TOPOLOGY

CHAPTER FIVE

INTERIOR POINT

$[a,b]^\circ = (a,b)$

$(a,b)^\circ = (a,b)$

$\mathbb{R}^\circ = \mathbb{R}$

$\mathbb{Q}^\circ = \phi$

$(\mathbb{Q}^c)^\circ = \phi$

$\mathbb{N}^\circ = \phi$

$\mathbb{Z}^\circ = \phi$

$A \subseteq X$

$p \in G \subseteq A$

neighbour of p.

interior point of A.

If $A \subseteq X$ then $p \in A$ is Int(A) if $\exists$ open set G containing p is a subset of A.

The reverse of interior point is neighbourhood.

Denoted by A° OR Int(A)

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. Consider the following topology on $X = \{a, b, c, d, e\}$:

$$\mathcal{T} = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, e\}\}$$

(i) Find the interior points of the subset $A = \{a, b, c\}$ of X. (ii) F

$a \in \{a\} \subseteq \{a,b,c\}$ ✓

$a \in \{a,b\} \subseteq \{a,b,c\}$ ✓

$a \in \{a,c,d\} \not\subseteq \{a,b,c\}$

$a \in \{a,b,c,d\} \not\subseteq \{a,b,c\}$

$a \in \{a,b,e\} \not\subseteq \{a,b,c\}$

So, a is an interior point of A.

$b \in \{a,b\} \subseteq \{a,b,c\}$

$b \in \{a,b,c,d\} \not\subseteq \{a,b,c\}$

$b \in \{a,b,e\} \not\subseteq \{a,b,c\}$

So, b is also interior point of A.

$c \in \{a,b,c,d\} \not\subseteq \{a,b,c\}$

$c \in \{a,c,d\} \not\subseteq \{a,b,c\}$

So, c is not interior point of A.

$$\boxed{A^{\circ} = \{a,b\}}$$

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Let $\{G_i\}$ be collection of all open set of A.

If $x \in A^\circ$ then $\exists$ $i_\circ \in G_{i_\circ} \subseteq \cup G_i$

THE INTEROR OF SET A IS THE UNION OF ALL OPEN SUBSETS OF A.

$\Rightarrow x \in G_{i_\circ}$

$\Rightarrow x \in \cup G_i$

$\Rightarrow A^\circ \in \cup G_i$

$\Rightarrow A^\circ \subseteq \cup G_i$ —(i)

on the other hand, if $y \in \cup G_i$ then

$\Rightarrow y \in G_{i_\circ}$

$y \in A^\circ$

$\Rightarrow \cup G_i \in A^\circ$

$\Rightarrow \cup G_i \subseteq A^\circ$ —(2)

$$\boxed{A^\circ = \cup G_i}$$

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THE INTEROR OF SET A IS OPEN.

$A^\circ \Rightarrow$ is open

Suppose that A is open.

then $x \in A$ $\exists$ A

such that

$x \in A \subseteq A$

$\Rightarrow A \subseteq A$

$\Rightarrow A \subseteq A^\circ$ —(i)

But

$A^\circ \subseteq A$ —(ii)

$\boxed{A = A^\circ}$

$\Rightarrow A^\circ$ is also open.

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Let $\{U_\alpha, \alpha \in I\}$ be collection of all open subsets of A.

Let $x \in \bigcup_{\alpha \in I} U_\alpha, \alpha \in I$

THE INTEROR OF SET A IS THE LARGEST OPEN SUBSET OF A i-e G IS AN OPEN SUBSET OF A.

$\Rightarrow x \in U_\alpha, \alpha \in I$

$\Rightarrow x \in U_\alpha, \alpha \in I \subseteq A$

$\Rightarrow x \in A^\circ$

$\Rightarrow \bigcup_{\alpha \in I} U_\alpha, \alpha \in I \subseteq A^\circ$ — (i)

Conversely, Let $x \in A^\circ$

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Suppose that A is open and let $x \in A$

$\Rightarrow x \in A \subseteq A$

THE INTEROR OF SET A IS OPEN IFF $A = A^0$

$\Rightarrow x \in A^\circ$

$\Rightarrow A \subseteq A^\circ \Rightarrow$ (i)

But $A^\circ \subseteq A \Rightarrow$ (ii)

from (i) and (ii)

$\boxed{A = A^\circ}$

Conseqvely, if $A = A^\circ$ since A° is open $\Rightarrow A$ is also open.