# GROUP THEORY

# GROUP THEORY

## CHAPTER ONE

## SET AND ITS ELEMENTS

set A, P(x)
statement true
for $\{x : x \in A : P(x)\}$

Denoted by a, b, x, y ..
Small letters

set having no ele. → empty set

→ Collection of well-defined and distinct objects

$X = \{a, b, c\}$
$a \in X$
$f \notin X$

set having only one ele. called singleton set

Denoted by capital letter A, B, X, Y ..

enclosed {}

3 ways of writting sets
- Descriptive way
- Tabular form
- Set builder Notation

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## CHAPTER ONE

$A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4\}$

If $A \not\subset B$

$A \subset B$ empty set and set itself is always subset of set

**SUBSETS**

part of set

If set A is a subset of B

$A \subset B$

$B \rightarrow$ super subset of A

$\boxed{2^n}$

$n \rightarrow$ no. of ele. in set

A contains always less than or equal no. of ele. in B

B contains A

$A = \{1, 2, 3\}$

$2^3 = 8$

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## CHAPTER ONE

$A \subset B$

$B \not\subset A$

$A \neq B$

$\longleftarrow$ $\boxed{\textbf{EQUAL SETS}}$ $\longrightarrow$

If set A and set B are equal then

$A \subset B$

$B \subset A$

$\Rightarrow A = B$

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## CHAPTER ONE

$A = \{1, 2, 3\}$ $B = \{1, 2, 4\}$

$A - B = \{3\}$

### DIFFERENCE OF TWO SETS

$A - B = \{x : x \in A, x \notin B\}$

Two sets A and B their difference is A-B

Set of those ele. in A but not in B

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$B = \{2, 3, 4, 5\}$ $A = \{2, 3, 4, 5, 6\}$

$B \subset A$

$A^c = A - B = \{6\}$

$\boxed{\text{COMPLEMENT OF SET}}$ → If $B \subset A$ then $A - B$ is said to be complement of A

$A^c$

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## CHAPTER ONE

$A=\{1,2,3\}$ $B=\{3,4,5\}$

$A \cap B$
$=\{3\}$

$\leftarrow A \cup B=\{1,2,3,4,5\}$

**UNION OF TWO SETS** $\rightarrow$ Two sets A and B then

$A \cup B=\{x : x \in A$ or $x \in B\}$

unity

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$A \cap B = \{3\}$

**INTERSECTION OF TWO SETS**

common of two sets

Two sets A and B then

$A \cap B$

$A \cap B = \{x : x \in A$ and $x \in B\}$

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$A = \{1, 2, 3\}$ $B = \{4, 5\}$

$A \cap B = \phi$

**DISJOINT SETS**

$A \cap B = \phi$

Two sets are said to be disjoint if they do not have nothing common

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## CHAPTER ONE

$A_\alpha = \{A_\alpha : \alpha \in I\}$

**INDEX FAMILY OF SETS**

→ Given a set $I$ for each $\alpha \in I$ there exist a set

$\bigcup_{\alpha \in I} A_\alpha = \{A_\alpha : \alpha \in I\}$

Collection of sets

$\bigcap_{\alpha \in I} A_\alpha = \{A_\alpha : \alpha \in I\}$

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## CHAPTER ONE

**UNIVERSAL SET** → Biggest set of all

U

All sets are subset of it

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## CHAPTER ONE

$(A \cap B)^c = A^c \cup B^c$

$$\boxed{\text{DE MORGAN LAW}}$$

$(A \cup B)^c = A^c \cap B^c$

LHS $(A \cup B)^c = \{x : x \notin A \cup B\}$

$= \{x : x \notin A \cap x \notin B\}$

$= \{x : x \in A^c \cap x \in B^c\}$

$= \{x : x \in A^c \cap B^c$

$\Rightarrow A^c \cap B^c$

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## CHAPTER ONE

$A = \{1, 2\}$ $B = \{a, b\}$

$A \times B = \{(1,a)(1,b)(2,a)(2,b)\}$

$R_1 = \{(1,a)(2,a)\}$

multiplying the set

**CARTESIAN PRODUCT OF SETS**

Two sets A and B

$(a, b) \in R$

$A \times B = \{(a,b) : a \in A, b \in B\}$

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## CHAPTER ONE

## RELATIONS

subset of cartesian product

$R \subseteq A \times B$

Binary relations

$(a, b) \in R$

$D_R = \{a : (a, b) \in R\}$

$R_R = \{b : (a, b) \in R\}$

$(a, b) \in R$

$a R b$

$(a, b) \notin R$

$a \not R b$

$R^c \rightarrow (A \times A) - R$

If relation from set A to B is empty then called empty relation

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## CHAPTER ONE

$\boxed{\textbf{IDENTITY RELATIONS}}$ → on any set in which the ele are mirror of each other only

$R = \{(1,1)(2,2)(3,3)\}$

1 R b   2R2   3R3

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## CHAPTER ONE

$R = \{(1,1)\ (2,1)\ (1,3)\}$

$R^{-1} = \{(1,1)\ (1,2)\ (3,1)\}$

# INVERSE OF RELATIONS

$R^{-1}$

Make :-

Domain of $R$ = Range of $R^{-1}$

Domain of $R^{-1}$ = Range of $R$

$R^{-1} = \{(b,a) : (a,b) \in R\}$

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## CHAPTER ONE

### $\boxed{\text{REFLEXIVE RELATIONS}}$

Reflexive relation contains identity relation

$R = \{(1,1)(1,2)(2,2)\}$

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## CHAPTER ONE

$A = \{1, 2, 3\}$

$R = \{(1,1)\,(2,2)\,(1,2)\,(2,1)\}$

$R^{-1} = \{(1,1)\,(2,2)\,(2,1)\,(1,2)\}$

Identity relation is always Reflexive Symmetric Transitive

**SYMMETRIC RELATIONS**

$\boxed{R = R^{-1}}$

If $(a,b) \in R$ then $(b,a) \in R$ $\forall\, a, b \in R$

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## CHAPTER ONE

$A = \{1, 2, 3\}$

$R = \{(1,1)\ (3,3)\ (3,2)\ (2,3)\ (1,2)$
$(2,1)\}$

## TRANSITIVE RELATIONS

If $(a,b) \in R$ and $(b,c) \in R$
the $(a,c) \in R$ , $\forall a,b,c \in R$

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## CHAPTER ONE

$A=\{1,2,3\}$

$R=\{(1,1)(2,2)(1,2)(2,3)\}$

$R^{-1}=\{(1,1)(2,2)(2,1)(3,2)\}$

$R \cap R^{-1}=\{(1,1)(2,2)\}$

## ANTI-SYMMETRIC RELATIONS

$R \cap R^{-1} = I_R$

$(a,b) \in R$

$(b,a) \in R$

$\Rightarrow a=b \,,\, b=a$

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## CHAPTER ONE

**EQUIVALENCE RELATIONS**

- Reflexive
- Transitive
- Symmetric

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## CHAPTER ONE

$[a]$ / $C_a$ ⟵ $\boxed{\textbf{EQUIVALENCE CLASS}}$ ⟶ exist for equivalence relation

$\{b \in A : (a,b) \in R\}$

set of those element of A which are related to fixed ele 'a'.

$[a]$

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## CHAPTER ONE

$$\bigcup_{\alpha \in I} A_\alpha = A \leftarrow \boxed{\textbf{PARTITION}} \rightarrow \text{collection of subsets } A_\alpha, \ \alpha \in I$$

$$\downarrow$$

$$A_\alpha \cap A_\beta = \phi$$

# GROUP THEORY

## CHAPTER ONE

Every function is a relation

In mapping, we get images.

but every relation $\not\Rightarrow$ function

function works like a machine

**FUNCTIONS**

unique type of Relations

Every mapping is a function

$\boxed{f(x)=y}$

$\forall x \in X \; \exists$ unique $y \in Y$, $X \rightarrow$ Domain

$Y \rightarrow$ Range / Co-Domain

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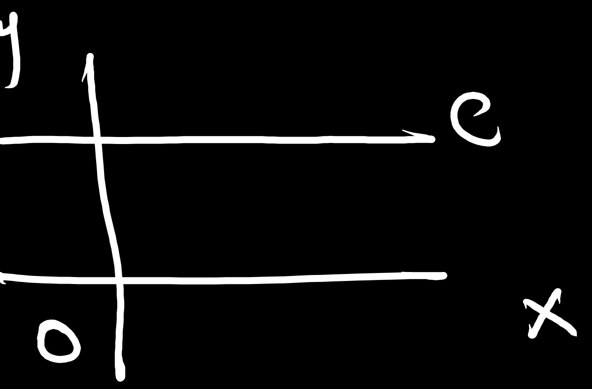
## CHAPTER ONE




### CONSTANT FUNCTIONS

Range is fixed

$f(x) = c$

Domain not fixed!

A function whose range in fixed no matter whats the input.

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## CHAPTER ONE

**SURJECTIVE MAPPING** → onto function.

CoDomain of func. = Range of function

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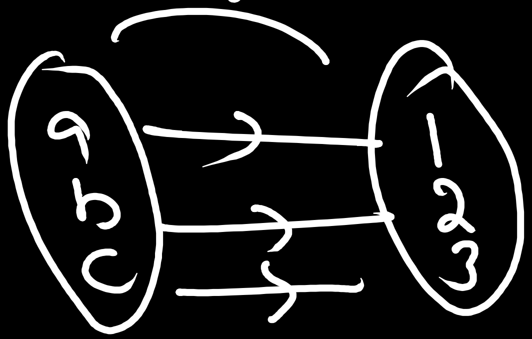
## CHAPTER ONE

$f: X \to Y$




**INJECTIVE MAPPING** → one-to-one mapping

$f: X \to Y$

every $x \in X$ have distinct image in $y \in Y$

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## CHAPTER ONE

$\boxed{\textbf{BIJECTIVE MAPPING}}$ → mapping that is both

→ one to one
and
→ onto

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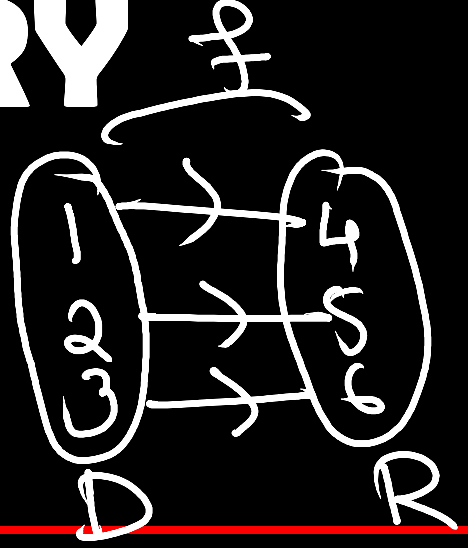
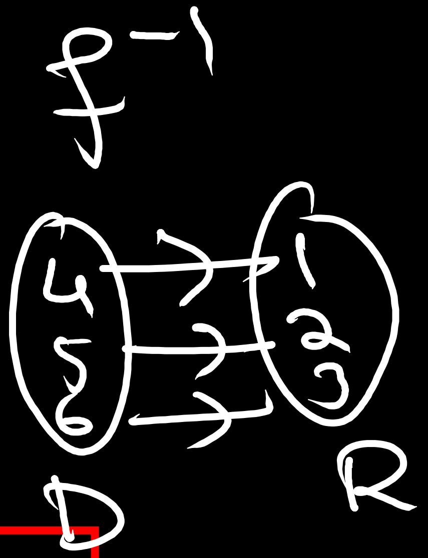
## CHAPTER ONE




**INVERSE MAPPING**

→ Domain → Rage

Range → Domain

$f : X \to Y$

$f^{-1} : Y \to X$

# GROUP THEORY

## CHAPTER ONE

$f(x) = x^2$

$g(x) = 2x$

$\boxed{\text{COMPOSITION OF FUNCTIONS}}$

$g \circ f = g(f(x))$

$f : X \to Y \quad g : Y \to Z$

$g \circ f : X \to Z$

$g \circ f = g(f(x))$

$= 2(x)^2$

$g \circ f = 2x^2$

# GROUP THEORY

## CHAPTER ONE

$* : X \times X \to X$

Inverse ele. exist.

closure property

Associative property

$a*(b*c) = (a*b)*c$

$(a,b) \in X \times X$

**BINARY OPERATION**

mapping / rule

function

$a*b$

Identity ele. exist

Domain of this func. $= X \times X$

Range of this func. $= X$

Commutative property

$a*b = b*a$