NUMERICAL ANALYSIS

SOLUTION OF NON-LINEAR Eqⁿ

"Matlab Kay Hum nay Roots find Karni hain Eqⁿ ki" BUS!

ITERATIVE METHODS AND CONVERGENCE

Approx. Solution by successive STEPS

- BISECTION METHOD
- FIXED-POINT METHOD
- Regula-Falsi METHOD
- SECANT METHOD
- NEWTON METHOD

ERROR

"EXACT value − APPROX. value" BUS!

Exact value = 2.345

Approx. value = 2.34

Error = 0.005

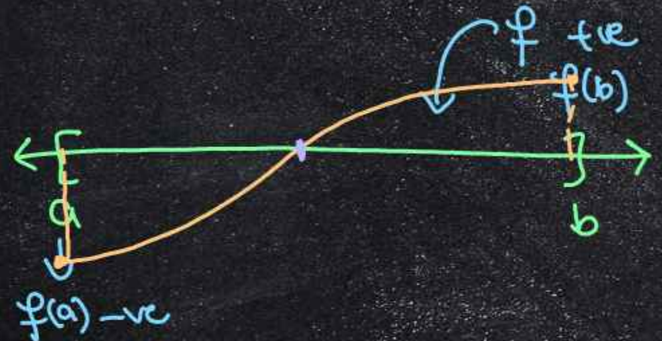
- ROUND-OFF ERROR — "occurs when a number is approximated due to limited storage space available inside computer"
- PYQ 20 TRUNCATION ERROR — "occurs due to usage of limited number of terms of infinite series"
- Absolute ERROR — "|EXACT value − APPROX. value|"
- Relative ERROR — "|(EXACT value − APPROX. value) / EXACT|"
- ERROR ESTIMATION — "The process of determining ERROR in numerical computation"
- FLOATING-POINT ARITHMETIC — "Representation of Real numbers in a finite number of bits using Scientific ERROR"

e.g 0.00123 in floating point is 1.23×10^{-3}

PYQ 2009

BISECTION METHOD

Let $f(x)=0$ be non-linear Eqn which is continous b/w (a) and (b) and $f(a)$ and $f(b)$ having opposite signs- Then Intermediate value theorem ensure that the no. p exist such that $f(p)=0$ b/w a and b.



To begin set $a_1=a$ and $b_1=b$

Let p_1 be midpoint of a and b

s.t $p_1=\frac{a_1+b_1}{2}$

If $f(p_1)=0$ then $p=p_1$, we are done!

If $f(p_1)\neq 0$ then p_1 as either same sign as a or b then select opposite subinterval-

find p_2 : $p_2=\frac{a_1+p_1}{2}$ if $f(a_1)f(p)<0$

then if $f(p_2)=0$, $p=p_2$, we are done

If $f(p_2)\neq 0$ then select opposite sign subinterval and so on...!

Continuing in this process, we can find Root of given Eqn...!

"Yehi Kaam BUS!!!"

PYQ 2015

Q: Find the Root of Eqn $e^x-3x=0$ Correct up to two decimal places with Bisection Method.

$f(x)=e^x-3x$

x	0	1
f(x)	1	-0.28

So, Root lies b/w 0 and 1

PYQ 2012

Determine the no. of iteration necessary to solve $f(x)=x^3+4x^2-10$ with accuracy 10^{-3} using $a_1=1$ and $a_2=2$

We will use approx.

$|p_n-p|=\frac{b-a}{2^n}<10^{-3}$

p_n → value

given $a_1=1$ and $b_1=2$

So, $|p_n-p|=\frac{2-1}{2^n}<10^{-3}$

$\Rightarrow 2^{-n}<10^{-3}$

$\Rightarrow \log_{10}2^{-n}<\log_{10}10^{-3}$

$\Rightarrow -n\log_{10}2<-3\log_{10}10$ ($\log_{10}10=1$)

$\Rightarrow n>3/\log_{10}2\approx 9.96$

Hence, 10 iteration are required to find accuracy

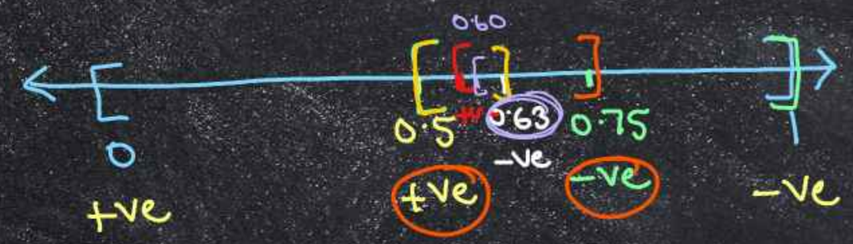
And $f(0) \cdot f(1) < 0$

So,

a	b	p	f(p)
0	1	0.5	0.15
0.5	1	0.75	-0.13
0.5	0.75	0.63	-0.01
0.5	0.63	0.57	0.06
0.57	0.63	0.60	0.02
0.60	0.63	0.62	0.00

So, 0.62 is our root of Eqn $e^x - 3x$

$|P_n - P| = \frac{b-a}{2^n} < 10^0$ of 10^{-3}.



$\frac{0+1}{2} = 0.5 \Rightarrow e^{0.5} - 3(0.5) = 0.15$

$\Rightarrow e^{0.75} - 3(0.75) = -0.13$

BISECTION METHOD'S Algorithms:

So, $f(x) = 0$ be non-linear Eqn (given)

So, $f(x)$ has continous curve b/w a and b and als $f(a)$ and $f(b)$ has opposite signs.

Then BISection Methode uses

Intermediate Value Theorem

- Choose $a_1 = a$ and $b_2 = a$ then find $p_1 = \frac{a_1 + b_2}{2}$.
- If $f(p_1) = 0$ then $p = p_1$, we are done.
- If $f(p_1) \neq 0$ then choose that Subinterval having opposite signs

Fixed Point Iteration

$g(x) = x$ → fixed point func.

Let $f(x)=0$ be non-linear Eqn
And f is continous
then $f(x)=0$ then algebraically converted into $g(x)=x$ then a/b then using iterative p schaum

$$x_{n+1} = g(x_n)$$

Baat Yeh hai Kay

- $x_0 \neq$ given
- interval find karo opposite sign wala wahan sey initial (x_0) approximation ki value milay gi!
- $g(x)$ wala func banao!
- $g'(x)$ aur $|g'(x_0)| < 1$ then cvg. hoga..!
- fixed iteration schaum $x_{n+1} = g(x_n)$ lagao

Jahan value repeat karay Wahan stop karo Jao..!

BUS ----!

PYQ 2016

Apply fixed point iteration to find root of $\sin x = 5x - 2$ upto (5) decimal places near 0.5 → x_0

$f(x) = \sin x - 5x + 2 = 0$

$\sin x + 2 = 5x$

$\frac{1}{5}(\sin x + 2) = x = g(x)$

$\Rightarrow g(x) = \frac{1}{5}(\sin x + 2)$ ✓

$g'(x) = \frac{1}{5}\cos x$

$|g'(0.5)| = |0.17552| < 1$

Fixed point iteration:

$$x_{n+1} = g(x_n)$$

$$x_{n+1} = \frac{1}{5}(\sin x_n + 2)$$

1st iteration

$$x_1 = \frac{1}{5}(\sin x_0 + 2) = \frac{1}{5}(\sin 0.5 + 2) = 0.49589$$

2nd iteration

$$x_2 = \frac{1}{5}(\sin x_1 + 2) = \frac{1}{5}(\sin(0.49589) + 2) = 0.49516$$

3rd iteration

$$x_3 = \frac{1}{5}(\sin(0.49516) + 2) = 0.49503$$

4th iteration

$x_4 = \frac{1}{5}(\sin(0.49503) + 2) = \boxed{0.49501}$

5th iteration

$x_5 = \frac{1}{5}(\sin(0.49501) + 2) = \boxed{0.49501}$

Hence $x_4 = x_5$ is the root

NEWTON-RAPHON METHOD

Let $f(x)=0$ be non-linear Eqn then

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

PYQ 2021, 2011, 2015

Find Newton iterative formula to find cube root of (N) then calculate cube Root of 13.

Let $x = \sqrt[3]{N}$

$\Rightarrow x^3 = N$

$\Rightarrow x^3 - N = 0$

$\Rightarrow f(x) = x^3 - N$

$f'(x) = 3x^2$

So $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$\boxed{x_{n+1} = x_n - \frac{x_n^3 - N}{3x_n^2}}$$

put $N = 13$

$$x_{n+1} = x_n - \frac{x_n^3 - 13}{3x_n^2}$$

$f(x) = x^3 - 13$

x	0	1	2	3
f(x)	-13	-12	-5	14

So Root lies b/w 2 and 3

$x_0 = 2$

$$x_{n+1} = \frac{2x_n^3 + 13}{3x_n^2}$$

$$x_1 = \frac{2(2)^3 + 13}{3(2)^2} = 2.4167$$

$$x_2 = \frac{2(2.4167)^3 + 13}{3(2.4167)^2} = 2.3531$$

$$x_3 = \frac{2(2.3531)^3 + 13}{3(2.4167)^2} = \boxed{2.3513}$$

$$x_4 = \frac{2(2.3513)^3 + 13}{3(2.3513)^2} = \boxed{2.353}$$

PYQ 2024

Find Root of $x^3-5x+3=0$ upto (3) decimal places using Newton-Raphon Method

$f(x) = x^3-5x+3$ ←

$f'(x) = 3x^2-5$

x	0	1
$f(x)$	3	-1

$[0,1]$

So, Root lies b/w 0 and 1 because $f(0)\cdot f(1) < 0$

$x_0 = 0$

So, Newton-Raphon iterative formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^3-5x_n+3}{3x_n^2-5}$$

1st iteration

$$x_1 = x_0 - \frac{x_0^3-5x_0+3}{3x_0^2-5} = 0 - \frac{0^3-5(0)+3}{3(0)^2-5}$$

$$x_1 = 0.600$$

2nd iteration

$$x_2 = (0.600) - \frac{(0.600)^3-5(0.600)+3}{3(0.600)^2-5} = 0.668$$

3rd iteration

$$x_3 = (0.668) - \frac{(0.668)^3-5(0.668)+3}{3(0.668)^2-5} = 0.654$$

4th iteration

$$x_4 = (0.654) - \frac{(0.654)^3-5(0.654)+3}{3(0.654)^2-5} = 0.657$$

5^{th} iteration

$$x_5 = (0.657) - \frac{(0.657)^3 - 5(0.657) + 3}{3(0.657)^2 - 5} = 0.657$$

is Root of $x^3 - 5x + 3$.

SECANT AND REGULA-FALSE METHOD

PYQ 2014, 2023

Find Root of $x^3 - 2x - 5 = 0$ up to 3 decimal places with Regula-False position Method.

IMPORTANT Things --- !

Bisection + Regula False Methods ARE CALLED BRACKETING METHODS

SECANT + FIXED POINT + NEWTON METHODS ARE CALLED Open Methods

Rate of Convergence

1-	Bisection Method	order 1
2-	Regula-False Method	order 1
3-	Secant Method	order 1.62
4-	Newton-Raphson	order 2

Operators

- PYQ 2013, 2015 — **Forward difference op.**
  $$\Delta(y_k) = y_{k+1} - y_k = f(x_{k+1}) - f(x_k)$$
- PYQ 2013, 2015 — **Backward difference op.**
  $$\nabla(y_k) = y_k - y_{k-1} = f(x_k) - f(x_{k-1})$$
- PYQ 2013 — **Shift operator**