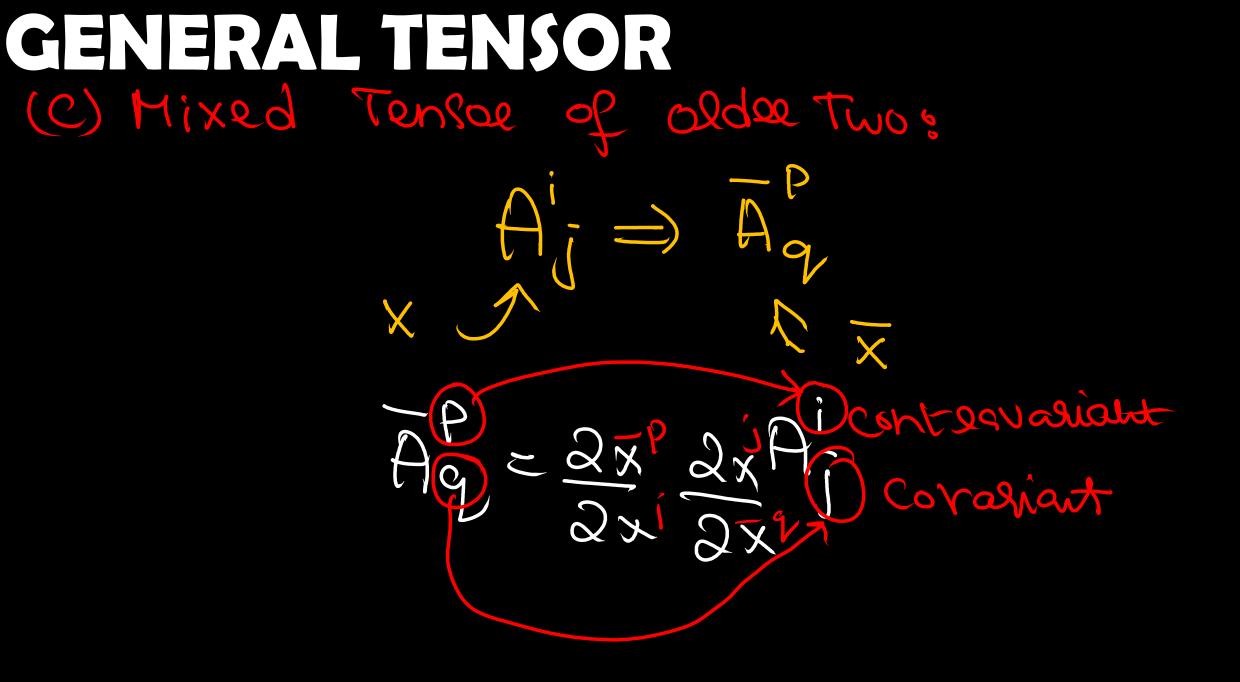
GENERAL TENSOR



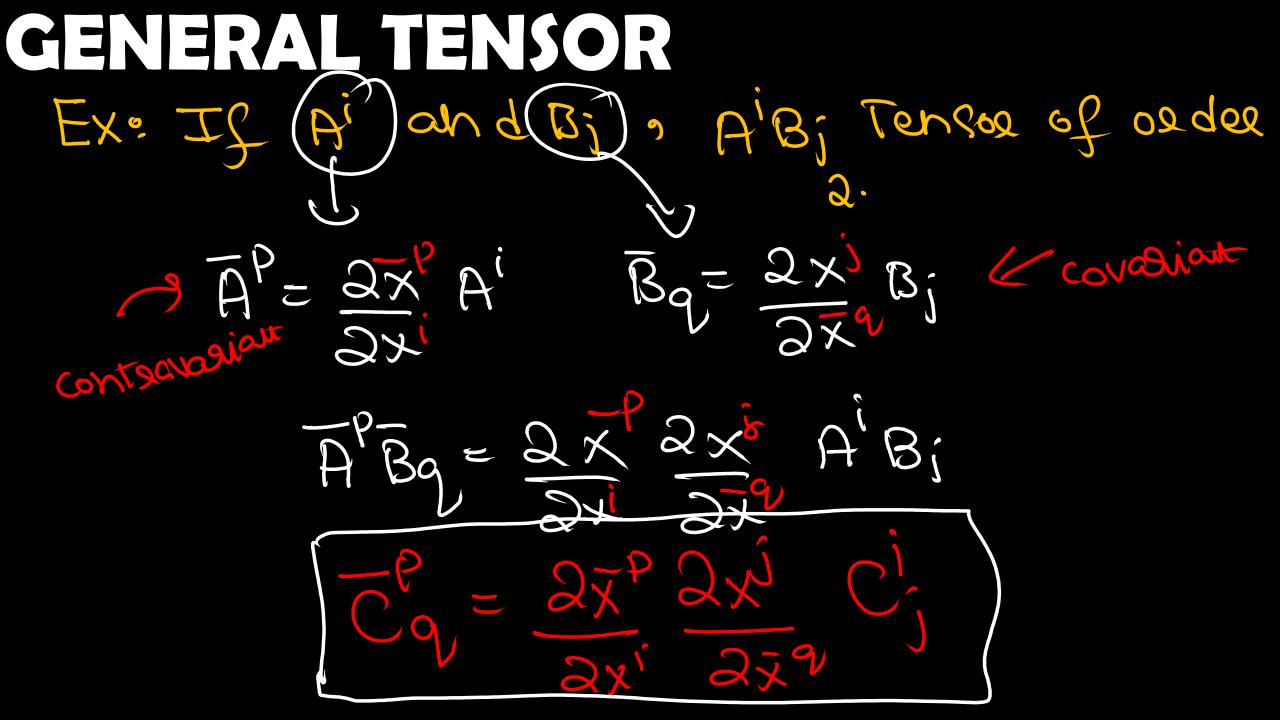
GENERAL TENSOR Second Older Tensol: (G) Contegraliant A'J -> x - cooldinate AP9 ZEODIDINATE

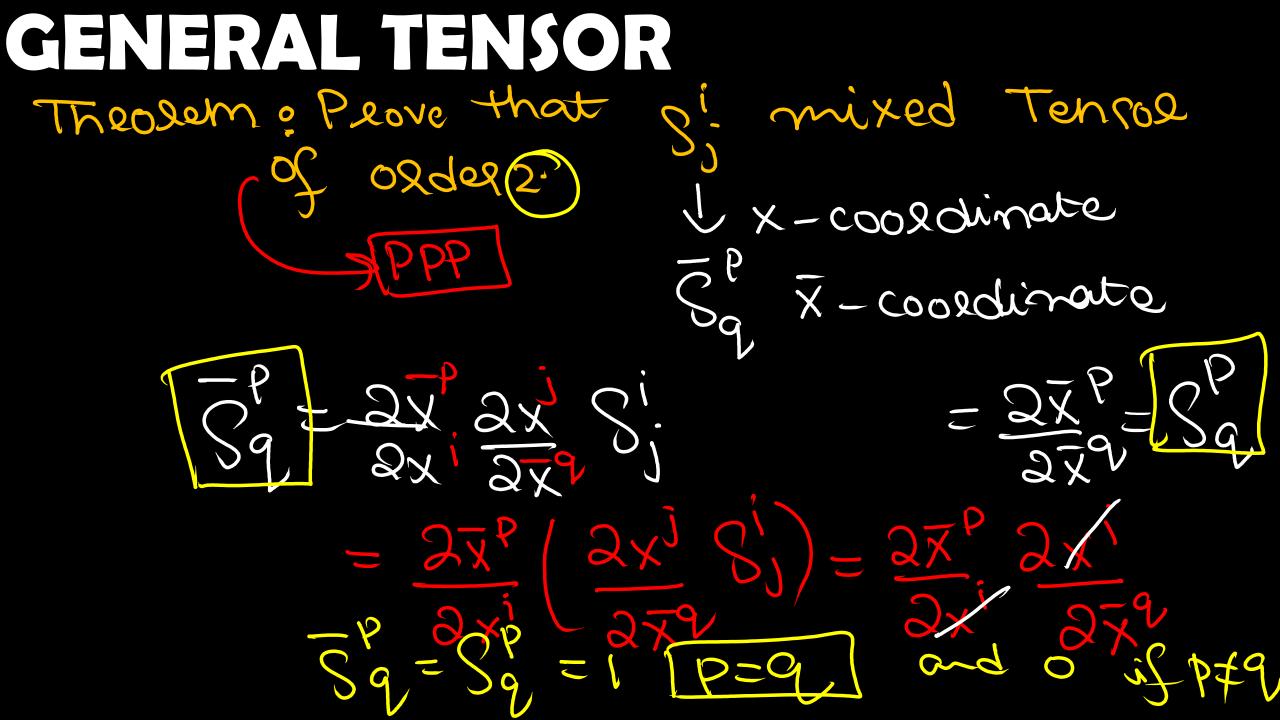
 $\overline{A}^{PQ} = \frac{2\overline{x}^{P}}{2\overline{x}^{i}}\frac{2\overline{x}^{2}A^{i}}{2\overline{x}^{j}}$

 $z \frac{2x}{2x} \frac{2x}{2x}$ Apj

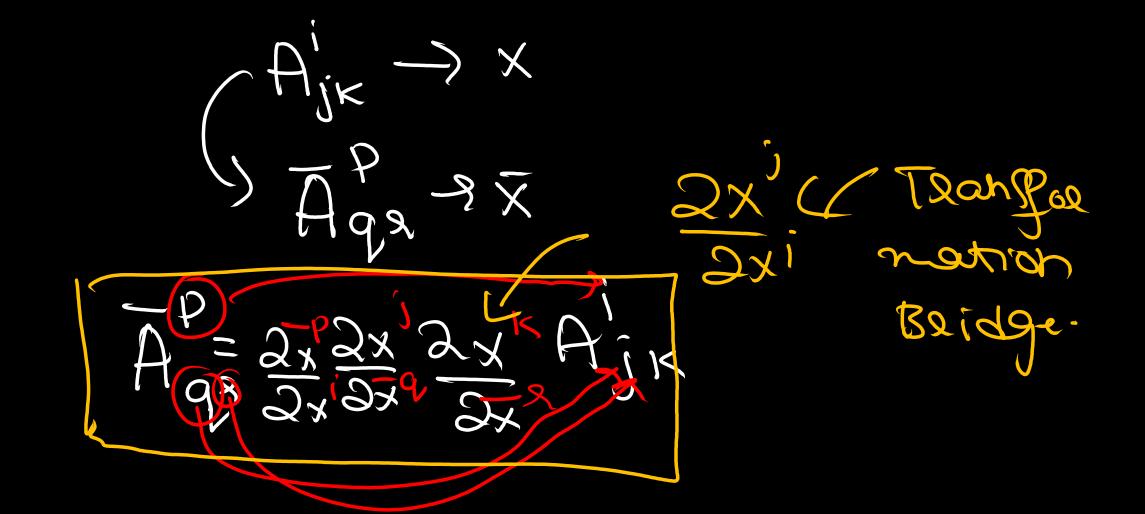


GENERALIENSOR are two contravarient BI Ai and Ex: IC (Rⁱ Bⁱ Contravaliat then úS Tensoos 9 $\overline{A}^{P}\overline{B}^{\gamma} = \frac{2}{2} \times \frac{2}{2} \times \frac{2}{2} \times \frac{1}{2} \times \frac{1}{$ 02d 22 2 P' Ā 2 2 B RJ $2x^{2}$ 2x 2



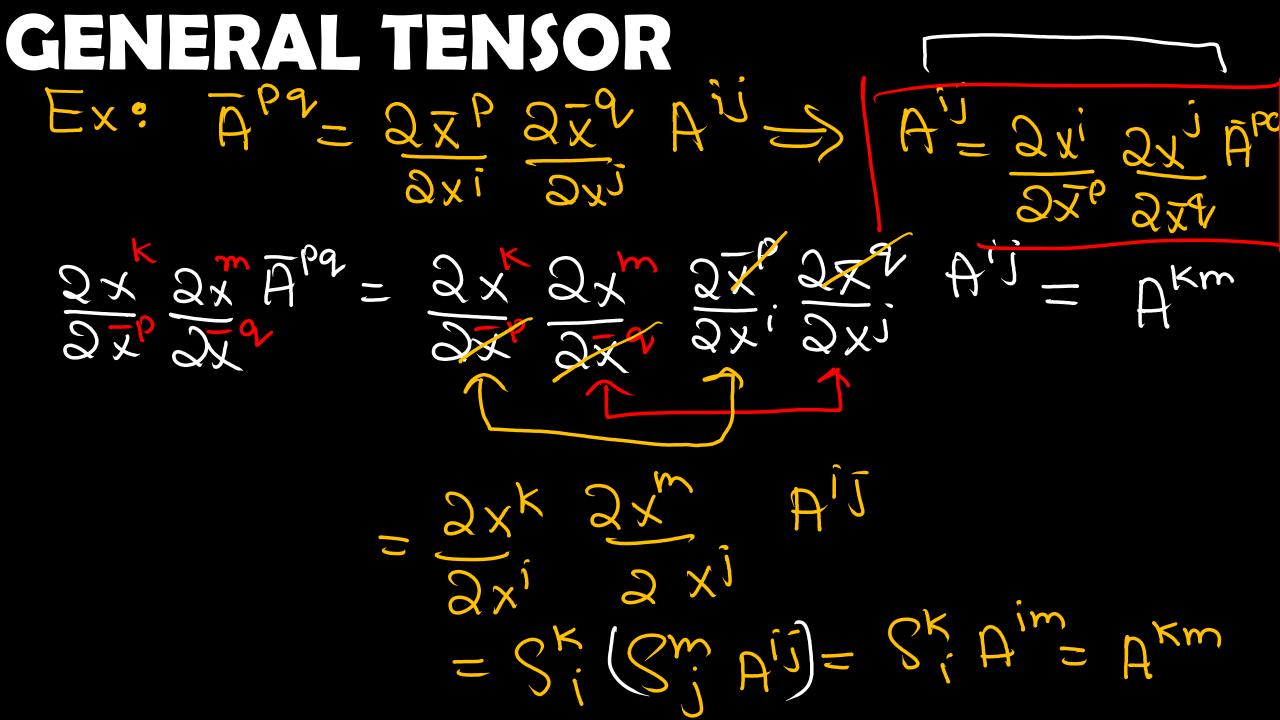


GENERAL TENSOR Transformation Dans-



GENERALIENSOR formation Tach Inverse Transformation Jano Threak then prove $\overline{A}' = \frac{2x}{2x^{j}}$ ĄJ If that P¹ 2X' axi QxJ 2^{χ} $2\overline{7}$ A = S $A^{j} =$ X P' $2 \times$

GENERAL TENSOR K Replace A' = AK 2 x A \bigcirc



GENERAL TENSOR

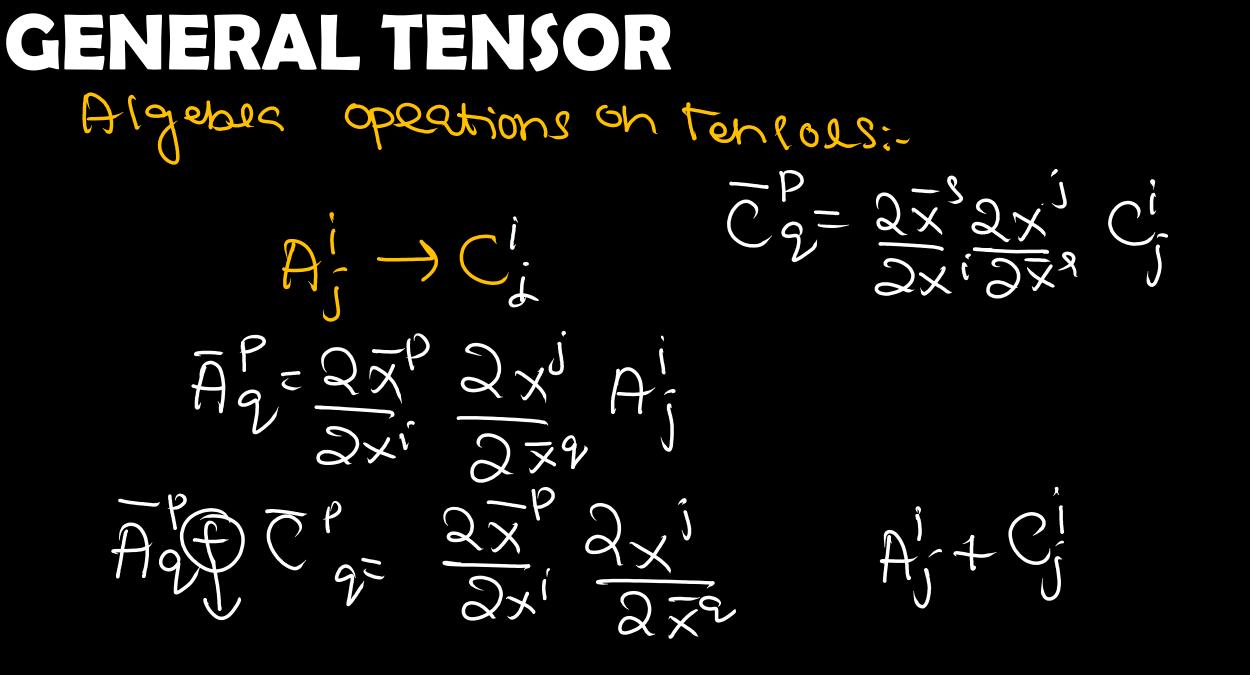
 $\frac{2x^{i}}{\partial x^{p}} = \frac{2x^{j}}{\partial x^{2}} = A^{p} = A^{ij}$

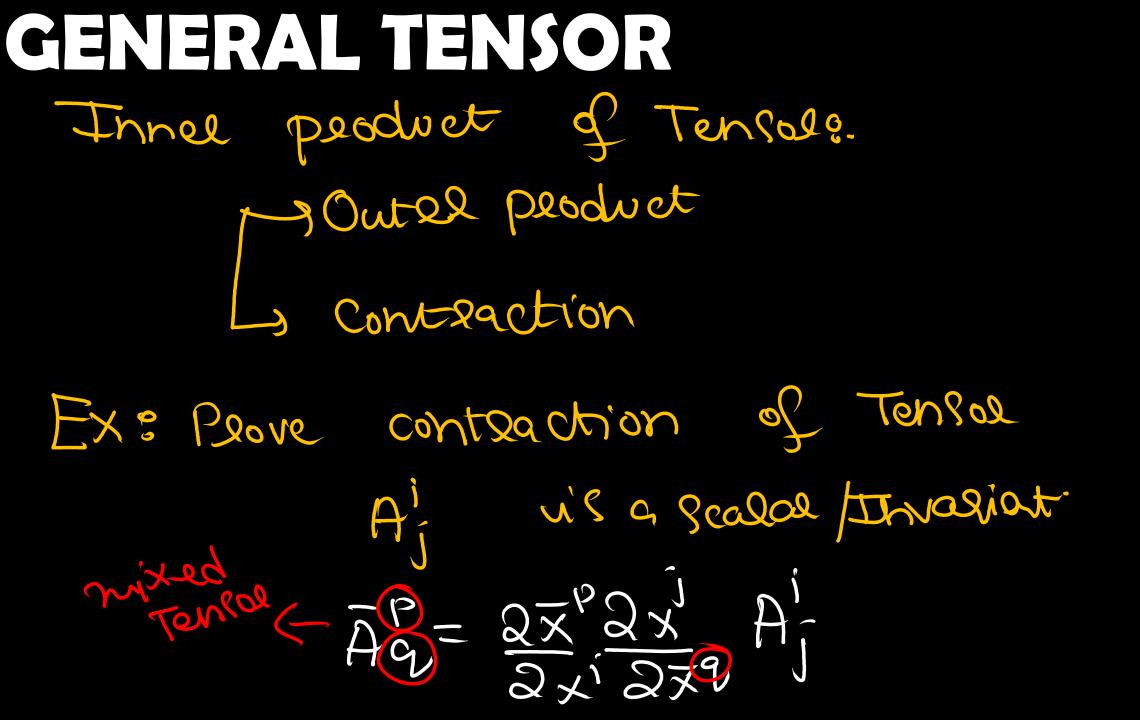
 $A^{ji} = \frac{ax^{i}}{ax^{i}} \frac{ax^{j}}{ax^{i}} \frac{\bar{A}^{pq}}{\bar{A}^{qq}}$

GENERAL TENSOR RANK OF TENSOR:-

Solumber of indices attached to it!

GENERAL TENSOR Ex: How many components does a Tensor have of Rank 5 in a space of 4 Dimension? No of components = (Dimensions) Rant $= (4)^{S} = [1024]$

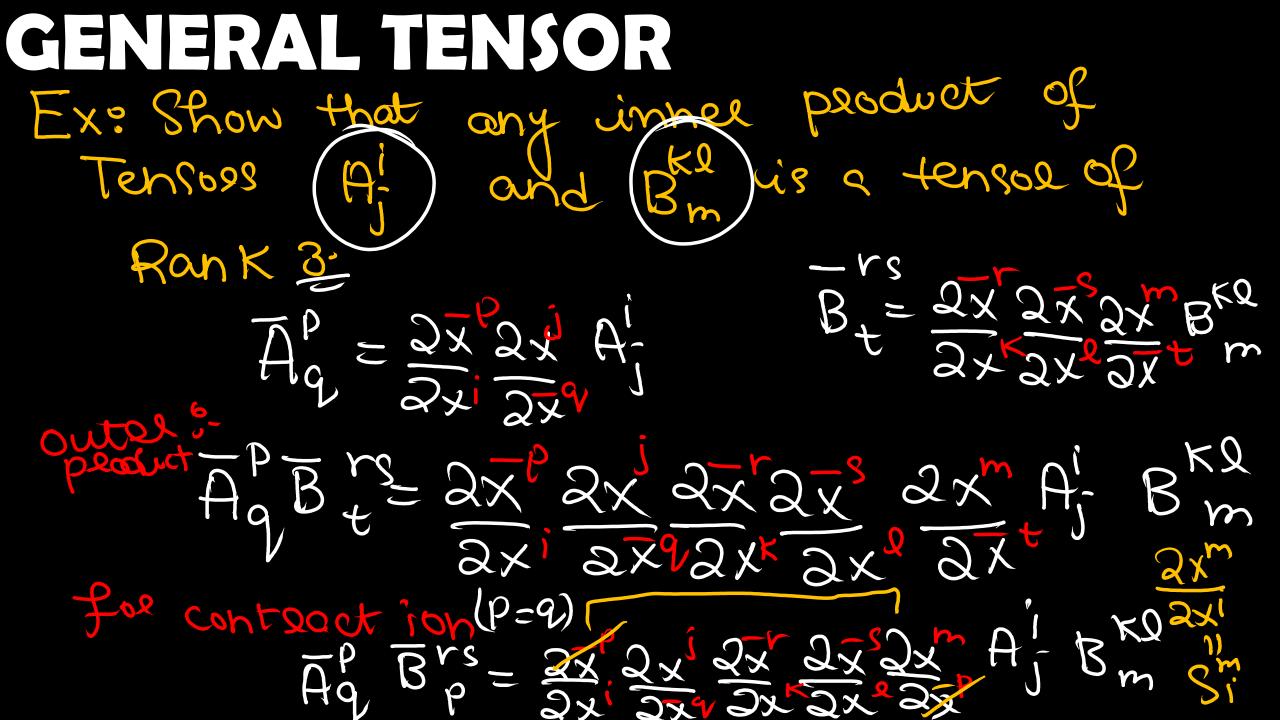




GENERAL TENSOR

P

d'V" A $= 2x^{j}A_{i}^{j}$ 2x' RJ $\mathbf{ }$ A';-J)



┓┓╡╏╽┨╸): Ag Brs = 2x^j 2x^r 2x^s S^m A' B^{kQ} 2x^q 2x^k 2x^k 3x^e <u>2xi</u> <u>axi</u> <u>2xs</u> <u>Aj</u> <u>Bkl</u> <u>2x</u> <u>2xk</u> <u>2xe</u> Qxs Ag Bkp Qxe $\frac{2x^{r}}{2x^{k}}$ 5 2x vor Rank alder 3.

GENERAL TENSOR QUOTIENT Theoremo_ (X) rinnel peoduct = Tensolany quantity EJX is Jehsol.

EX? Ajj-) quantity in X-cooldinater, Ai B'= Cj, B'-) coatlavalient tengos Cj=> Convagiout. Peore that Aij is Covariant Tensoe of order Z. Aij > App B=Bi, Cq= 2xt Ci $\begin{array}{rcl} \widehat{A}pq \ \widehat{B}^{P} = & \widehat{C}q, \\ \widehat{A}pq \ \widehat{B}^{P} = & \frac{2}{2}x^{j} \\ & \widehat{A}pq \ \widehat{B}^{P} = & \frac{2}{2}x^{j} \\ & 2\overline{x}^{p} \end{array}$

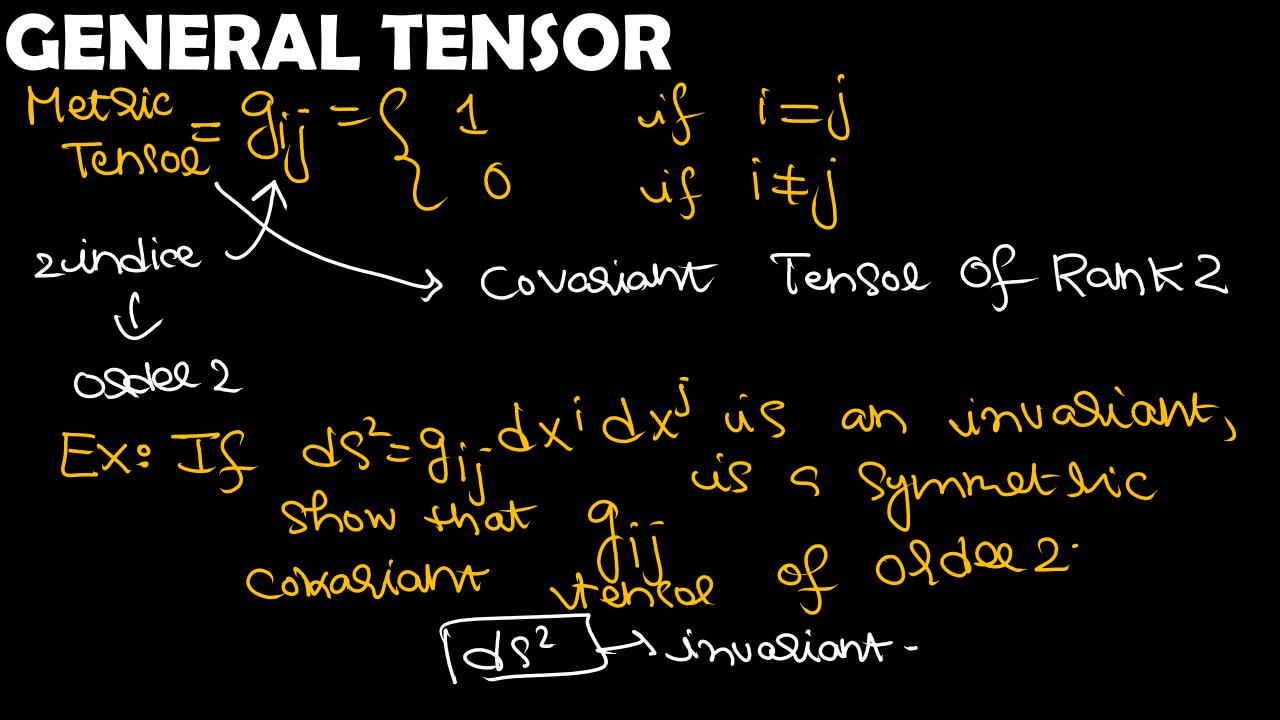
ax j Cj P B Apg, $\underline{B}_{k} = \frac{3 \times 1}{3 \times 1} B$ Using ZXX $\overline{G}^{p} = 2 \overline{\chi}^{i} = 2 \overline{\chi}^{p} \overline{G}^{x}$ i Bax axp azq z Q x J $Q \overline{x}^{9}$ dX $= Q \chi^{j}$ $\widehat{\mathcal{B}}$ Apr 9 Ĩ P P 2xª 2x

 $\widehat{Apg} \stackrel{\text{BP}}{=} \frac{2x^{j}}{3x^{2}} \frac{2x^{j}}{3x^{p}} \frac{2x^{j}}$ $= \frac{1}{8} \left(\frac{1}{8} - \frac{1}{2} - \frac{1}{2} \frac{1}{2} \frac{1}{2} - \frac{1}{2} \frac{1}{2} \frac{1}{2} - \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} - \frac{1}{2} \frac{1}$ $\sum \frac{-\beta}{\beta} \neq c$ Covasiant. Apg <u>ax</u>P Older 2

Symmetric And Skew-Symmetric Tensoss: Theolom: Every Tensoe can be expressed as sum of two tensoes, one is symmetric and other is Skew-Symmetric $= \frac{1}{2} (A^{ij} + A^{ji}) + \frac{1}{2} (A^{ij} - A^{ji})$ S = B' + C' Stow-Sympetsic symmetsic 1 en rol

GENERALIENSOR $B^{i}J = \frac{1}{2}(A^{i}J + A^{j}) = \frac{1}{2}(A^{j} + A^{j}) = B^{j}$ $\sum_{i} B_{i} i i S Symmetric$ $C_{i} = \frac{1}{2} (A_{i} A_{i} (A_{i})) = \frac{1}{2} (A_{i} A_{i})$ =) c'i is Skow symethic

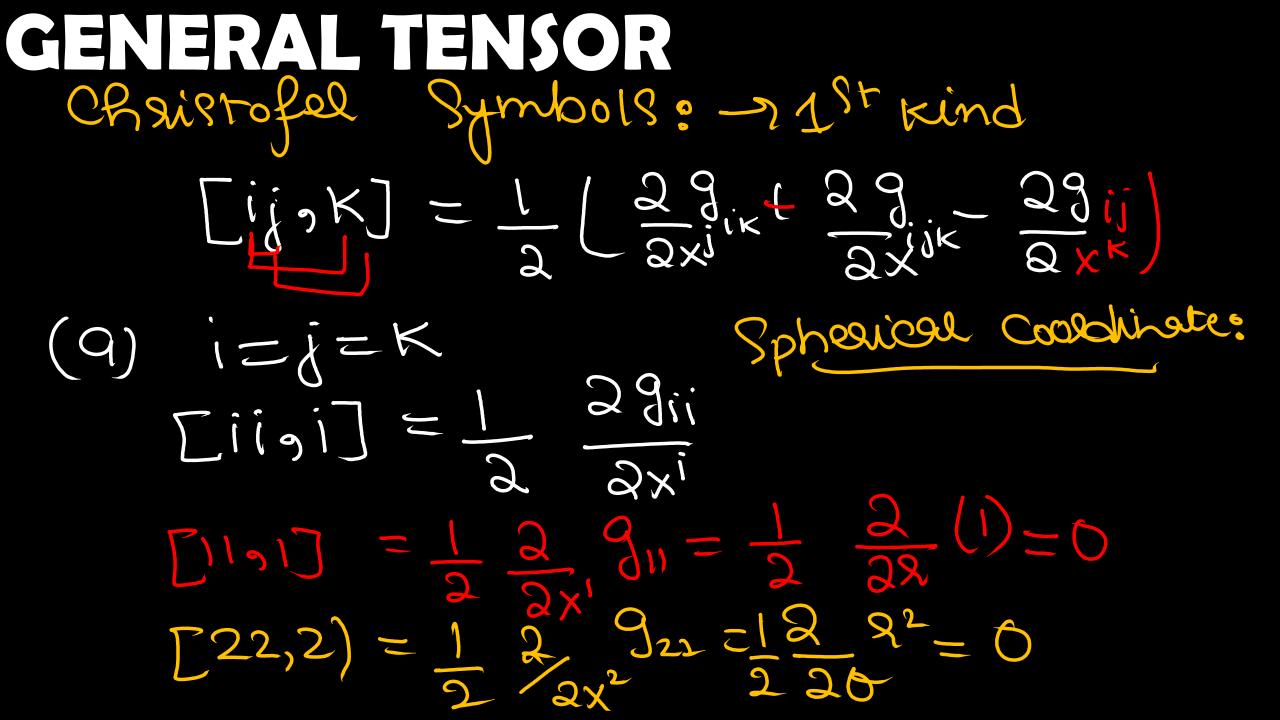
element And Metric Tensorgina (X, y, z)) Dine element) (dg²) $ds^2 = dx + dy^2 + dz^2$ $dx = \frac{2x}{2y} dy + \frac{2x}{2y} dy^2 + \frac{2x}{2y} dy^3$ dsz gij dxidx (í j =1,2---h) $ds^2 = g_{ij} dx^i dx^j$ metric



082 dxi dxj dxidxj dxP= gijdxidxj Covagiant Lehsol 229 of 0 6200 à QX $2\overline{x}$ $\partial \overline{\mathbf{x}}$ dxpdxp

Metsic Tenrol in Cycindelcal Coldinates $(\mathcal{A}, \Theta, \mathbb{Z}) \rightarrow (\mathcal{X}, \mathcal{X}, \mathcal{X})$ $\begin{array}{c} q = \chi^2 \\ \tau^2 \end{array}$ g = 1 J_{33} $g_{12} = g_{21} = g_{23} = g_{32} = g_{32} = g_{32} = g_{33} = g$ montarie Poam = R2 \bigcirc

Metzie Tenroe in Spherical Cooldinates $(\chi, \Theta, \Phi) \longrightarrow (\chi, \chi^2, \chi^3)$ $\frac{g_{11}=1}{g_{22}} = \frac{g_{22}}{g_{23}} = \frac{g_{22}}{g_{33}} = \frac{g_{22}}{g_{33}}$ $g_{12} = g_{21} = g_{13} = g$ metsnie =/ 000 form / 020 lessin20



GENERAL TENSOR (b) $i = j \neq k$ $\sum_{i=1}^{i} K_{i}^{2} = -\frac{1}{2} \frac{2}{2} \frac{2}{x^{k}}^{3ii}$ $\begin{bmatrix} (1_{9}2] = -\frac{1}{2} \frac{2}{2x^{2}} = -\frac{1}{2} \frac{2}{2x^{2}} = -\frac{1}{2} \frac{2}{20} = -\frac{$ $(C) \quad i = K \neq i$ $\begin{bmatrix} ij & i \end{bmatrix} = \underbrace{l}_{2} \underbrace{2}_{Xj} \underbrace{3}_{ii}$ (d) $i \neq j \neq k$ $[[ijnk] \geq 0]$

GENERALTENSOR Chaistofell of 2^{hd} kind: { m } z gmk Lijgk f 1st kind (b) $i = j \pm m$ z = -1 + 2(x)z = 1 + 2(x) $\begin{cases} m \\ iii \end{cases} = -\frac{1}{2} \frac{q}{2} \frac{g}{3} \frac{g}{$ $=-\frac{1}{2}22(-2)$

GENERAL TENSOR (c) $i = m \neq j$ $\begin{cases} i \neq j \\ j \neq j \end{cases} = \frac{1}{2} = \frac{2}{2} \times j^{2} = \frac$

(d) $i \neq m \neq j$ $\sum_{i \neq j} = 0$