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$\boxed{Ext(A) = Int(A^c)}$

interior of complement of A.

$\boxed{Ext(A^c) = Int\,A}$

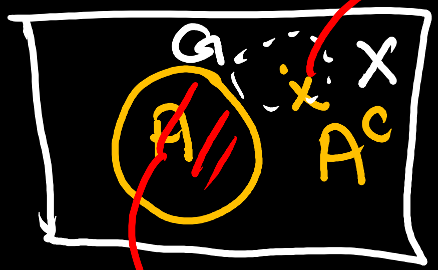
EXTERIOR POINT

(X, τ)

$A \subseteq X$

then any pt. $x \in X$ then $\exists$ open set G s.t.

$x \in G \subseteq A^c$



Ext(A)

Int(A)

$x \in G \subseteq A^c$

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BOUNDARY POINT

(X, τ), $A \subseteq X$

any $x \in X$ is bd(A) if $\exists$ G containing x

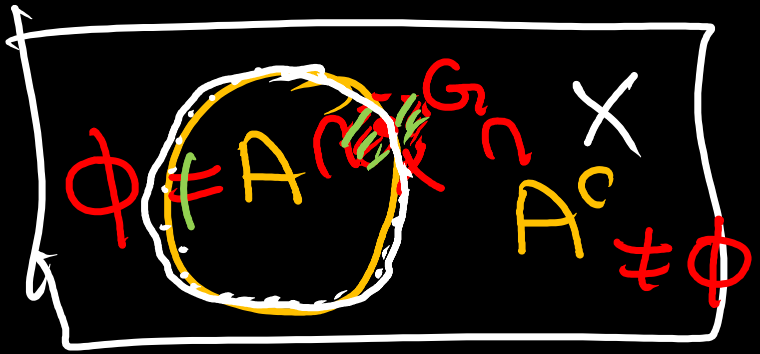
$x \in G \cap A \neq \phi$ and $x \in G \cap A^c \neq \phi$

Bd(A) does not belong to A and A^c

Frontier point

Bd(A)/Fr(A)/ S(A)

Set of points which donot belong to A and A^c



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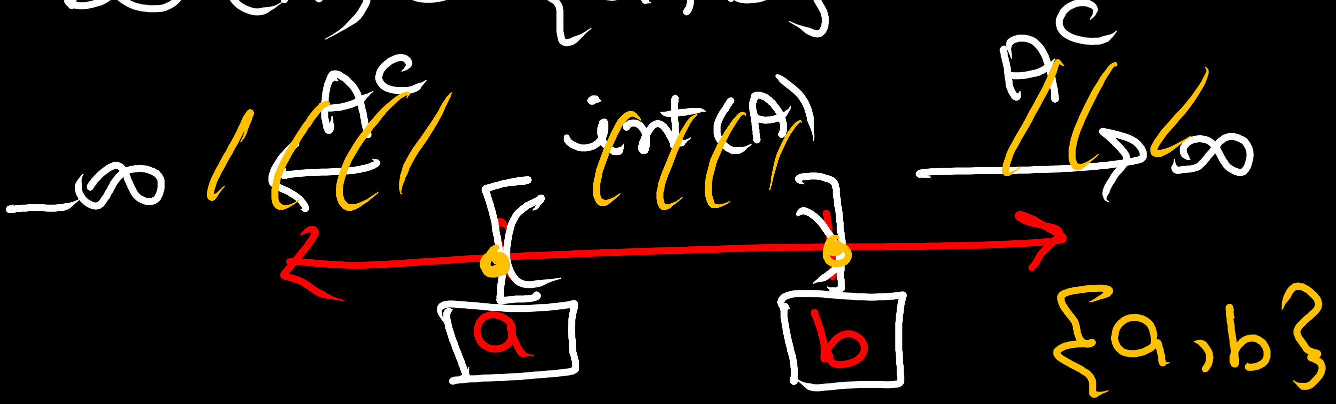
Example 5.1: Consider the four intervals $[a, b]$, (a, b), $(a, b]$ and $[a, b)$ whose endpoints are a and b. The interior of each is the open interval (a, b) and the boundary of each is the set of endpoints, i.e. $\{a, b\}$.

$$\left.\begin{array}{l} A = [a, b] \\ A = [a, b) \\ A = (a, b] \\ A = (a, b) \end{array}\right\} \quad \begin{array}{l} \operatorname{int}(A) = (a, b) \\ \operatorname{Bd}(A) = \{a, b\} \end{array}$$



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$\text{int}(A) \to x \in A$ $\quad$ $a \in \{a\} \subseteq A^c$

$\text{ext}(A) + \text{Bd}(A) \to x \in X$

int(A)

$a \in G \subseteq A$

Example 5.2: Consider the topology

$$\mathcal{T} = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$$

on $X = \{a, b, c, d, e\}$, and the subset $A = \{b, c, d\}$ of X. The points c and d are each interior points of A since

$$c, d \in \{c, d\} \subset A$$

where $\{c, d\}$ is an open set. The point $b \in A$ is not an interior point of A; so $\text{int}(A) = \{c, d\}$. Only the point $a \in X$ is exterior to A, i.e. interior to the complement $A^c = \{a, e\}$ of A; hence $\text{int}(A^c) = \{a\}$. Accordingly the boundary of A consists of the points b and e, i.e. $b(A) = \{b, e\}$.

$X = \{a, b, c, d, e\}$ $\quad$ $A = \{b, c, d\}$ $\quad$ $\boxed{A^c = \{a, e\}}$

$b \in \{b, c, d, e\} \not\subseteq \{b, c, d\}$

$b \in X \not\subseteq \{b, c, d\}$ $\quad$ b is not interior A

$c \in \{c, d\} \subseteq \{b, c, d\}$ $\quad$ c is int(A)

$c \in \{a, c, d\} \not\subseteq \{b, c, d\}$

$c \in \{b, c, d, e\} \not\subseteq \{b, c, d\}$ $\quad$ $c \in X \not\subseteq \{b, c, d\}$

d is also int (A)

int (A) = {c, d}

ext (A) = {a}

Bd(A) = pts does not belong Int A
and ext (A)

Bd (A = {b, e}

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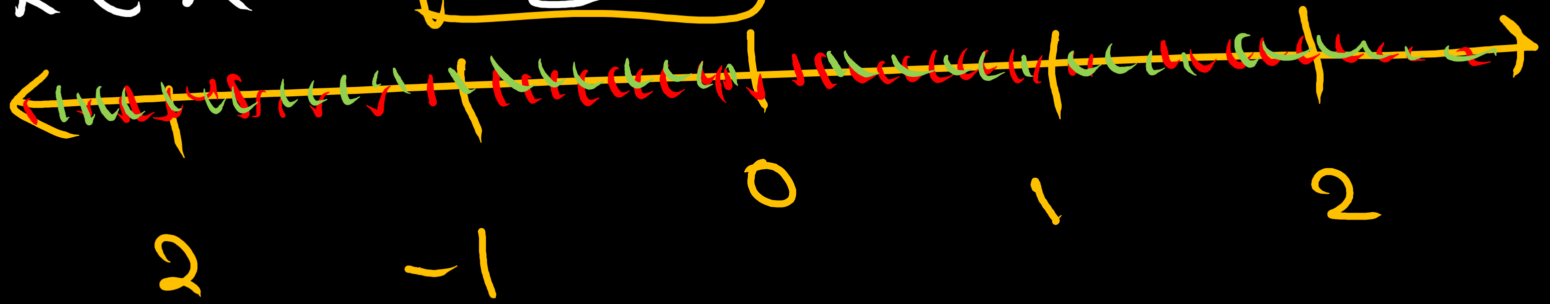
Solved problem finished!

Example 5.3: Consider the set $\mathbf{Q}$ of rational numbers. Since every open subset of $\mathbf{R}$ contains both rational and irrational points, there are no interior or exterior points of $\mathbf{Q}$; so $\operatorname{int}(\mathbf{Q}) = \emptyset$ and $\operatorname{int}(\mathbf{Q}^c) = \emptyset$. Hence the boundary of $\mathbf{Q}$ is the entire set of real numbers, i.e. $\mathrm{b}(\mathbf{Q}) = \mathbf{R}$.

$\operatorname{int}(A) \to x \in A \to \boxed{x \subset A}$

$\operatorname{ext}(A) \to x \in X \to \boxed{X \subseteq A^c}$



$$\operatorname{int}(\mathbb{Q}) = \operatorname{int}\mathbb{Q}^c = \operatorname{ext}(\mathbb{Q}) = \operatorname{ext}(\mathbb{Q}^c) = \emptyset$$

$$\operatorname{Bd}(\mathbb{Q}) = \operatorname{Bd}(\mathbb{Q}^c) = \boxed{\mathbb{R}}$$

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always reemember that $\to \phi$, $\to$ whole space X $\downarrow$ both open and closed $\downarrow$ Clopen Set.

32. Let A be a non-empty proper subset of an indiscrete space X. Find the interior, exterior and boundary of A.

$(X, \tau) = \tau = \{\phi, X\}$

$A \neq X$

proper subset of $A = \phi$

$\text{int}(A) = \phi$

$\text{ext}(A) = \phi$

$\text{bd}(A) = X$

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32. Let A be a non-empty proper subset of an indiscrete space X. Find the interior, exterior and boundary of A.

Solution:

X and $\emptyset$ are the only open subsets of X. Since $X \neq A$, $\emptyset$ is the only open subset of A; hence $\text{int}(A) = \emptyset$. Similarly, $\text{int}(A^c) = \emptyset$, i.e. the exterior of A is empty. Thus $b(A) = X$.

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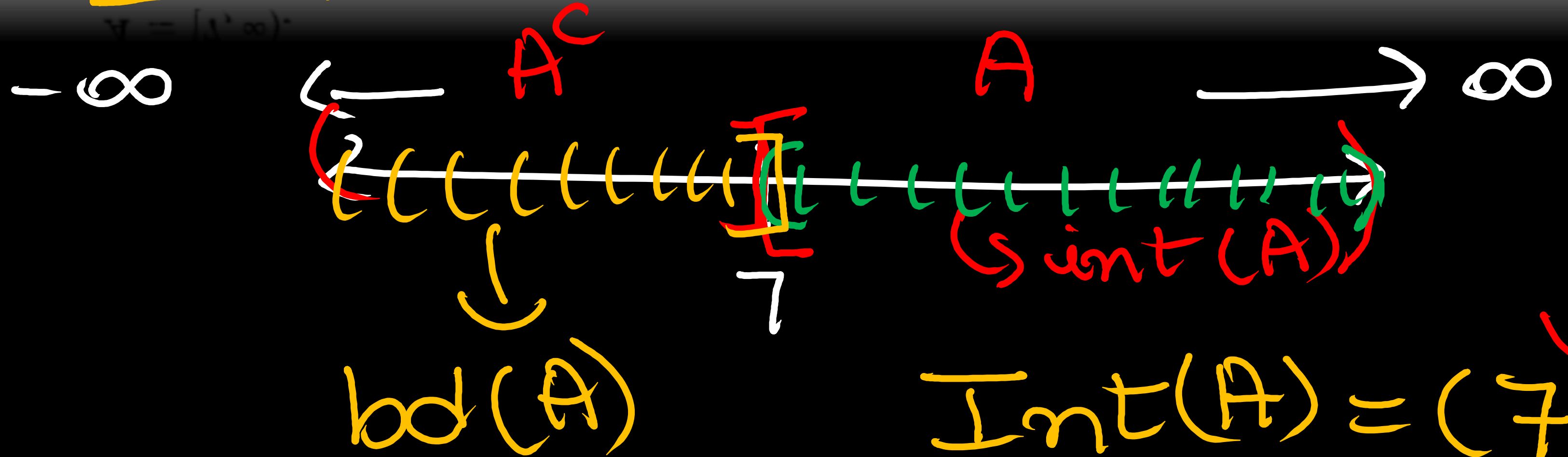
$x \in X \in G \subseteq A^c$

33. Let $\mathcal{T}$ be the topology on $\mathbf{R}$ consisting of $\mathbf{R}$, $\emptyset$ and all open infinite intervals $E_a = (a, \infty)$ where $a \in \mathbf{R}$. Find the interior, exterior and boundary of the closed infinite interval $\boxed{A = [7, \infty)}$.



open sets.

$\mathrm{Int}(A) = (7, \infty)$

$\mathrm{Ext}(A) = \emptyset$

$\mathrm{bd}(A) = (-\infty, 7]$

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33. Let $\mathcal{T}$ be the topology on $\mathbf{R}$ consisting of $\mathbf{R}$, $\emptyset$ and all open infinite intervals $E_a = (a, \infty)$ where $a \in \mathbf{R}$. Find the interior, exterior and boundary of the closed infinite interval $A = [7, \infty)$.

Solution:

Since the interior of A is the largest open subset of A, $\text{int}(A) = (7, \infty)$. Note that $A^c = (-\infty, 7)$ contains no open set except $\emptyset$; so $\text{int}(A^c) = \text{ext}(A) = \emptyset$. The boundary consists of those points which do not belong to $\text{int}(A)$ or $\text{ext}(A)$; hence $\text{b}(A) = (-\infty, 7]$.

Supplementary problems finished!

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INTERIOR, EXTERIOR, BOUNDARY

75. Let X be a discrete space and let $A \subset X$. Find (i) $\text{int}(A)$, (ii) $\text{ext}(A)$, and (iii) $b(A)$.

$X = P(X)$

$A \subset X$

$\text{int}(A) = A$

$\text{ext}(A) = A^c$

$\text{bd}(A) = \phi$

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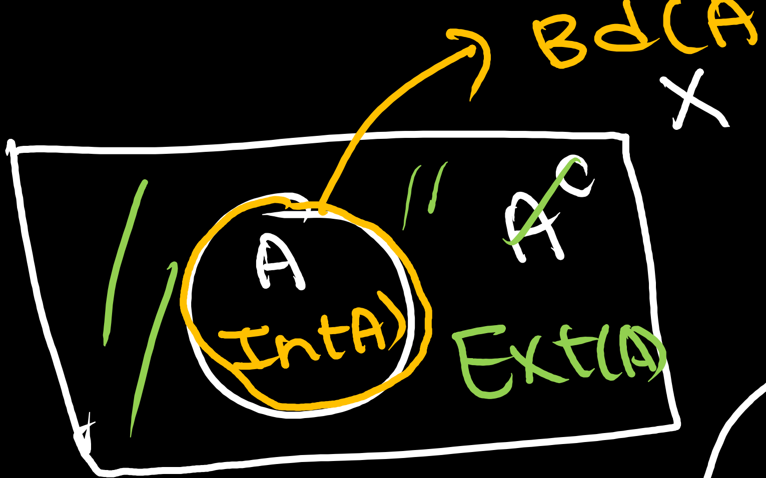

$A \subseteq X$, A is closed iff

$Bd(A) \subseteq A$

$Bd(A)$ does not contained in A and A^c

$Bd(A) \cap A \neq \phi$

$Bd(A) \cap A^c \neq \phi$

IMPORTANT RESULTS

$A^\circ \cup Ext(A) \cup Bd(A) = X$

$A^\circ \cap Ext(A) \cap Bd(A) = \phi$

$A^\circ \cap Ext(A^c) = \phi$

$A^\circ \cap Bd(A) = \phi$

$Ext(A^c) \cap Bd(A) = \phi$

$A \subseteq X$, A is open then $A \cap Bd(A) = \phi$

$Bd(A) = \phi$ iff A is both open and closed