TOPOLOGY

CHAPTER FIVE

NEIGHBORHOODS

If p having more than one neighbourhood the p has neighbourhood system

$p \in G \subseteq N \subseteq X$

"N is a neighbourhood of p"

If $p \in X$. A subset N of X is called nhbd iff N is a superset of open set G containing p.

N_p

The reverse relation is "p is a interior point of N"

TOPOLOGY

CHAPTER FIVE

Consider the following topology on $X = \{a, b, c, d, e\}$:

$$\boxed{\mathcal{T}} = \{\cancel{X}, \cancel{\emptyset}, \{\cancel{a}\}, \{\cancel{a, b}\}, \{\cancel{a, c, d}\}, \{\cancel{a, b, c, d}\}, \{a, b, e\}\}$$

List the neighborhoods (i) of the point e, (ii) of the point c.

$N_e = \{X, \{a,b,e\}, \{a,b,c,e\}$
$\{a,b,d,e\}\}$

$\boxed{e} \in X, \{a,b,e\}$

→ supersubset of X is X

→ supersubset of $\{a,b,e\}$ is $\{a,b,e\}$

" " " is $\{a,b,c,e\}$

is $\{a,b,d,e\}$

TOPOLOGY

CHAPTER FIVE

NEIGHBORHOODS IN INTERVALS

$a \in \mathbb{R}$

$\delta > 0$

Every open interval is a open set.

-ve

+ve

$(a-\delta, a+\delta)$

→ open interval.

So, each point of open interval have neighbourhood.

(a,b) $a \in (a,b) \subseteq (a,b)$ $x \in X$

→ closed interval $[a,b]$

→ we can only find nhdb of points b/w endpoints of interval

$b \in (b-\delta, b+\delta) \not\subseteq [a,b]$

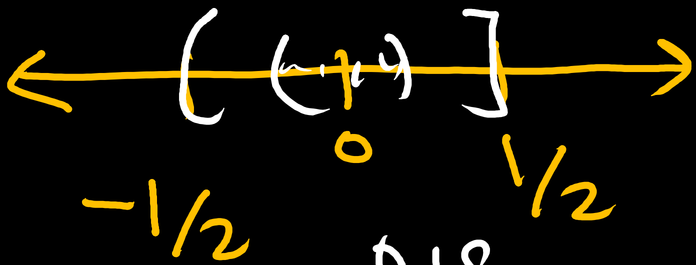
We cannot find nhbd at end points of closed intervals.

TOPOLOGY

CHAPTER FIVE

. Determine whether or not each of the following intervals is a neighborhood of 0 under the usual topology for the real line **R**. (i) $(-\frac{1}{2}, \frac{1}{2}]$, (ii) $(-1, 0]$, (iii) $[0, \frac{1}{2})$, (iv) $(0, 1]$.

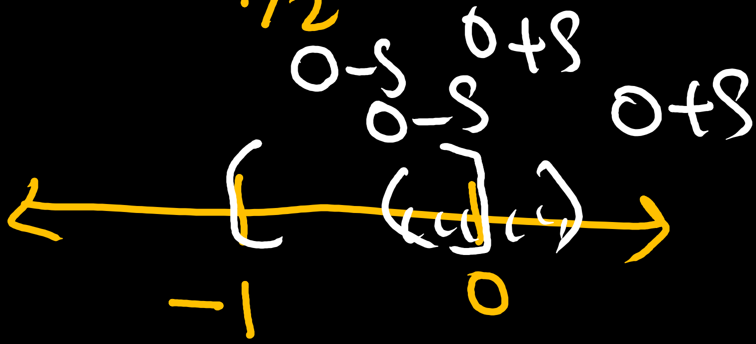
(i)



, $0 \in (0-\delta, 0+\delta) \subseteq (-1/2, 1/2]$

So, $(-1/2, 1/2]$ is a nbhd of 0.

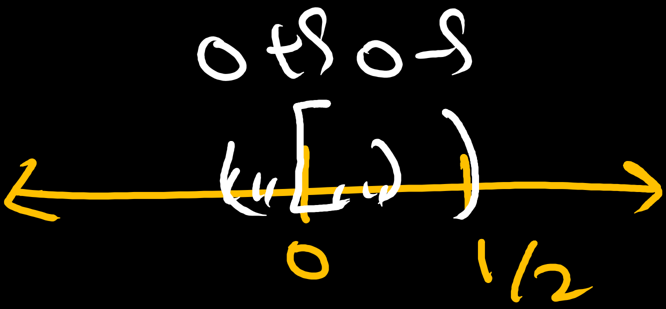
(ii)



$0 \in (0-\delta, 0+\delta) \not\subseteq (-1, 0]$

So, $(-1, 0]$ is not nbhd of 0.

(iii)



$0 \in (0-\delta, 0+\delta) \subseteq [0, 1/2)$

TOPOLOGY

CHAPTER FIVE

RELATION BETWEEN INTERIOR POINT AND NEIGHBORHOOD

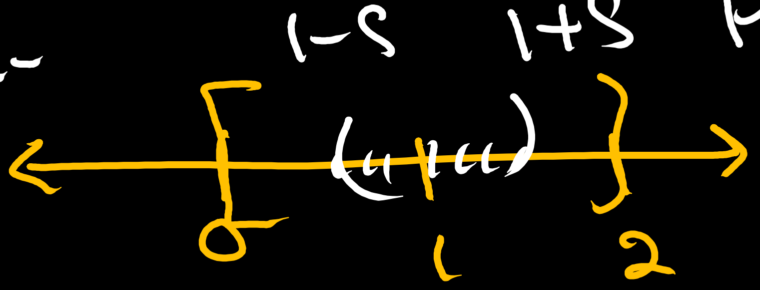
→ if N is a nhbd of p then "p is a interior of N"

I cannot find interior pt. of closed interval.

→ we can find interior pt at end point of open interval.

→ $[0,2]$

Ex:-



1 is the interior point of $[0,2]$

$1 \in (1-\delta, 1+\delta) \subseteq [0,2)$

$[0,2)$ is a nhbd of 1

TOPOLOGY

CHAPTER FIVE

EXAMPLES

$\mathbb{N} = \{1, 2, 3 \ldots\}$

So, I cannot find the nhbd of any point in $\mathbb{N}$

$\rightarrow$ set of th $\neq \phi$ set of interior.

$\mathbb{R}^\circ = \mathbb{R}$

$\mathbb{Q}^\circ = \phi$

$(\mathbb{Q}^c)^\circ = \phi$

TOPOLOGY

CHAPTER FIVE

NEIGHBORHOOD AXIOMS

$\longrightarrow N_p = \{N_1, N_2 \cdots\}$

N_p is a neighbourhood system:-

$\boxed{1}$ N_p is not empty and p belongs to each member N_p.

$\boxed{2}$ The intersection of any two members of N_p belongs to N_p.

$\boxed{3}$ Every superset of a member N_p is also belongs to N_p.

$\boxed{4}$ Each member $N \in N_p$ is a superset of a member of G where G is neighbourhood of each of it's p